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Nonlinearities in the exchange rates returns and volatility

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Abstract

Recent findings of nonlinearities in financial assets can be the product of contamination produced by shifts in the distribution of the data. Using the BDS and Kaplan tests it is shown that, some of the nonlinearities found in foreign exchange rate returns, can be the product of shifts in variance while other do not. Also, the behavior of the volatility is studied, showing that the ARFIMA modeling is able to capture long memory, but, depending on the *proxy* used for the volatility, is not always able to capture all the nonlinearities of the data

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1. Introduction

The purpose of this paper is to investigate whether the foreign exchange rates behave nonlinear. At the same time some methodology issues in detecting nonlinear behavior will be discussed. Simulating studies [1–6] have shown that the BDS and the Kaplan tests have power against a large class of alternatives, so they will be used in this paper. Also, these tests have been widely applied to investigate the behavior of financial time series as in Refs. [1,7], most of them yielding to the acceptance of nonlinearity in

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financial time series. There are two main explanations for the nonlinearity of financial returns. Concretely, one explanation for the nonlinear dependence in exchange rates is that they come from a deterministic process that looks random (e.g. chaotic process). A second explanation is that exchange rates changes are nonlinear stochastic functions of their own past. In this sense, Hinich and Patterson [8] show that stock prices are realizations of nonlinear stationary stochastic processes, also Hsieh [7] finds that rejections of linearity in stock returns are due to neglected conditional heteroskedasticity and cannot be attributed to structural changes.

Following the second explanation, some of the models used for asset prices and volatility assume that the unconditional distribution of assets rates is constant over time, which means that returns are stationary. This is the case of the autoregressive conditional heteroskedasticity models or ARCH processes.

In this investigation we will use a modification of the test proposed by Lima [9] that attempts to discriminate the findings of nonlinearity caused for intrinsic mechanisms, from those due to nonstationarities in the data. We will show that some of the findings of nonlinearity are due to possible shifts in distribution, that is nonstationarities of exchange rates while others are not.

Also, we use this methodology to study the behavior of the volatility using the ARFIMA models that are able to capture the long memory of this variable.

2. Testing nonlinearity

Among the tests of nonlinearities, the BDS and the Kaplan tests have been proven as very powerful and it will be used to test nonlinearities in exchange rate series in our paper.

2.1. The BDS test

The Brock, Dechert and Scheinkman (BDS) [10] is a test for independence based on the estimation of the correlation integral at various dimensions. It has power against virtually all types of linear and nonlinear departures so it does not currently provides a direct test either for nonlinearity or for chaos.

The BDS follows asymptotically a normal distribution with zero mean and unit variance under the null hypothesis of independence. Hence the hypothesis of nonlinearity and chaos are nested within the alternative hypothesis, which includes both non-independent linear and non-independent nonlinear processes.

Only when all the linear possibilities have been removed from the data by prefiltering, the test can be interpreted as a test of nonlinearities. The filtering can be done fitting the data with the proper ARMA [11] model, because the residuals of the model should be in principle linear independent, and any dependence found in the residuals must be nonlinear.

The BDS uses the correlation function that has two arguments, the embedding dimension m and the size of dimensional distance ε. The proper choice of the two parameters is studied in detail in Kanzler [12]. We will use his recommendations and his tables of the BDS distribution to test the results.

The BDS statistic for an embedding dimension m and dimensional distance ε can be estimated consistently on a sample of T observations by

$$w_{m,T}(\varepsilon) = \sqrt{T-m+1}\,\frac{C_{m,T}(\varepsilon)C_{1,T-m+1}(\varepsilon)^m}{\sigma_{m,T}(\varepsilon)}\,, \tag{1}$$

where the correlation integral is calculated as the average of all products of m histories:

$$C_{m,T}(\varepsilon) = \frac{2}{(T-m+1)(T-m)}\sum_{s=m}^{T}\sum_{t=s+1}^{T}\prod_{j=0}^{m-1} I_\varepsilon(x_{s-j},x_{t-j})\,, \tag{2}$$

where I_ε is the Heaviside function and the estimated variance $\sigma^2_{m,T}(\varepsilon)$ is calculated by

$$\sigma^2_{m,T}(\varepsilon) = 4\left[k^m + 2\sum_{j=1}^{m-1} k^{m-j}[C_{1,T}(\varepsilon)]^{2j} + (m-1)^2[C_{1,T}(\varepsilon)]^{2m} - m^2k[C_{1,T}(\varepsilon)]^{2m-2}\right]\,, \tag{3}$$

where

$$k_T(\varepsilon) = \frac{2}{T(T-1)(T-2)}\sum_{t=1}^{T}\sum_{s=t+1}^{T}\sum_{r=s+1}^{T} [I_\varepsilon(x_t,x_s)I_\varepsilon(x_s,x_r) + I_\varepsilon(x_t,x_r)I_\varepsilon(x_r,x_s) + I_\varepsilon(x_s,x_t)I_\varepsilon(x_t,x_r)] \tag{4}$$

The BDS statistic follows asymptotically the standard normal distribution:

$$\lim_{n\to\infty} w_{m,T}(\varepsilon) \sim \mathrm{N}(0,1)\,. \tag{5}$$

As mentioned before the null hypothesis of the BDS test is that the data are i.i.d., thus the rejection of the null with ARMA model prefiltered data means that the data are not independent and identically distributed, so (as pointed out in [9]) any change in the data distribution will generate the rejection of linearity. In order to test how sensitive is this test to nonstationarities due to possible shifts in the unconditional variance over time, BDS will be calculated for a data set of 3000 observations with two different variances, created with three random samples of 1000 observations drawn from N(0, 1), N(0, 2), and N(0, 1) distributions respectively. The BDS is calculated beginning with a data set of 100 observations and adding 10 data to the set of observations under analysis. Fig. 1 shows the evolution of the BDS and the confidence intervals at the 5% significance level. It can be observed that before the data set has reach 1000 observations, the BDS does not allow the rejection of the null, but as the variance change, the BDS jumps out the confidence intervals and the null is rejected for the rest of the data sets. That means that shifts in the variance produce a jump in the result of the BDS test,

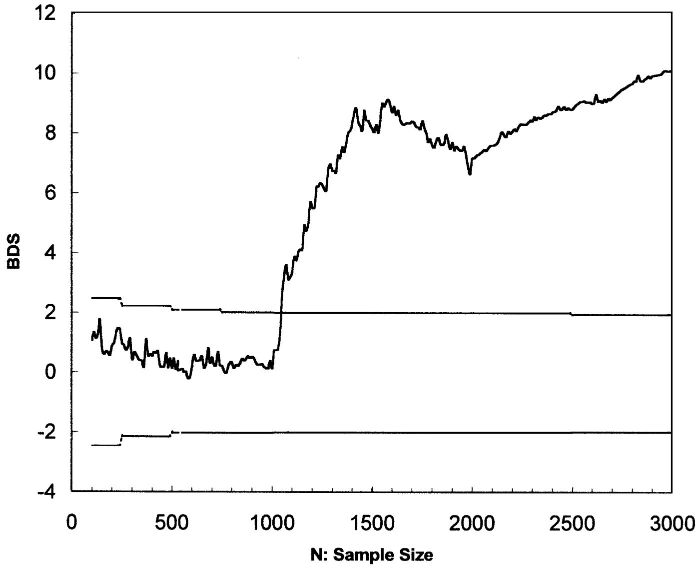

Fig. 1. BDS statistics for two-variance data set. Dashed lines represent the 95% confidence intervals.

Table 1
BDS test for a stochastic time series with 3000 observations: 1–1000 IID $N(0,1)$; 1001–2000 IID $N(0,2)$; 2001–3000 IID $N(0,1)$

N	1–500	1–1000	1–1500	1–2000	1–2500	1–3000
$m=4$	−0.657	−1.940	13.216*	13.216*	15.062*	16.783*
N	1–500	500–1000	1000–1500	1500–2000	2000–2500	2500–3000
$m=4$	−0.657	−1.171	0.653	0.014	0.627	−1.720

*Statistical significance at 5%.

yielding to the rejection of the null hypothesis. But if sub-samples of the same variance are taken, the null hypothesis cannot be rejected as shown in Table 1.

2.2. The Kaplan test

The Kaplan test [13,14] was initially formulated for the detection of determinism in the underlying dynamics of a time series, though thereafter it has been used to test the hypothesis of stochastic or deterministic nonlinearities in the generating process of a time series (see Refs. [4–6]).

This test is based on the continuity properties of the trajectories described by a deterministic dynamical system in the phase space: when there is a deterministic law governing the evolution of the state variable, then if two points are very close together in the phase space, their images (resulting from the iteration of the system) will be close also; on the contrary, if the underlying system is stochastic, the images of two nearby points could be very separate in the phase space, since in a stochastic process a single point may be followed by very different images.

The Kaplan test is related with the average distance between two points and their respective images. More formally stated, the procedure is as follows. Using the Takens theorem and choosing properly the time delay τ and the embedding dimension m, it is possible to define the vector $x_t = (x_t, x_{t-\tau}, x_{t-2\tau}, \ldots, x_{t-(m-1)\tau})$, to reconstruct the behavior of the (unknown) deterministic dynamical system generating the time series. In such a case, there is a continuous recursive function $\mathbf{x}_{t+p} = \mathbf{f}(\mathbf{x}_t)$, where $\mathbf{x}_{t+p}$ is the image of the m-dimensional point $\mathbf{x}_t$. For a given choice of t, p and m the distance between two points, $\delta_{ij} = |\mathbf{x}_i - \mathbf{x}_j|$, and the distance between their images $\varepsilon_{i,j} = |\mathbf{x}_{i+p} - \mathbf{x}_{j+p}|$ for all pairs of time subscripts (i, j) are calculated. The average of the values of $\varepsilon_{i,j}$ over those (i, j) satisfying $\delta_{ij} < r$ is defined to be $E(r)$. The Kaplan's test statistic K is the limit of $E(r)$ as $t \to 0$. For a perfect deterministic system with continuous $\mathbf{f}$, one expects to have $K \to 0$ as $r \to 0$ (that is, a point $\mathbf{x}_i$ may have only an image). On the other hand, if the underlying system is a stochastic process K will have a strictly positive lower bound (that is, a point $\mathbf{x}_i$ may have different images).

Estimations of K can be obtained through the linear regression:

$$\varepsilon_{ij} = K + \beta\delta_{ij} + u_{ij} , \tag{6}$$

where u is an error term since the estimated constant K can be interpreted as the average value of ε_{ij}, $E(r)$, when $r = 0$. There are, however, some limitations when one uses this test to detect determinism. These limitations come from the fact that when a system is chaotic, although deterministic, there will be a positive inferior level for the estimated K that will be related with the sensitive dependence on the initial conditions (other factors, as measurement errors, may also make possible a positive K value to appear). Thus, to test the hypothesis of determinism, one has to estimate the K statistic for the original series and to compare it with the one obtained under the alternative hypothesis of a purely stochastic process. Unfortunately, and due to the multitude of possible stochastic processes that are included in the alternative hypothesis, the Kaplan test cannot be used to verify the hypothesis of determinism. Nevertheless, it is possible to use the Kaplan statistic to test the less restrictive hypothesis of linearity against the alternative of nonlinearity (deterministic or stochastic). In order to do that, the statistics from an adequately large number of linear processes that plausibly might have produced the data have to be calculated, and compared with the estimated K statistic obtained from the original time series. This procedure involves the production of general linear stochastic process surrogates for the original time series. Kaplan uses surrogate series with the same histogram, amplitude and autocorrelation function as the original time series. With these surrogate series one can estimate the expected values for the K statistic under the hypothesis of linearity (KS). This hypothesis of linearity is rejected if the value of the statistic from the surrogates is never small enough relative to the value of the statistic computed from the original data ($K < KS$).

Table 2
Kaplan test for a stochastic time series with 3000 observations: 1–1000 IID $N(0,1)$; 1001–2000 IID $N(0,2)$; 2001–3000 IID $N(0,1)$

N	1–500	1–1000	1–1500	1–2000	1–2500	1–3000
KS	1.10	1.15	1.53	1.77	1.66	1.58
*KS*min	1.11	1.15	1.53	1.77	1.66	1.58
Ktest	1.13	1.16	1.52*	1.76*	1.65*	1.56*
N	1–500	500–1000	1000–1500	1500–2000	2000–2500	2500–3000
KS	1.10	1.14	2.16	2.35	1.15	1.11
*KS*min	1.11	1.14	2.16	2.36	1.16	1.11
Ktest	1.13	1.17	2.24	2.46	1.18	1.17

Sample period N; embedding dimension $m = 1$; time delay for the reconstruction $\tau = 1$; time delay for the calculation of the image $p = 1$; *Ksmean*: K statistic mean for the 20 surrogate series; *Ksstd*: K statistic standard deviation for the 20 surrogate series; *Ks*min: minimum K statistic from the 20 surrogate series; *KS*: $Ksmean - 2Ksstd$; *Ktest*: K statistic for the original time series. The linear null hypothesis is rejected when $Ktest < KS$ and/or $Ktest < Ks\min(*)$.

The distribution of this statistic is not tabulated. However, Kaplan proposes two methods to compute the minimum value of *KS* consistent with the hypothesis of linearity. The first is the minimum value of K estimated from a finite sample of surrogate time series, and impute that to the population of surrogates (*Ks*min). The second involves the calculation of the mean and standard error of the value of K from the finite sample of surrogates and then subtract a multiple (conventionally 2) of the standard error from the mean to get an estimate of the population minimum. In our applications we use 20 surrogate time series using the previous procedure proposed by Kaplan (linear stochastic time series with the same histogram, amplitude and autocorrelation function as the original time series). The values of the K statistic are calculated by the constant estimate in regression (6) using all the points satisfying $\delta_{ij} < r = 1.5\sigma$, where σ is the standard deviation of the original time series.

As the BDS test, the Kaplan one may also be used to detect nonstationarities due to shifts in the unconditional variance over time. Table 2 shows the results of this test when it is applied to the same time series used in the previous section drawn from the three random samples N(0,1), N(0,2) and N(0,1). It can be observed that linearity is rejected when the data set has reached 1000 observations (with the first change in the distribution of the stochastic generation process). The results in Table 2 also show that the Kaplan test accept the hypothesis of stationarity when applied on each sub-sample, although to the absolute value of the K statistic increase with the volatility of the time series.

We can summarize graphically the results of the Kaplan test by using the statistic $\min\{Ks; Ks\min\} - Ktest$. Thus, the null hypothesis of linearity will be rejected when $\min\{Ks; Ks\min\} - Ktest > 0$. The results for the previous two-variance series are represented in Fig. 2.

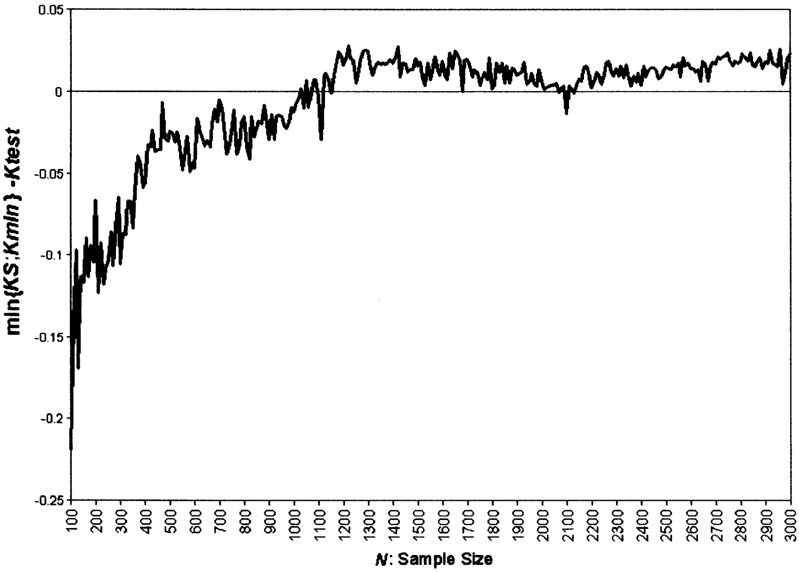

Fig. 2. Kaplan test for two-variance time series. The K-statistic is calculated for the sample period from 1 to N with steps of 10 observations ($m = 1, \tau = 1, p = 1$).

3. Application to foreign exchange rate returns

The data set under study consists on daily prices of three foreign currencies in term of US dollars. The three currencies are: German Mark (DM), Japanese Yen (JY) and British Pound (BP) that are particularly attractive as they represent the most actively traded and quoted foreign currencies. All the data set have a total of 2895 observations from 2 January of 1990 to 31 May 2001. The returns are calculated by taking the logarithm differences between successive trading days.

In order to use the BDS and the Kaplan as tests for nonlinearities in returns, all the linear possibilities should have been removed from the data by prefiltering. So prior to the application of the test all the series were detrended and passed by a standard linear filter that consists of removing the day-of-the-week and month-of-the-year effects, also an AR filter were used to remove linear correlations.

Table 3 shows the results of the application of the BDS to these filtered return time series for embedding dimensions from 2 to 5 and a value of ε of 1.5 units of the standard deviation of the data. In Table 4 the results of the Kaplan test can be observed. For all the cases the BDS and the Kaplan tests yield to the rejection of linearity of the time series. These results does not differ from the results obtained before with exchange rate time series [15].

Now, to test the effects of the shifts of distribution, the stability analysis of both tests will be carried up for the exchange rate series. Fig. 3 shows the results for $m = 2$ and $\varepsilon = 1.5$ standard deviation. It can be observed that for the DM and BP short after

Table 3
BDS test for the different currencies

m	ε/σ	DM	JY	BP
2	1.5	5.97*	5.99*	7.98*
3	1.5	7.00*	6.77*	9.99*
4	1.5	7.59*	7.90*	11.18*
5	1.5	8.21*	9.87*	12.49*

*Statistical significance at 5%.

Table 4
Kaplan test for the different currencies

DM	1	2	3	4	5	6	7	8	9	10
KS	0.715	0.703	0.692	0.661	0.673	0.622	0.643	0.573	0.568	0.494
*Ks*min	0.717	0.700	0.701	0.665	0.684	0.626	0.649	0.563	0.592	0.532
Ktest	0.703*	0.657*	0.618*	0.586*	0.524*	0.495*	0.514*	0.536*	0.531*	0.569
BP	1	2	3	4	5	6	7	8	9	10
KS	0.614	0.608	0.589	0.574	0.560	0.555	0.525	0.488	0.463	0.404
*Ks*min	0.615	0.605	0.592	0.578	0.563	0.556	0.550	0.461	0.484	0.411
Ktest	0.603*	0.538*	0.494*	0.441*	0.394*	0.368*	0.343*	0.353*	0.309*	0.358*
JY	1	2	3	4	5	6	7	8	9	10
KS	0.762	0.745	0.737	0.718	0.672	0.648	0.694	0.638	0.525	0.548
*Ks*min	0.761	0.744	0.744	0.717	0.681	0.660	0.674	0.646	0.520	0.528
Ktest	0.736*	0.674*	0.607*	0.551*	0.552*	0.548*	0.489*	0.430*	0.452*	0.524*

*Statistical significance at 5%.

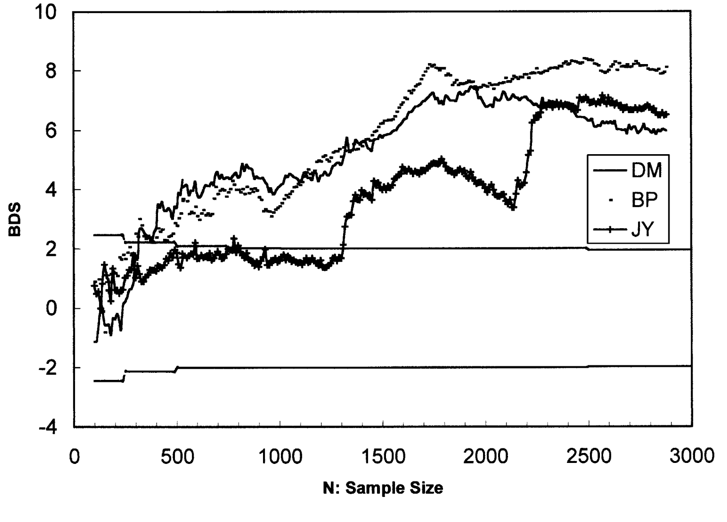

Fig. 3. BDS statistics of the exchange rate returns. Dashed lines represent the 95% confidence intervals.

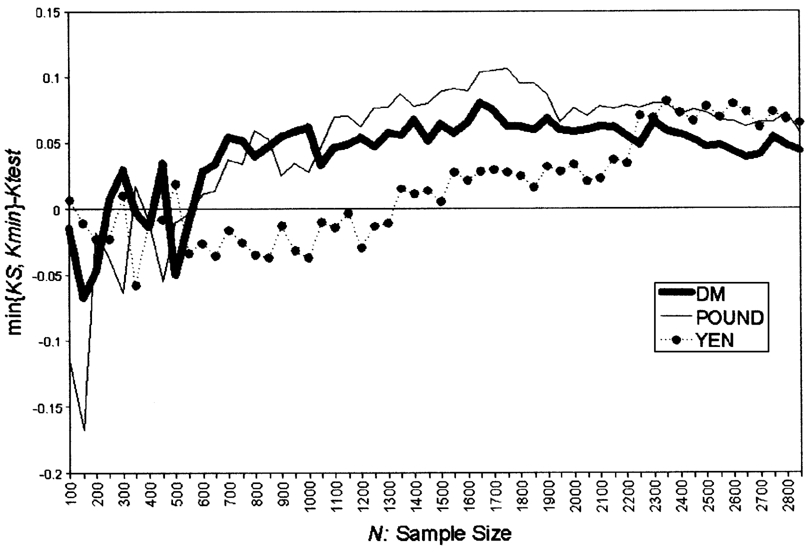

Fig. 4. Kaplan test for the exchange rate returns time series. The K-statistic is calculated for the sample period from 100 to N with steps of 50 observations ($m = 2, \tau = 1, p = 1$). The null hypothesis of linearity is rejected when $\min\{KS; Ks\min\} - Ktest > 0$.

initialization the sample path of the test statistic crosses the 95% confidence interval, but for the JY until the data set has reach 1310 observations the BDS keeps under the confidence intervals, hence the null would not be rejected at the 5% significance level, and jumps from there outside of the interval. The same result about linearity is achieved when the Kaplan test is used as it can be seen in Fig. 4.

These results for the JY case may be revealing a change in conditional variance. The 1310 observation corresponds to February 1995, when the economic weaknesses of Japan led to a sharp realignment of yen–dollar exchange rates. At the beginning of 1995 the US and Japanese central banks and ministries of finance co-operated, the Bank of Japan lowered its interest rate, weaken the yen, the discount rate was cut from 1.75 in March 1995 to 0.50 per cent in August. From April 1995 to May 1997 the yen declined nearly 40 per cent. The BDS seem to be very sensitive to this fact.

4. Application to foreign exchange rate volatility

The study of the behavior of the volatility requires other kind of approximation. The squared of returns and the log of the squared returns can be used as a *proxy* of the volatility, and it is well known [16–21] that both exhibit long-range correlations with persistence. Families of models that exhibit this kind of behavior are the ARFIMA (p,d,q) models:

$$\phi(L)(1-L)^d x_t = \theta(L)u_t \quad u_t \backsim \text{i.i.d.}(0,\sigma^2), \tag{7}$$

Table 5
Results from the ARFIMA estimation

	r_t^2		
	DM	BP	JY
Model	$(1,d,1)$	$(1,d,1)$	$(1,d,1)$
d	0.32 (5.83)*	0.30 (6.45)*	0.31 (5.89)*
AR1	0.47 (8.71)*	0.30 (3.04)*	0.63 (9.16)*
MA1	−0.68 − (11.5)*	−0.52 − (4.24)*	−0.76(−13.1)*

	$\ln r_t^2$		
	DM	BP	JY
Model	$(1,d,1)$	$(1,d,1)$	$(1,d,1)$
d	0.29 (5.90)*	0.28 (6.33)*	0.26 (5.92)*
AR1	0.39 (6.73)*	0.36 (5.69)*	0.33 (4.48)*
MA1	−0.62 (−8.91)*	−0.58 (−7.80)*	−0.54 (−6.17)*

*t-values are in parentheses, statistical significance at 5%.

where L is the lag operator, d is the fractional differencing parameter and all the roots of $\phi(L)$ and $\theta(L)$ lie outside the unit circle. For any real number d, the fractional difference operator $(1-L)^d$ is defined through a binomial expansion

$$(1-L)^d = 1 - \mathrm{d}L + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \cdots \tag{8}$$

and for $0.5 < d < 0.5$ the process is stationary.

The estimation requires writing the spectral density function $f(w)$ in terms of the parameter of the model and calculates the autocovariance function at lag k

$$\gamma(k) = \frac{1}{2\pi}\int_0^{2\pi} f(w)\mathrm{e}^{\mathrm{i}wk}\,\mathrm{d}w. \tag{9}$$

Then the parameters of the model are estimated by the exact maximum likelihood method [22,23]. This method uses all the information, long and short term, of the series and allows the calculation of all the parameters of the model. It requires the correct specification of the ARMA structure to obtain the final ARFIMA specification. In this investigation we have estimated all the possible models for $p=0,1,2,3$ and $q=0,1,2,3$. The best model have been selected using the Akaike Information Citerion (AIC), the Bayesian Information Citerion (BIC) as well as the likelihood-based criteria. Table 5 shows the estimation results for the chosen model for both the squared returns and the log-squared returns.

In this case, the BDS and the Kaplan tests will be used as a test of misspecification of the ARFIMA model for the volatility. Using the detrended fluctuation analysis (DFA [24,25]) it is possible to verify that the residuals of the estimated models do not exhibit long-range correlations as shown in Table 6, if there is no correlation or only short correlation DFA statistics is 0.5, but if there is long-range power-law correlations then DFA statistics is different form 0.5.

Table 6
Results for de DFA statistics over the residuals of the ARFIMA models

	r_t^2			$\ln r_t^2$		
	DM	BP	JY	DM	BP	JY
DFA	0.51	0.51	0.50	0.51	0.50	0.51

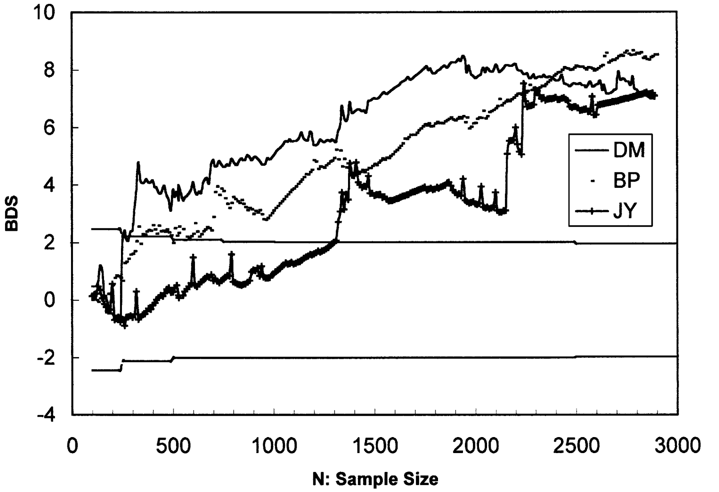

Fig. 5. BDS of the model for the squared returns. Dashed lines represent the 95% confidence intervals.

Running the BDS and the Kaplan tests over the residual data, two different behaviors for the squared returns and the log-squared returns can be observed. In the first case shown in Fig. 5 the sample path of the test statistic is similar to the one obtained for returns, that is, for the BP and DM the BDS statistics jumps quickly out of 95% confidence interval, where the null is rejected, revealing the existence of non-ARFIMA nonlinearities, not captured by the model. But the JY is inside the confidence interval until the data set has 1310 observations, as it is explained before, this behavior can be a sign of a jump in conditional variance.

The behavior of the residuals of the log-squared returns is very different, as shown in Fig. 6.

The BDS test keeps inside the confidence interval in all the three cases, meaning that the ARFIMA model is able to capture both long-memory and nonlinearities for this series of volatility.

The Kaplan test results for the residuals of the ARFIMA models have not any significative difference with the BDS test (Figs. 7 and 8), so the same conclusions can be applied (There are differences only in the case of the DM log-squared returns, because of the Kaplan test rejects the null hypothesis of linearity).

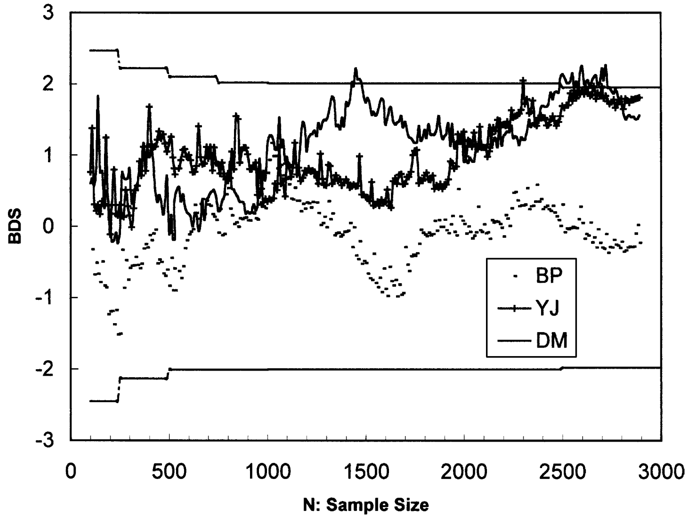

Fig. 6. BDS of the model for the log-squared returns. Dashed lines represent the 95% confidence intervals.

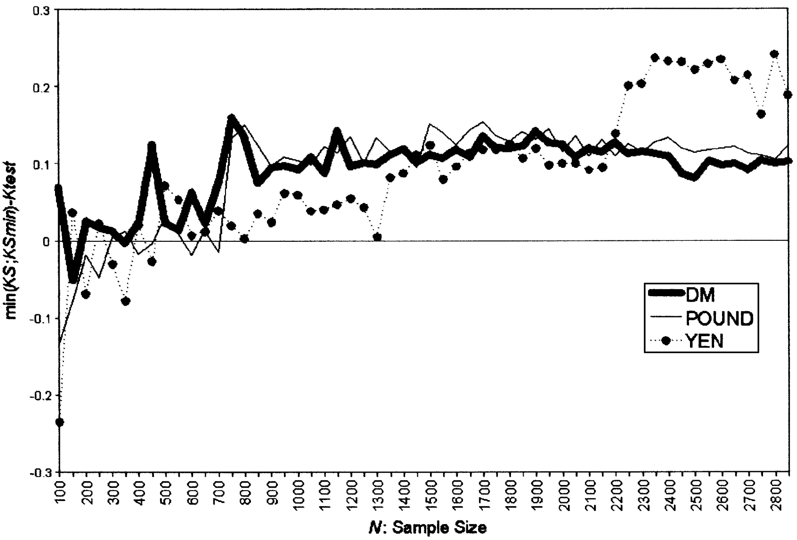

Fig. 7. Kaplan test for exchange rate volatility time series (residuals squared returns). The K-statistic is calculated for the sample period from 100 to N with steps of 50 observations ($m = 2, \tau = 1, p = 1$). The linear null hypothesis is rejected when $\min\{KS; Ks\min\} - Ktest > 0$.

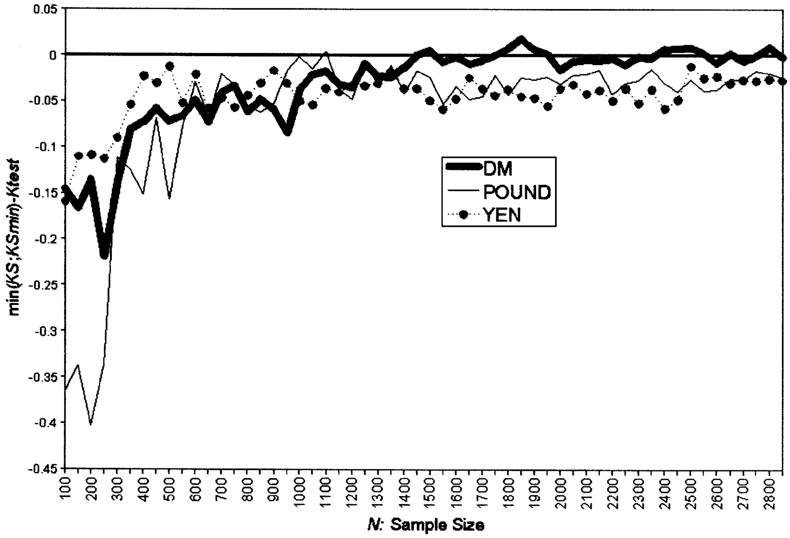

Fig. 8. Kaplan test for exchange rate volatility time series (residuals log squared returns). The K-statistic is calculated for the sample period from 100 to N with steps of 50 observations ($m = 2, \tau = 1, p = 1$). The linear null hypothesis is rejected when $\min\{KS; Ks\min\} - Ktest > 0$.

5. Conclusions

Recent research has put forward the idea that both financial assets returns, and volatilities are nonlinear processes. This paper investigates the impact of nonstationarities on the testing of nonlinearities. For returns time series nonlinearities are found for the exchange rates DM/$ and BP/$, but for the JY/$ a possible shift in conditional variance yields to a rejection of nonlinearity for the hole data set. The behavior of volatility is studied through the behavior of the residuals of the ARFIMA estimated model. Two different behavior are found depending on the *proxy* of volatility used. For the residuals of the squared returns, similar behavior as the returns time series is found, that means that the ARFIMA model is not able to capture the nonlinearities of the time series under study. However for the log-squared returns the residuals of the ARFIMA model seems to be i.i.d. which means that this kind of model is able to capture, both long-memory and nonlinearities of the series under study.

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