Chaos and nonlinear forecastability in economics and finance†

By Blake LeBaron

Department of Economics, University of Wisconsin – Madison, 1180 Observatory Drive, Madison, Wisconsin 53706, U.S.A.

Both academic and applied researchers studying financial markets and other economic series have become interested in the topic of chaotic dynamics. The possibility of chaos in financial markets opens important questions for both economic theorists as well as financial market participants. This paper will clarify the empirical evidence for chaos in financial markets and macroeconomic series emphasizing what exactly is known about these time series in terms of forecastability and chaos. We also compare these two concepts from a financial market perspective contrasting the objectives of the practitioner with those of the economic researchers. Finally, we will speculate on the impact of chaos and nonlinear modelling on future economic research.

1. Introduction

It has been almost ten years since economists began searching for chaotic dynamics in economic time series. This study has yielded deeper understandings of the dynamics of many different series, and has led to the development of several useful tests for nonlinear structure. However, the direct evidence for deterministic chaos in many economic series remains weak. This paper will survey the existing results and give some intuition about why they probably were not unexpected. We also argue that chaotic dynamics still needs to be taken seriously for economic systems, but the tools and methods will differ from those used in the past.

The possibilities of chaos in economic systems brought an enormous amount of initial interest. The concepts of limited forecastability and complex dynamical properties has very strong intuitive appeal for economics. From forecasting movements in foreign exchange and stock markets, to understanding international business cycles, chaos in economics had a broad range of potential applications. This led to an explosion of empirical work searching for possible chaos in all types of economic and financial time series. These studies have found little or no evidence for chaos in any economic time series, but they have turned up a surprising amount of unexplained nonlinear structure in many series. In hindsight most of these results should have been expected to some extent. Researchers looking at macroeconomic and financial series face certain constraints which make the likelihood of directly seeing chaos small. In macroeconomics the problem of short and

† This paper was produced from the author's disk by using the TEX typesetting system.

Phil. Trans. R. Soc. Lond. A (1994) **348**, 397–404

Printed in Great Britain

noisy time series coming from a system whose dynamics and measurement probes may be changing over time impedes the ability to precisely estimate nonlinear processes. In financial markets traders' ability to perceive complex patterns and trade against them reduces the strength of these patterns yielding series which, although not completely random, are probably some of the most difficult to forecast of all real world time series.

We argue that even though the case for economic chaos appears weak, the issue is still a very open question for economic research. Many economic questions are concerned with the fraction of observed fluctuations coming from exogenous shocks versus underlying structures in the system. In macroeconomics this would be related to the amount of business cycle fluctuations attributable to economic structures that are part of the system versus outside shocks. Similarly, in financial markets we are concerned with the amount of price movements and trading activity coming from the flow of new information into the system, versus the system generating this through a dynamic of trading and learning. Analysis along these lines will require a greater attention to economic theory along with new empirical tests which put a great emphasis on how noise is processed in an economic environment.

These issues will be covered in detail in the next three sections. First, the results from macroeconomics will be discussed. Second, financial forecasting will be reviewed. The final section will discuss some of the most interesting directions for future work in economics.

2. Macroeconomics

The most likely candidates for nonlinear dynamics in economics were macroeconomic time series. These series which exhibit a large amount of coherent structure through business cycle fluctuations seemed like natural place to look for unseen determinism.

The first tests for chaotic dynamics in macro economic series used several different diagnostics. Many began the search with an application of the Grassberger–Procacia (1982) dimension estimation algorithm. Others used different nonlinear diagnostics such as bispectral methods developed in Rao & Gabr (1984). Papers such as Ashley & Patterson (1989), Barnett & Chen (1986), Brock & Sayers (1988), Frank & Stengos (1989) all examined several different macro time series. For a more extensive list see Brock *et al.* (1991), Jaditz & Sayers (1993) and Lorenz (1989). Most authors came to the same conclusion. The series were probably not deterministic chaos, but many showed evidence for interesting nonlinear structure. Barnett & Chen (1986) caused some controversy when they claimed chaos had been found in a monetary index. However, their findings were disputed in Ramsey *et al.* (1990) which concluded that their claims about chaos were premature. The recent results in Barnett & Hinich (1993) still find strong evidence for nonlinearities in these monetary series which are not easily explained.

Initially, traditional invariants such as information dimension, entropy, or Lyapunov exponents were estimated. Eventually, some of the studies included a test statistic for independence, the BDS test (Brock *et al.* 1988). This statistic tested the independence of a time series using the fact that for any independent series, x_t,

$$P(d^m(x_t, x_s) < \epsilon) = P(d^1(x_t, x_s) < \epsilon)^m,$$

$$d^m(x_t, x_s) = \max_{j=0,m-1} |x_{t+j} - x_{s+j}|.$$

$P()$ is the probability. This tests the much more restrictive null hypothesis that the series is independent and identically distributed. It is not a test for chaos. It is useful because it is a well defined, and easy to apply test which has power against any type of structure in a series. This feature can be viewed as both a cost and a benefit. On the one hand it can detect many types of nonlinear dependence that might be missed by other tests. On the other hand, rejection using this test is not very informative. One extension of this test is to use it as a residual diagnostic. The idea is that a certain linear or nonlinear model specification can be tested by looking at

$$\hat{e}_t = x_t - f(x_{t-1}, \ldots, x_{t-k}),$$

where $f()$ is some model specification. The diagnostic test for independence is then applied to $\hat{e}_t$, the estimated model residuals. There is a well defined asymptotic distribution theory for the BDS test which is worked out in (Brock *et al.* 1988), which can be applied to model residuals as well as raw series. However, using residuals for diagnostic testing should be done with some caution. Brock (1986) suggests that taking linear residuals of chaotic processes may scramble orbits and reduce the power of tests trying to detect them. Brock's conjecture was borne out in a recent paper by Theiler & Eubank (1993).

Most of these series suffer from a major limitation of economic data, short, noisy, and possibly non-stationary time series. The number of points available are probably small by any of the data point requirements that have been previously suggested (Smith 1992; Ruelle 1990). Most macro series range from about 100 to 800 data points, depending on length and frequency. Most of the previously cited studies check the reliability of their results given the small amount of data available by comparing estimates with distributions from simulated stochastic processes. This methodology draws much of its inspiration from bootstrap techniques of Efron (1979). Bootstrap related tests are very similar to those put forth in Theiler *et al.* (1992). In general, this approach has helped keep researchers honest about what they can confidently claim to have found, and it has probably kept down the number of false positive tests of chaos.

A second related branch of research has attempted to directly fit nonlinear specifications to macroeconomic data. This approach has generally been more successful at finding strong evidence for nonlinearities in these series. Papers such as Hamilton (1989), Potter (1990), and Terasvirta & Anderson (1992) find evidence for nonlinear behaviour in aggregate macro time series. They are generally supportive of the conjecture that business cycles behave differently during expansions and contractions, and that there are differences in the impact on future growth from positive and negative shocks today. Two recent papers directly use a nonlinear forecasting framework to evaluate nonlinearities in macro economic time series. Granger *et al.* (1993), and Jaditz & Sayers (1993), find that a nonlinear forecasting framework does not add much in terms of out of sample forecast performance. This may suggest some problems in terms of stability or stationarity of the previously documented results.

3. Finance

Chaos and its implications for forecasting has been even more hotly debated for financial markets. Financial series provide potentially longer and cleaner series on which to do estimation and out of sample testing. Also, the obvious potential monetary gains to forecasting stock price and foreign exchange rate series have drawn a lot of interest. Tests similar to those used for macroeconomic time series have been applied here as well. Frank & Stengos (1988), Hsieh (1989, 1991), Mayfield & Mizrach (1992), Peters (1991), and Scheinkman & LeBaron (1989) are just of few of the many papers looking for nonlinear structure in financial series. The results often find strong evidence for nonlinear dependence, but no convincing evidence for chaotic dynamics. Peters (1991) is one exception since he claims to have found low-dimensional chaos in a monthly S&P series. However, his results are not backed up by any convincing simulations to assess statistical significance.

The issue of whether a financial series is indeed chaotic may not be of great importance to a financial forecaster who is only interested in adjusting dynamic trading strategies according to apparent predictability in time series. The fact that the previously mentioned diagnostics all found evidence for some kind of nonlinear structure should be a tantalizing indicator for financial forecasters, but they don't give much advice on where to look for this predictability. One of the largest deviations from pure randomness in financial series is volatility persistence. Return movements are very hard to forecast, but magnitudes of the movements are predictable. This has led to a large literature on trying to model this phenomenon (Engle 1982; Bollersley *et al.* 1990). The fact that stock returns exhibit this sort of structure alone is somewhat of a mystery, but the puzzle was further strengthened by LeBaron (1992*a*, *b*), who showed that return autocorrelations in the stock and foreign exchange markets were changing depending on an estimate of recent volatility. Financial series followed a process that looked like,

$$r_t = \log(p_t) - \log(p_{t-1}), \quad r_t = f(\sigma_t^2)r_{t-1} + \epsilon_t, \quad \sigma_t^2 = (1/N)\sum_{i=1}^{N} r_{t-i}^2,$$

where p_t is the price at day t, and N is a window over which conditional variance is estimated. It turns out that $f()$ is a decreasing function of conditional variance indicating that local predictability in the series is higher during periods of lower volatility. This phenomenon can be used to achieve some small out of sample improvements in forecasts, especially in weekly foreign exchange series. Besides the importance of improved out of sample forecasts coming from a nonlinear model, this phenomenon shares an interesting property with many chaotic series. The forecastability of the process is not uniform across its range of movement. There may be periods when forecasts are very good, and periods in which forecasting is almost impossible. Economic forecasters should keep this fact in mind when building and evaluating forecasting models.

One final feature of financial forecasting which may be related to nonlinearities concerns the analysis of technical trading rules. These rules are heuristic forecasting methods which traders claim give them an edge in forecasting the movements of financial markets. Once generally accepted as being worthless by much of the academic community several papers have reopened this debate. Papers such as Brock *et al.* (1992), LeBaron (1990), Levich & Thomas (1993), Sweeney (1986), Schulmeister (1987), and Taylor (1993) present evidence that there is predictive power contained in some of the rules used by technical traders. The predictability appears to be greatest for foreign exchange markets where the magnitude of trading profitability makes up for reasonable estimates of the costs of trading in these market. Many of the most successful rules used are related to moving average rules which attempt to follow long range trends. They recommend that a trader buy when the price is above a long range moving average,

$$p_t > \frac{1}{N} \sum_{i=0}^{N-1} p_{t-i},$$

and sell when the price is below.

To summarize these results, there is interesting evidence of potential predictability in many of these series. Before one concludes that there is lots of money to be made forecasting financial series several cautions should be considered. First, the actual implementation of a forecasting rule for trading may involve unforeseen costs, and prices taken from recorded data-sets may not actually be tradeable. Second, taking on some of these dynamic strategies may involve exposure to extensive risks. Large expected returns might be included with a high probability of the strategy losing a considerable amount of money.

This nonlinear forecastability still ignores the original question about chaos in these series. Given the support for some kind of nonlinear structure the question of chaos still appears very interesting. It is possible that identifying chaos for actual returns series may run up against another barrier. The problem here is not the lack of data since for some high frequency financial series millions of data points are available. The problem may be related to how much forecastability can be left around in a financial time series. To estimate a Lyapunov exponent a researcher needs some idea of forecast degradation over a short horizon. This implies that over the shortest horizon good forecasts were available to analyse how their performance drops after several periods have gone by. Forecasts of this high quality may be unreasonable for financial time series.

As an example of what a small amount of predictability means in a financial market the weekly British pound/U.S. dollar foreign exchange series was used. It was sampled every Wednesday from January 1974 through July 1992 at noon New York time. A trader with access to just the correct sign forecast to this series would still have to work hard to claim that the series was chaotic, but the trader would probably not care much, since the daily return to a strategy of borrowing pounds or dollars, and lending in the other currency according to the directional forecast would yield a return of about 1% per week, or 68% compounded over a year. Even if the trader were charged a large transaction cost of 0.5% per trade, the strategy would at worst earn about 30% per year, assuming trades are made every week. If a positive Lyapunov exponent was reliably estimated it would probably imply even greater predictability over the short horizon. It is unlikely that such predictable structure exists for any financial time series.

These are only approximations and conjectures, but they emphasize an important point. For financial series there may be an extremely wide gap between successful nonlinear forecasting, and actual identification of chaotic dynamics. The researcher looking for chaos in some of these series probably should cast the search a little wider than just looking at raw price movements.

4. Toward the future of chaos in economics

Many of the results up till now have been very negative about the presence of chaos in economic time series. This final section turns more upbeat in suggesting several untried and promising paths for the future.

The most obvious extension is to examine multivariate data-sets. Examples of this would include trading volume and returns series from financial markets (LeBaron 1993), GNP product accounts, and
spatial and geographic information on growing economies. These remain relatively uncharted for nonlinear empirical studies in economics.

A second important direction is to have economic theory play a bigger role in nonlinear empirical studies. Much early work has been atheoretic with few connections from empirical results to underlying economic theories. This can be a plus or a minus at times, but in terms of chaotic dynamics the connections drawn from theories to data have been far too rare. The theorist operating in this area will need to use new techniques, since old standards of parameter estimation and diagnostic testing may fail (Geweke 1993). It is probably likely that extensive simulations from economic models will have to be compared with empirical data. However, the features matched may not be those from traditional chaos analysis. There have been a few papers which have ventured to bridge the theory and empirical gap for chaotic dynamics (Chavas & Holt 1991; De Grauwe *et al.* 1993; Mosekilde & Larsen 1988), but the area remains open for further study.

Extending the set of statistical tools used is also a useful direction. Tests that are tuned and adjusted more towards economic questions will be very important. Are business cycle fluctuations driven more by randomness or inherent structures in the economy? Is trade in financial markets self-generating? Continuing to explore these issues along with applications of some of the most recently developed tests such as those in Lee *et al.* (1993), Dechert & Gencay (1992), Nychka *et al.* (1992), Sugihara & May (1990), and Yao & Tong (1994), should be an area for very fruitful research.

Another new area of research is to look more closely at large scale interconnected systems as metaphors for economic behaviour. This complex systems approach is used in Bak *et al.* (1992) and Brock & LeBaron (1993) in modelling some macroeconomic phenomenon. These papers, in modelling the individual components of an economy explicitly, are going after one of the key elements missing in macroeconomics, the connection of micro and macro dynamics. Both these papers are part of a movement to begin to understand some of the macro time series aspects of large scale interconnected economic systems.

In some ways we are only beginning to cut the surface of how to analyse real world data in light of the knowledge about what kinds of dynamics can be generated by nonlinear processes. We have attempted to directly move tests from one field to another in an attempt to gain new insights into our home fields. At times this has brought in fresh ideas, and added to the development of some new diagnostics. However, this transfer should not be the end of the story. We probably should rethink what the important problems were that we were attacking and readjust our searches in these directions. We also need to move closer to understanding and estimating theoretical models for explaining the empirical puzzles that we are interested in.

References

Ashley, R. A. & Patterson, D. M. 1989 Linear versus nonlinear macroeconomics: a statistical test. *Int. Economic Rev.* **30**, 685–704.

Bak, P., Scheinkman, J., Chen, K. & Woodford, M. 1993 Aggregate fluctuations from independent sectoral shocks: self-organized criticality in a model of production and inventory dynamics. *Ricerche Economiche* **47**, 3–30.

Barnett, W. & Chen, P. 1986 The aggregation-theoretic monetary aggregates are chaotic and have strange attractors. In *Dynamic Econometric Modelling: Proc. Third Austin Symp. in Economics* (ed. W. Barnett, E. Berndt & H. White). Cambridge University Press.

Barnett, W. A. & Hinich, M. 1993 Has chaos been discovered with economic data? In *Nonlinear dynamics and evolutionary economics* (ed. R. H. Day & Ping Chen), pp. 254–265. Oxford University Press.

Bollerslev, T., Chou, R. Y., Jayaraman, N. & Kroner, K. F. 1990 ARCH modeling in finance: a review of the theory and empirical evidence. *J. Econometrics* **52**(1), 5–60.

W. A. Brock. Distinguishing random and deterministic systems: abridged version. *J. Economic Theory* **40**, 168–195.

Brock, W. A., Dechert, W. D., Scheinkman, J. A. & LeBaron, B. 1988 A test for independence based on the correlation dimension. Technical report, University of Wisconsin, Madison.

Brock, W. A., Hsieh, D. & LeBaron, B. 1991 *Nonlinear dynamics, chaos, and instability: statistical theory and economic evidence.* Cambridge: MIT Press.

Brock, W. A., Lakonishok, J. & LeBaron, B. 1992 Simple technical trading rules and the stochastic properties of stock returns. *J. Finance* **47**, 1731–1764.

Brock, W. A. & LeBaron, B. 1993 Using structural modelling in building statistical models of volatility and volume of stock market returns. Technical report, University of Wisconsin – Madison.

Brock, W. A. & Sayers, C. L. 1988 Is the business cycle characterized by deterministic chaos? *J. Monetary Economics* **22**, 71–90.

Chavas, J. P. & Holt, M. T. 1991 On nonlinear dynamics: the case of the pork cycle. *Am. J. Agricultural Economics* **73**, 819–828.

De Grauwe, P., Dewachter, H. & Embrechts, M. 1993 *Exchange rate theory: chaotic models of foreign exchange markets.* Oxford: Blackwell.

Dechert, W. D. & Gencay, R. 1992 Lyapunov exponents as a nonparametric diagnostic for stability analysis. *J. appl. Econometrics* **7**, S41–S60.

Efron, B. 1979 Bootstrap methods: another look at the jackknife. *Ann. Statistics* **7**, 1–26.

Engle, R. F. 1982 Autoregressive conditional heteroskedasticity with estimates of the variance of united kingdom inflation. *Econometrica* **50**, 987–1007.

Frank, M. & Stengos, T. 1989 Measuring the strangeness of gold and silver rates of return. *Rev. Economic Stud.* **56**, 553–568.

Frank, M. & Stengos, T. 1988 Some evidence concerning macroeconomic chaos. *J. Monetary Economics* **22**, 423–438.

Geweke, J. 1993 Inferences and forecasting for deterministic non-linear time series observed with measurement error. In *Nonlinear dynamics and evolutionary dynamics* (ed. R. H. Day & P. Chen). Oxford University Press.

Granger, C. W. J., Terasvirta, T. & Anderson, H. M. 1993 Modeling nonlinearity over the business cycle. In *Business cycles, indicators, and forecasting* (ed. J. H. Stock & M. W. Watson), pp. 311–326. University of Chicago Press.

Grassberger, P. & Procaccia, I. 1982 Characterization of strange attractors. *Phys. Rev. Lett.* **50**, 346–349.

Hamilton, J. D. 1989 A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica* **57**, 357–384.

Hsieh, D. 1989 Testing for nonlinear depedence in daily foreign exchange rates. *J. Business* **62**, 339–368.

Hsieh, D. 1991 Chaos and nonlinear dynamics: applications to financial markets. *J. Finance* **46**, 1839–1878.

Jaditz, T. & Sayers, C. 1993*a* Is chaos generic in economic data? *Int. J. Bifurcation Chaos* **3**, 745–755.

Jaditz, T. & Sayers, C. 1993*b* Using out of sample forecasting performance to evaluate model specification. Technical Report, Bureau of Labor Statistics, Washington, D.C.

LeBaron, B. 1990 Technical trading rules and regime shifts in foreign exchange. Technical Report, University of Wisconsin – Madison.

LeBaron, B. 1992*a* Some relations between volatility and serial correlations in stock market returns. *J. Business* **92**.

LeBaron, B. 1992*b* Forecast improvements using a volatility index. *J. appl. Econometrics* **7**, S137–S150.

LeBaron, B. 1993 The joint dynamics and stability of stock prices and volume. Technical Report, The University of Wisconsin – Madison.

Lee, T., White, H. & Granger, C. W. 1993 Testing for neglected nonlinearity in time series models: a comparison of neural network methods and alternative tests. *J. Econometrics* **56**, 269–290.

Levich, R. M. & Thomas, L. R. 1993 The significance of technical trading-rule profits in the foreign exchange market: a bootstrap approach. *J. Int. Money Finance* **12**, 451–474.

Lorenz, H. W. 1989 *Nonlinear dynamical economics and chaotic motion.* Berlin: Springer-Verlag.

Mayfield, E. S. & Mizrach, B. 1992 On determining the dimension of real-time stock price data. *J. Business Economic Statist.* **10**, 367–374.

Mosekilde, E. & Larsen, E. R. 1988 Deterministic chaos in the beer production–distribution model. *System Dynamics Rev.* **4**, 131–147.

Nychka, D., Ellner, S., Gallant, A. R. & McCaffrey, D. 1992 Finding chaos in noisy systems. *Jl R. statist. Soc.* B **52**, 399–426.

Peters, E. 1991 A chaotic attractor for the s+p 500. *Financial Analysts J.*, pp. 55–81 (March–April).

Potter, S. 1990 Nonlinear time series and economic fluctuations. PhD thesis, University of Wisconsin.

Ramsey, J. B., Rothman, P. & Sayers, C. L. 1990 The statistical properties of dimension calculations using small data sets: some economic applications. *Int. Economic Rev.* **31**, 991–1020.

Subba Rao, T. & Gabr, M. M. 1984 Introduction to bispectral analysis and bilinear time series models. Details? Publisher?

Ruelle, D. 1990 Deterministic chaos: the science and fiction. *Proc. R. Soc. Lond.* A **427**, 241–248.

Scheinkman, J. A. & LeBaron, B. 1989 Nonlinear dynamics and stock returns. *J. Business* **62**, 311–338.

Schulmeister, S. 1987 An essay on exchange rate dynamics. Technical Report, Wissenschaftszentrum Berlin fur Sozialforschung, Berlin, Germany.

Smith, R. L. 1992 Estimating dimension in noisy chaotic time series. *Jl R. statist. Soc.* B **54**, 329–351.

Sugihara, G. & May, R. M. 1990 Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series. *Nature, Lond.* **344**, 734–741.

Sweeney, R. J. 1986 Beating the foreign exchange market. *J. Finance* **41**, 163–182.

Taylor, S. J. 1992 Rewards available to currency futures speculators: compensation for risk or evidence of inefficient pricing? *Economic Record* **68**, 105–116.

Terasvirta, T. & Anderson, H. M. 1992 Characterizing nonlinearities in business cycles using smooth transition autoregressive models. *J. appl. Econometrics* **3**.

Theiler, J. & Eubank, S. 1993 Don't bleach chaotic data. *Chaos* **3**, 771–782.

Theiler, J., Eubank, S., Longtin, A., Galdrikian, B. & Farmer, J. D. 1992 Testing for nonlinearity in time series: the method of surogate data. *Physica* D **58**, 77–94.

Yao, Q. & Tong, H. 1994 Quantifying the influence of initial values on nonlinear prediction. *Jl R. statist. Soc.* B **56**, 301–325.