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PHYSICA A

Scaling in the market of futures

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Abstract

The price time series of the Italian government bonds (BTP) futures is studied by means of scaling concepts originally developed for random walks in statistical physics. The series of overnight price differences is mapped onto a one-dimensional random walk: the *bond walk*. The analysis of the root mean square fluctuation function and of the auto-correlation function indicates the absence of both short- and long-range correlations in the bond walk. A simple Monte Carlo simulation of a random walk with trinomial probability distribution is able to reproduce the main features of the bond walk. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

The relationship between economics and the physical sciences has a long and interesting history. Outstanding economists of the past explicitly inspired their work from the principles of Newtonian physics and statistical mechanics, attracted by the success of these theories. It is, for instance, the case of Vilfredo Pareto who carried on a mathematical approach to political economy by exploiting his background in the physical sciences and engineering [1].

However, despite the existence of many problems of common interest, the interaction between statistical physicists and economists has never been strong [2]. Only in recent times, physicists and economists started talking to each other more and more frequently. An important milestone in this dialogue was the publication of the proceedings of a meeting held in Santa Fe in 1988 and organized by the Santa Fe Institute [3].

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The renewed interest of physicists for economics was a consequence of the "explosion" of non-linear (or chaotic) science. Many scholars started to ask themselves whether there was any predictability of financial time series, a problem of enormous practical importance for investors in the financial market. These developments were marked by a bit of folklore. Many non-specialized journals, newspapers and magazines wrote article about or interviewed J. Doyne-Farmer, who, together with his old friend Norman Packard, left the academic research to establish the Prediction Company, in order to gamble on the financial markets [4]. In the meantime many reports appeared on Wall Street hiring scientists familiar with statistics and time-series analysis [5]. The papers distributed at the recent meeting on *Forecasting financial markets* show that the techniques outlined in the Santa Fe meeting are now, less than ten years later, quite well established in the field of financial analysis [6].

Among the various approaches, the one making use of *scaling concepts* has been successful especially for the characterization of financial time series. Scaling concepts provide a unifying and very useful tool for the investigation of phenomena in physics, chemistry and biology [7–9]. The use of scaling concepts in the economic and social sciences can be traced back at least to the work of Zipf [10], who discovered the rank-frequency statistics taking now his name. Essentially, Zipf found that the frequency of a word in a written document is inversely proportional to the rank order of that word. Zipf tried to justify this discovery in terms of a variational principle (*the principle of least effort*) analogous to the successful variational principles in mechanics or optics. A recent discussion on the origin of Zipf's law can be found in Ref. [11].

The usefulness of scaling concepts in the analysis of financial markets has been shown by a series of papers written by Mantegna and Stanley [12–16]. Analyzing the time evolution of the S&P 500 index, they proved that the simplest model describing the dynamics of prices in speculative markets is the truncated Lévy flight [12–14]. Other works have focused on the analogy between the price dynamics in the foreign exchange market and three-dimensional turbulence [17]. However, such a result does not hold true for the S&P 500 index [15].

This paper is devoted to a simple analysis of the time series of the Italian long-term bond (BTP: *Buoni del Tesoro Poliennali*) futures. The focus is on the difference between the closing price and the opening price of the next day. Overnight price differences are rather an intriguing subject. If you ask a practitioner analyst, he or she will almost always find a reason for the price difference based on the major overnight economical and political events. Moreover, usually, the volume of contracts exchanged at the morning opening price is one of the largest in the day. The paper is organized as follows. In Section 2, the overnight price difference is mapped onto a simple random flight and the scaling behavior of its mean square fluctuation is computed. Finally, in Section 3, I summarize the main result of the present paper and outline the direction of future work.

2. Futures and scaling properties

The Italian Government bond futures are contracts based on the Italian Government bonds and exchanged at the London International Financial Futures and Options Exchange (LIFFE). In Fig. 1, the time evolution of the BTP-future prices for 1377 days, from 18 September 1991 to 20 February 1997 is presented. The dashed line represents the opening prices, whereas the continuous line represents the closing prices. The two curves almost superimpose.

As I said in the introduction, the following analysis will focus on overnight price differences. In Fig. 2, the 1366 points of the overnight price difference are plotted. The week-end and holiday variations are considered as overnight differences. In order to further simplify the problem, I shall address the question of whether the next morning price is greater, equal, or lower than the previous evening price. Then, it is possible to map the overnight price difference time series onto a one-dimensional random walk. If the price increases, I let a walker move up ($u(i)=+1$) one unit length; if the price decreases, the walker will move down ($u(i)=-1$) one unit length. Finally, if the price does not change, the walker does not move ($u(i)=0$). This procedure defines a walk (the *bond walk*), perfectly analogous to the DNA walk which has been used to investigate the presence of long-range correlations in DNA sequences [18]. The only difference is that, here, also the case of non-moving walkers is taken into account. Following Ref. [18], I compute the displacement of the walker after the tth step

$$y(t)=\sum_{i=1}^{t} u(i)\,. \tag{1}$$

This quantity is plotted in Fig. 3. In order to characterize the correlations, the root mean square fluctuation around the average displacement, $F(t)$, can be computed:

$$F^2(t)=\langle \Delta y(t)^2\rangle - \langle \Delta y(t)\rangle^2\,, \tag{2}$$

where $\Delta y(t)=y(t_0+t)-y(t_0)$, and $\langle\cdot\rangle$ is the average over all initial steps t_0. $F(t)$ obeys the following scaling law [1,18]:

$$F(t)\propto t^{\alpha}\,. \tag{3}$$

If the walk is completely random, that is the correlations are zero on average except in the origin, then the scaling exponent has the value $\alpha=\frac{1}{2}$; the same value is attained if there are short-range correlations [1,8,18]. For long-range (ideally infinite-range) correlations the scaling exponent takes a value different from $\frac{1}{2}$ [18]. In order to estimate the scaling exponent, I have divided the time series into 12 non-superimposing sub-series containing 100 points each. For each sub-series, I got an estimate of $F(t)$ for t between 1 and 35. In Fig. 4, the average value of $F(t)$ is plotted with its error computed according to Student's t-distribution. The linear best fit of the data gives

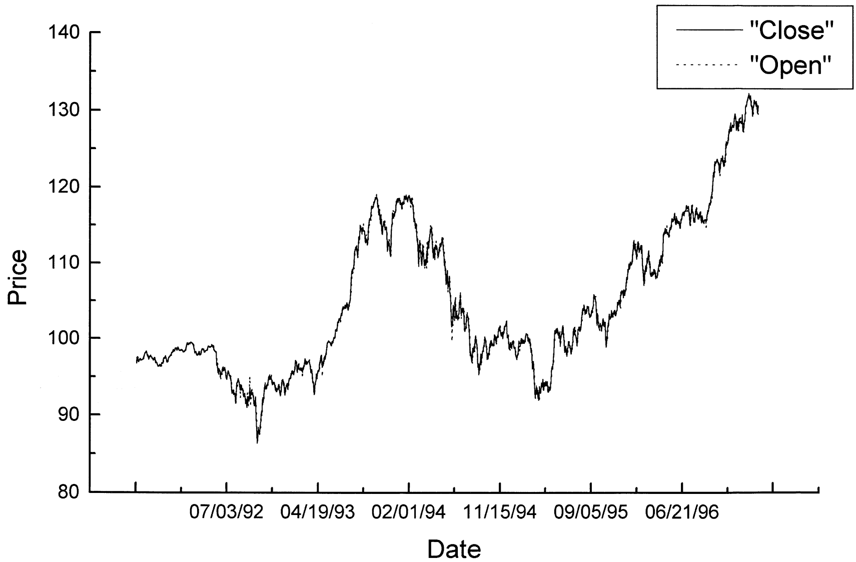

Fig. 1. Price variation of the BTP future as a function of time. The opening (dashed line) and closing (continuous line) prices are sampled every day. The two curves almost always superimpose.

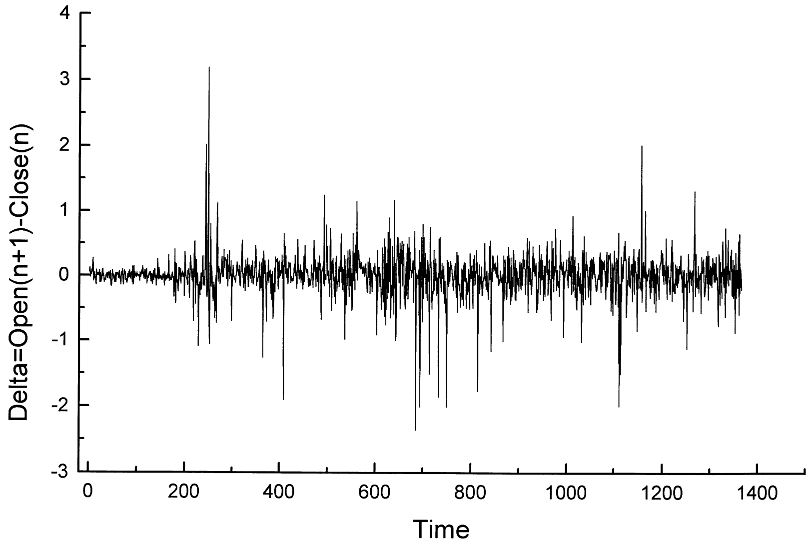

Fig. 2. Overnight price difference time series. Most of the overnight price variations are below 1 Italian Lira.

a scaling exponent $\alpha = 0.49$. Therefore, the presence of long-range correlations in the bond random walk can be ruled out. With the analysis performed so far it is impossible to exclude the presence of short-range correlations. To this purpose, a direct estimate of the auto-correlation function is necessary. In Fig. 5, an estimate of the auto-correlation

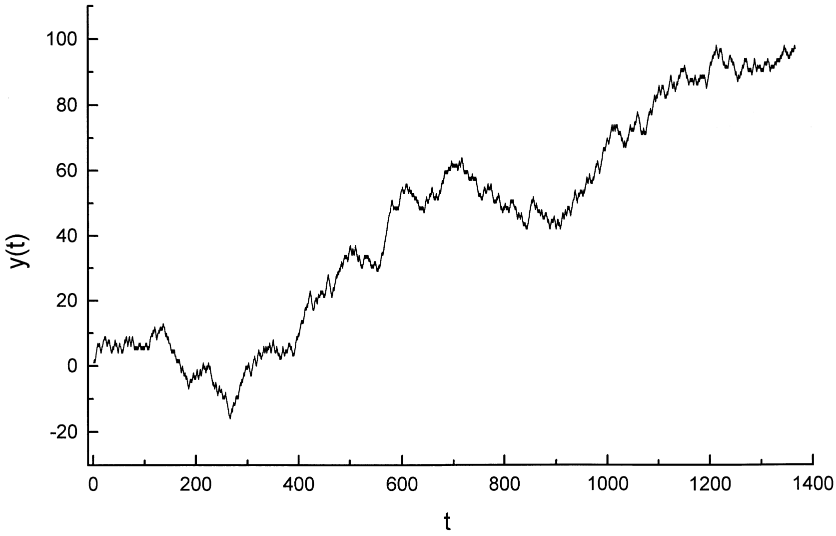

Fig. 3. Displacement $y(t)$ for the bond walk defined in the text.

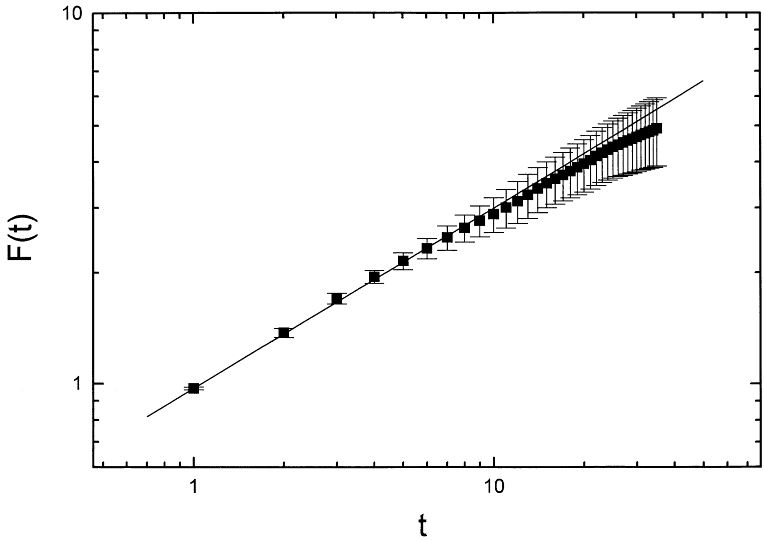

Fig. 4. Root mean square fluctuation function $F(t)$ for the bond walk. The straight line is a linear fit of the data and its slope is nearly $\frac{1}{2}$ (see text for more details).

function $C(\tau)$ is presented. The following unbiased estimator has been used [19]:

$$C(m) = \frac{1}{N-m} \sum_{n=0}^{N-m-1} u(n+m)u(n) - \left[\frac{1}{N} \sum_{n=0}^{N-1} u(n) \right]^2 , \tag{4}$$

where N is the number of points. The second term is the square of the average of $u(i)$.

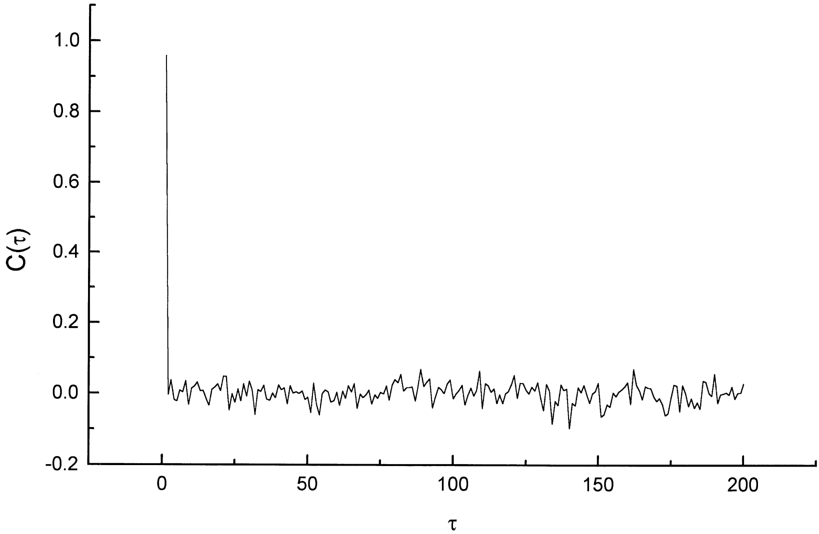

Fig. 5. Correlation function $C(\tau)$ for the bond walk. Only $C(0)$ is significantly greater than 0.

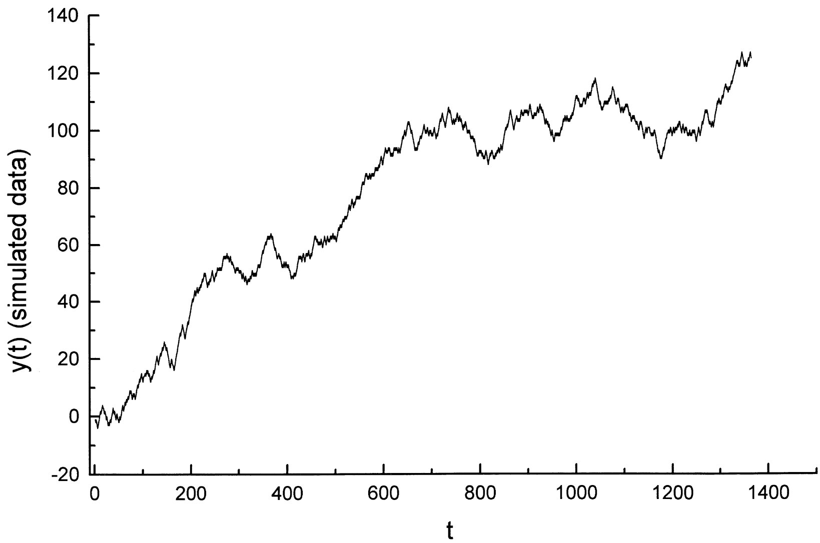

Fig. 6. Displacement $y(t)$ for the simulation described in the text.

It must be remarked that $C(0)<1$ because some $u(i)$'s are 0 and the average value of $u(i)$ is not 0. A glance to the auto-correlation function is sufficient to conclude that no short-range correlations seem to be present. Indeed, it turns out that $C(\tau) \ll C(0)$ for $\tau \geqslant 1$.

I shall now show that a simple one-dimensional random walk, where the walker has probability p_1 of moving upwards, p_2 of moving downward and p_3 of staying still, is

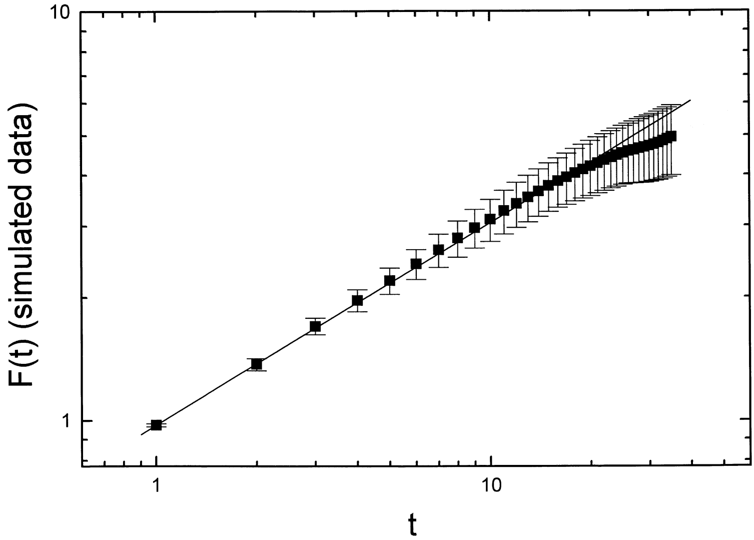

Fig. 7. Root mean square fluctuation function for the simulated walk. The straight line is a linear fit of the data and its slope is nearly $\frac{1}{2}$.

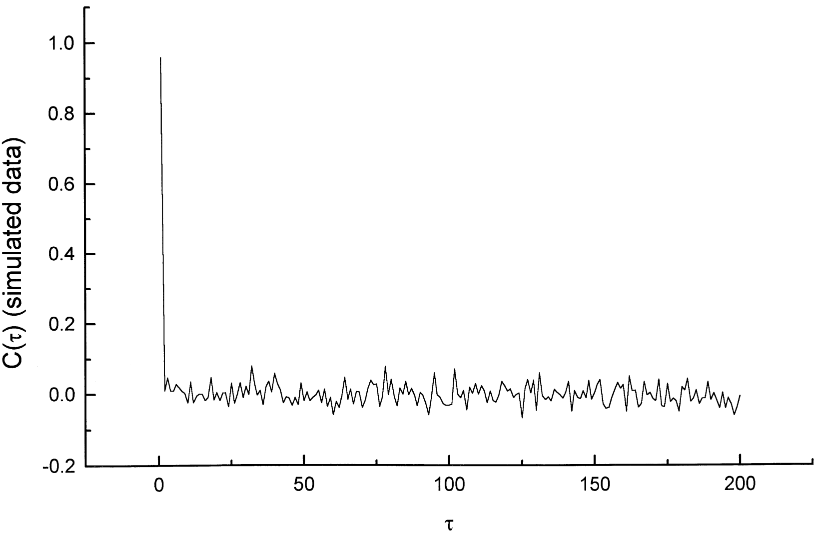

Fig. 8. Correlation function $C(\tau)$ for the simulated walk. Also here only $C(0)$ is significantly greater than 0.

sufficient to reproduce the behavior of the bond walk. In the latter the walker moves up 707 times (51.7%), down 610 times (44.7%), and does not move 49 times (3.6%). It is natural to assume p_1, p_2, and p_3 equal to the respective frequencies measured in the bond walk. The result of a Monte Carlo simulation of the bond walk is plotted in Fig. 6. At every step of the simulation, $+1$ is selected with probability $p_1 = 0.517$,

-1 is selected with probability $p_2 = 0.447$ and 0 is selected with probability $p_3 = 0.036$. The simulated displacement y has the same upward trend of the real displacement. In Fig. 7, the estimate of $F(t)$ is plotted. The scaling law of Eq. (3) is obeyed with $\alpha = 0.49$. Finally, in Fig. 8, the auto-correlation function computed according to Eq. (4) is given for the simulated walk. Again no short-range correlations are present, as expected.

3. Conclusions

In the present paper, I have investigated some of the scaling properties of the Italian government bond (BTP) future price time series. In particular, I have considered the overnight price differences and I have shown that no long- or short-range correlations seem to be present. In order to reach this conclusion, the time series of overnight differences has been mapped onto a one-dimensional random walk and the root mean square displacement fluctuations and the correlation function of the walk have been analyzed. The features of the bond random walk are well reproduced by a random walk with trinomial probability distribution. Also in this instance, scaling concepts taken from statistical mechanics have proven useful for the analysis of a financial time series. I am currently investigating whether the truncated Lévy flight introduced by Mantegna and Stanley is able to reproduce the dynamics of the tick-by-tick time series.

Acknowledgements

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