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Modeling highly volatile and seasonal markets: evidence from the Nord Pool electricity market

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Summary. In this paper we address the issue of modeling spot electricity prices. After analyzing factors leading to the unobservable in other financial or commodity markets price dynamics we propose a mean reverting jump diffusion model. We fit the model to data from the Nord Pool power exchange and find that it nearly duplicates the spot price's main characteristics. The model can thus be used for risk management and pricing derivatives written on the spot electricity price.

Key words. Electricity price, Mean reversion, Wavelet transform, Jump diffusion model

Introduction

The power industry's basic function is to convert fuel and primary energy into electricity and transport it to its customers. Because it is very costly to store electricity, the supply has to be matched with demand in real time.

Traditionally, centralized regulation of the electricity supply industry was considered necessary to ensure security of supply and efficient production. Efficiency was achieved through economics of scale. The power sector was characterized by a highly vertically integrated market structure with little competition. However, during the last decade many countries have restructured and deregulated their power sectors to introduce competition. These changes have already taken place in Europe (England and Wales, Norway, Sweden, Finland, Spain, Denmark, the Netherlands, Germany, Poland, Austria), in the Americas (parts of the US, Chile, Argentina, Peru, Bolivia, Colombia) and in the Asia/Pacific basin (Australia, New Zealand, Japan).

Introducing competition was believed to improve cost efficiency, increase diversity of fuel supply and provide additional benefits to the consumer. Microeconomic theory states that the price should decline when competition is introduced to the market (McConnel and Brue 1998). The power markets

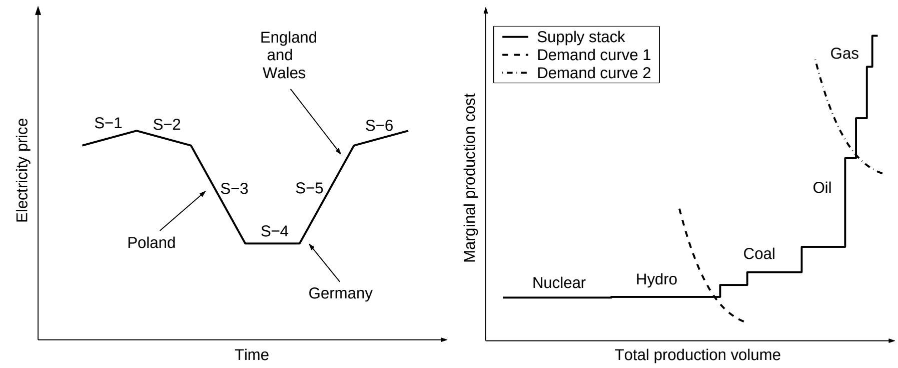

Fig. 1. *Left panel:* Stages of the "road from state to private monopoly" – state monopolies (S-1), declaration of competition (S-2), deregulation and decline of prices (S-3), tranquility period (S-4), mergers and effects of market power (S-5) and formation of private monopolies (S-6). Presumed stages in England and Wales, Germany and Poland are also depicted (see also Szalbierz and Weron 2000). *Right panel:* A schematic supply stack with two potential demand curves superimposed on it. The spot price, given as the intersection between demand and supply, is not very sensitive to demand shifts when the demand is low (curve 1), since the supply stack is typically flat in the low-demand region. However, when demand is high only small increments in demand can have huge effects on the price (curve 2).

in many countries have, however, reacted differently. They have followed (or are following) what could be called a "road from state to private monopoly" (Haas 2000, Szalbierz and Weron 2000) – after an initial fall, the wholesale prices rose back to pre-liberalization levels, see the left panel of Fig. 1.

In a competitive market utilities cannot automatically pass costs to customers. This has the effect of increasing uncertainty and risks born by the investors. Electricity is changing from a primarily technical business, to one in which the product is treated in much the same way as any other commodity, with trading and risk management as key tools to run a successful business. All this calls for adequate models of price dynamics capturing the main characteristics of electricity prices. In what follows we address the issue of modeling spot prices, because spot prices are one of the key factors in strategic planning and decision support systems of a majority of market players and are the underlying instrument of a number of power derivatives.

When supply meets demand

The supply stack is the ranking of all generation units of a given utility or of a set of utilities in a given region. This ranking is based on many factors, such as the marginal cost of production and the response time. The utility will typically first dispatch nuclear and hydro units, if available, followed by coal units. These types of plants are generally used to cover the so-called base load, whereas oil-, gas-fired and hydro-storage plants are used to meet peak-demand. Plants with low or moderate marginal costs often exhibit low flexibility, implying that the response time is long (up to a few hours) or that some constant amount of electricity has to be produced all the time. The supply stack is not static in time, since there are many factors influencing it, eg. fluctuation of fuel prices (oil and gas) or outages of plants (due to regular maintenance operations, transmission constraints or unforeseen breakdowns).

Demand, on the other hand, exhibits seasonal fluctuations, which are essentially due to climate conditions. In Europe the demand-peak normally occurs in the winter due to excessive heating. In other geographical regions, like southern states in the US, demand peaks in the summer, since humidity and heat initiate extensive use of air-conditioning. Electricity demand is also not uniform throughout the week. It peaks during weekdays' working hours and is low during nights and weekends (due to low industrial activity). Moreover, unexpected weather conditions can cause sudden and dramatic shocks with demand typically falling back to its normal level as soon as the underlying weather phenomenon is over.

The spot price is given as the intersection between demand and supply, see the right panel of Fig. 1. It is not very sensitive to demand shifts when the demand is low, since the supply stack typically is flat in the low-demand region. When demand is high, however, only small increments in demand can cause huge price spikes.

Nord Pool

The spot electricity market is actually a day-ahead market. A classical spot market would not be possible, since the system operator needs advanced notice to verify that the schedule is feasible and lies within transmission constraints. The spot is normally an hourly contract with physical delivery and is not traded on a continuous basis, but rather in the form of a conducted once per day auction. It is the underlying of most derivatives.

Several countries have deregulated their power markets and a few power exchanges have been established in the last decade. Yet, typically there are only one or two years of "stationary" data available due to the changes which are constantly taking place in many power markets. In our analysis, therefore, we will use spot prices from Nord Pool which is generally regarded as the most mature and "stable" power market in the world.

The Norwegian deregulation came into force in 1991 when grid owners were compelled to open their grids to competition. In 1993 Nord Pool started its business as a power exchange for the Norwegian market. In 1996 Sweden was integrated into the exchange, followed by Finland (1998) and western Denmark (1999). Nord Pool offers two types of standardized contracts – physically settled spot contracts and financially settled futures, forward, option and other specialized contracts. Every day is divided into 24 hourly spot contracts. Before noon, the previous day, all participants send in their bids for each hour. From the bids Nord Pool obtains the aggregated supply and demand curves. The auction system is used in order to increase liquidity. The system price is calculated as the equilibrium point for each of the 24 hours. It is a theoretical price in the sense that it assumes that no congestions will occur and is the same in the whole Nordic area (Nord Pool 2001).

Spot price dynamics

The goal of this paper is to propose a model for electricity spot price dynamics. The traditional approach to modeling price processes of stochastic (random) nature in finance is to apply diffusion-type stochastic differential equations (SDE's) of the form

$$dX_t = \mu(X,t)dt + \sigma(X,t)dB_t, \tag{1}$$

where $\mu(X,t)$ is the drift, $\sigma(X,t)$ is the volatility (scaling factor) and dB_t are the increments of standard Brownian motion. Some phenomena are modeled by a set of related SDE's, leading to so called multi-factor models. On the other hand, both μ and σ can be defined by SDE's of their own, leading to stochastic drift and/or volatility models. Probably the best known member of this family of processes is geometric Brownian motion (with $\mu(X,t) = \mu X$ and $\sigma(X,t) = \sigma X$), which was used already in the 1960's for modeling stock price movements. In this paper we will construct a model of electricity spot prices in line with the diffusion-type approach.

According to economic theory the price of any good is determined by matching demand with supply (McConnel and Brue 1998). Yet, electricity spot prices exhibit a behavior not observed in other financial or commodity markets. Is the power market a counterexample? Or is electricity so unique that the supply and demand curves intersect in a way causing this odd behavior? We have already given some hints as to the answer to these questions. However, for the sake of completeness, we will shortly review the most important factors leading to such a behavior.

Mean reversion

Energy spot prices are in general regarded to be mean reverting (Schwartz 1997). Among other financial time series spot electricity prices are perhaps the best example of anti-persistent data. In Weron and Przybyłowicz (2000) and Weron (2002) the R/S analysis, detrended fluctuation analysis and periodogram regression methods were used to verify this claim. Here we apply the Average Wavelet Coefficient (AWC) method of Simonsen et al. (1998), that has proven useful in particular when dealing with multi-scale time series (Simonsen 2003). This method utilizes the wavelet transform in order

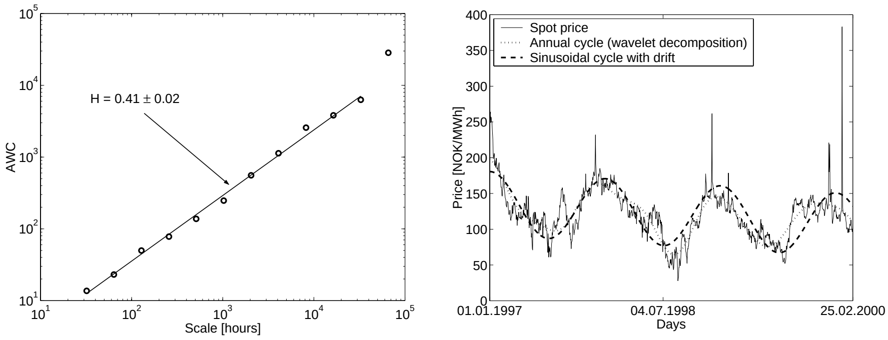

Fig. 2. *Left panel:* The AWC statistics, $W[p](a)$ vs. scale a on a log-log paper, of the hourly Nord Pool electricity spot price since May 4, 1992 until December 31, 2000. The scaling region $a > 24$ hours corresponds to a Hurst exponent $H = 0.41 \pm 0.02$. *Right panel:* Nord Pool market daily average system prices since January 1, 1997 until February 25, 2000. Superimposed on the plot is the annual cycle obtained through a wavelet decomposition technique and its approximation by a sinusoid with a linear trend. Observe that the sinusoid approximates the annual cycle quite well justifying its use in the analysis.

to measure the temporal self-affine correlations, i.e. it measures the Hurst exponent H. This is done by transforming the time series of spot electricity prices p_t into the wavelet-domain, $\mathcal{W}[p](a,b)$, where a denotes the scale parameter, and b is the location (Percival and Walden 2000). The AWC method consists of finding a representative (wavelet) "energy" or amplitude for a given scale a. This is usually done by simply taking the arithmetic average of $|\mathcal{W}[p](a,b)|$ over all location parameters b corresponding to one and the same scale a. We can therefore construct, from the wavelet transform of p_t, the AWC spectrum $W[p](a)$ that only depends on the scale. If p_t is a self-affine process characterized by the exponent H, this spectrum should scale as (Simonsen et al. 1998)

$$W[p](a) = \langle |\mathcal{W}[p](a,b)| \rangle_b \sim a^{H+1/2}. \tag{2}$$

So, if the signal is self-affine and we plot $W[p](a)$ against a the points should constitute a line with slope $H+1/2$ on a log-log paper. The application of this method to Nord Pool spot prices from the period May 4, 1992 – December 31, 2000 is depicted in the left panel of Fig. 2. For time intervals ranging from a day to almost four years the Hurst exponent $H = 0.41 \pm 0.02$ indicating mean reversion. For time intervals of less than 24 hours, however, H is above 0.5, the level of the Hurst exponent for Brownian motion (Simonsen 2003).

Mean reversion is typically modeled by having a drift term that is negative if the spot price is higher than the mean reversion level and positive if it is lower. To make things simple we will start with probably the simplest mean reverting model originally proposed for describing interest rates dynamics.

The Vasicek (1977) model, also referred to as an arithmetic Ornstein-Uhlenbeck process, is described by the following SDE

$$dX_t = (\alpha - \beta X_t)dt + \sigma dB_t = \beta(L - X_t)dt + \sigma dB_t. \tag{3}$$

This is a one-factor model that reverts to the mean $L = \frac{\alpha}{\beta}$ with β being the magnitude of the speed of adjustment. The second term is responsible for the volatility of the process. The conditional distribution of X at time t is normal with mean $E[X_t] = \frac{\alpha}{\beta} + (X_0 - \frac{\alpha}{\beta})e^{-\beta t}$ and variance $Var[X_t] = \frac{\sigma^2}{2\beta}(1 - e^{-2\beta t})$. These relations imply that $E[X_t] \to L = \frac{\alpha}{\beta}$ as $t \to \infty$. Starting at different points the Vasicek model trajectories tend to reverse to the long run mean and stabilize in the corridor defined by the standard deviation of the process. The equilibrium level L can be also made time dependent to reflect the fact that electricity prices tend to revert to different levels over the year.

Seasonal fluctuations

As we have already mentioned, demand follows seasonal fluctuations, caused mainly by climate conditions. In some countries also the supply side shows seasonal variations in output. Hydro units, for example, are heavily dependent on precipitation and snow melting, which varies from season to season. These seasonal fluctuations in demand and supply are directly translated into the seasonal behavior of spot electricity prices. In the right panel of Fig. 2, we have plotted the Nord Pool market daily average system prices since January 1, 1997 until February 25, 2000. The choice of the period used in the analysis is not incidental – 1996 was a dry year with exceptionally high electricity prices and the rest of the year 2000 was used for out-of-sample testing of the Asian options pricing model (Weron 2003). Superimposed on the plot is the annual cycle obtained through a wavelet decomposition technique, i.e. a technique consisting of removing several layers of noise and leaving out only the large scale wavelets (Percival and Walden 2000, Simonsen et al. 2002). As it turns out, in this period the annual cycle can be quite well approximated by a sinusoid with a linear trend. This is in line with the approach of Pilipovic (1997), who suggests fitting a proper sinusoidal function (eg. a sum of two cosine functions with distinct periods) to spot prices. However, such an approach would not be suitable for the California market where demand and, to some extent, the spot prices are rather flat throughout the year with a hump in the summer (Weron et al. 2001, Nowicka-Zagrajek and Weron 2002). Another method of modeling seasonality consists of fitting a piecewise constant function of a one year period, where for each month one tries to determine an average value out of the whole analyzed time series (Bhanot 2000, Lucia and Schwartz 2002). Although flexible, this method lacks smoothness, which may have a negative impact on statistical inference of the deseasonalized price process.

Jumps

In addition to mean reversion and strong seasonality on the annual, weekly and daily level, spot electricity prices exhibit infrequent, but large jumps. The spot price can increase tenfold during a single hour. This is the effect of non-storability of electricity. Electricity to be delivered at a specific hour cannot be substituted for electricity available shortly after or before, since it has to be consumed at the same time as it is produced. Jumps in the spot prices are an effect of extreme load fluctuations, caused by severe weather conditions often in combination with generation outages or transmission failures. These spikes are normally quite short-lived, and as soon as the weather phenomenon or outage is over, prices fall back to a normal level (Kaminski 1999).

The "jumpy" character of electricity prices, calls for spot price modeling, which is not continuous. One approach, suggested in Deidersen and Trück (2002), reduces to substituting Brownian motion with eg. a positively skewed α-stable Levy motion (Janicki and Weron 1994, Rachev and Mittnik 2000). However, this would lead to purely discontinuous price paths and would limit control of the intensity of the jumps. Another approach is to introduce to eqn. (3) a jump component $J_t dq_t$ (Johnson and Barz 1999, Clewlow and Strickland 2000), where J_t is a random jump size, eg. a lognormal random variable $\log J_t \sim N(\mu, \rho^2)$, and q_t is eg. a Poisson random variable with intensity κ. Eydeland and Geman (2000) propose a similar model, where – to account for the fact that jumps tend to be more severe during high price periods – the jump part is given by $J_t X_t dq_t$.

Electricity prices tend to rapidly revert to their normal level after a jump. In the models mentioned above the price is forced back by the mean reversion after a jump, which may be not fast enough. Geman and Roncoroni (2002) suggest using mean reversion coupled with downward jumps. Alternatively, a positive jump may be always followed by a negative jump to capture the rapid decline of electricity prices after a spike. On the daily level, i.e. when analyzing average daily prices, the latter approach seems to be a better solution since spikes typically do not last more than a day.

The model

As stated previously, the annual cycle can be quite well approximated by a sinusoid (see the right panel of Fig. 2). Instead of making the equilibrium level L time dependent, we incorporate this periodicity into our model in the form of an external, deterministic sinusoidal function $S_t = A \sin\left(\frac{2\pi}{365}(t+B)\right) + Ct$, where A, B and C are constants (obtained through a least squares fit).

Like demand, spot electricity prices are not uniform throughout the week. The intra-week and intra-day variations of demand caused by different level of working activities translate into periodical fluctuations in electricity prices. However, in the present analysis we will not address the issue of intra-day variations and will analyze only daily average prices. We deal with the intra-week variations by preprocessing the data using the moving average technique, which reduces to calculating the weekly profile s_t, i.e. an average week, and subtracting it from the spot prices (see eg. Weron et al. 2001).

Despite their rarity, price spikes are the very motive for designing insurance protection against electricity price movements. This is one of the most serious reasons for including jump components in realistic models of electricity price dynamics. We also do so with our model. Reflecting the fact that on the daily scale spikes typically do not last more than one time point (i.e. one day) we let a positive jump be always followed by a negative jump of about the same magnitude. This is achieved by letting the stochastic part X_t be independent of the jump component $J_t dq_t$. For the sake of simplicity we let J_t be a lognormal random variable $\log J_t \sim N(\mu, \rho^2)$ and q_t be a Poisson random variable with intensity κ. The jump component is estimated from the logarithm of the deseasonalized, with respect to the weekly and annual cycles, prices $d_t = \log(p_t - s_t - S_t)$ through a two-step procedure. First, all jumps – defined as price increments exceeding 3 standard deviations of all price changes – are removed from d_t. Next, the intensity κ and the distribution of the magnitude J_t of the jumps is estimated from these few selected points (7 in the whole analyzed data set).

Putting all the facts together, our model has the following form

$$p_t = s_t + S_t + e^{J_t dq_t + X_t}, \tag{4}$$

where X_t is defined by eqn. (3). The exponent in the last term of eqn. (4) reflects the fact that the marginal distribution of X_t is Gaussian, whereas the deseasonalized, with respect to the weekly and annual cycles, and "spikeless" spot prices can be very well described by a lognormal distribution, see the left panel of Fig. 3. The parameters of the Vasicek SDE (3) are estimated using a Matlab implementation (Cliff 2000) of the Generalized Method of Moments (Hansen 1982).

To verify the adequacy of our model we fitted it to Nord Pool market daily average system prices from the period January 1, 1997 – January 15, 2000, see the right panel of Fig. 2. Next, we simulated price trajectories starting January 15, 2000, see the right panel of Fig. 3. The statistical characteristics of these simulated paths closely resembled the original spot price, allowing us to use the model for pricing Nord Pool's Asian options written on the spot electricity price (Weron 2003).

Conclusions

The liberalization of the power markets has created additional risks and new challenges for players in the market. The uniqueness of electricity – such as highly fluctuating demand, non-flexible supply, transmission congestion

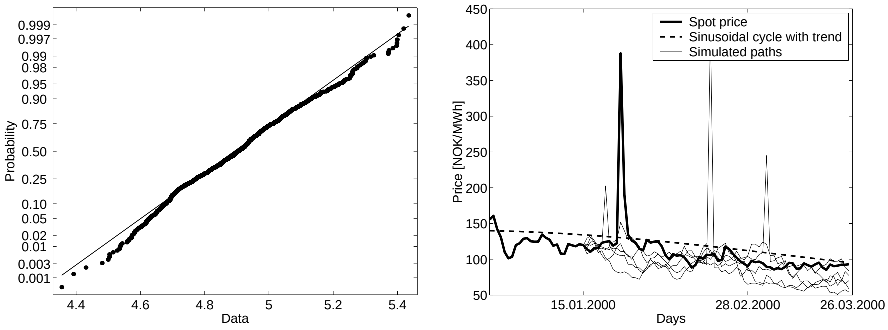

Fig. 3. *Left panel:* The normal probability plot of the logarithm of the deseasonalized, with respect to the weekly and annual cycles, and "spikeless" spot prices (i.e. $\log(p_t - s_t - S_t) - J_t dq_t$). The dots form a straight line indicating a Gaussian distribution. In fact, the Bera-Jarque test (Spanos 1993) indicates that we cannot reject normality at the 2% level, whereas for the logarithm of the deseasonalized (but with spikes) prices the Bera-Jarque test lets us reject normality even at the 0.5% level. *Right panel:* The true spot price trajectory and model simulated paths.

issues and its non-storability – distinguishes the power market from other financial or commodity markets and calls for new models of price dynamics capturing mean reversion, seasonality and price spikes. The number of papers addressing these problems is still scarce and the suggested solutions are usually not universal or unsatisfactory. In this paper we have proposed a model which closely resembles the spot price trajectories from the Nord Pool power exchange. We believe that it can be successfully used to price a number of derivatives in the Nordic power market.

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