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Testing for Linear and Nonlinear Granger Causality in the Stock Price-Volume Relation

CRAIG HIEMSTRA and JONATHAN D. JONES*

ABSTRACT

Linear and nonlinear Granger causality tests are used to examine the dynamic relation between daily Dow Jones stock returns and percentage changes in New York Stock Exchange trading volume. We find evidence of significant bidirectional nonlinear causality between returns and volume. We also examine whether the nonlinear causality from volume to returns can be explained by volume serving as a proxy for information flow in the stochastic process generating stock return variance as suggested by Clark's (1973) latent common-factor model. After controlling for volatility persistence in returns, we continue to find evidence of nonlinear causality from volume to returns.

THIS ARTICLE USES LINEAR and nonlinear Granger causality tests to examine the dynamic relation between daily aggregate stock prices and trading volume. Causality tests can provide useful information on whether knowledge of past stock price movements improves short-run forecasts of current and future movements in trading volume, and vice versa. We provide empirical support for the argument made by Gallant, Rossi, and Tauchen (1992) that more can be learned about the stock market through studying the joint dynamics of stock prices and trading volume than by focusing only on the univariate dynamics of stock prices. In addition, our analysis produces stylized facts about how daily aggregate stock prices and trading volume are intertemporally related, which may prove useful to future theoretical and empirical work on the stock market.

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Most of the empirical work on the stock price-volume relation focuses on the contemporaneous relation between trading volume and stock returns (see Karpoff (1987)). Studies that explicitly test for causality between stock prices and trading volume (Rogalski (1978), Smirlock and Starks (1988), Jain and Joh (1988), and Antoniewicz (1992)) rely exclusively on traditional linear Granger causality tests. Although such tests have high power in uncovering linear causal relations, their power against nonlinear casual relations can be low (see Baek and Brock (1992a) and Hiemstra and Jones (1993)). For this reason, traditional Granger causality tests might overlook a significant nonlinear relation between stock returns and trading volume.

In this article, we use both linear and nonlinear causality tests to study daily Dow Jones stock returns and percentage changes in New York Stock Exchange (NYSE) trading volume. Consistent with the empirical work of Kim, Nelson, and Startz (1991), who document a structural break in the generating mechanism for aggregate stock returns at the end of 1946, the causality tests are conducted over the 1915 to 1946 and 1947 to 1990 periods. We employ the traditional Granger test to investigate the presence of linear predictive power between stock prices and trading volume. The nonlinear Granger causality test used here is based on nonparametric estimators of temporal relations within and across time series. It is a modified version of Baek and Brock's (1992a) nonlinear Granger causality test. The modified test relaxes Baek and Brock's assumption that the time series to which the test is applied are mutually independent and individually independent and identically distributed. Instead, it allows each series to display weak (or short-term) temporal dependence. When applied to the residuals of vector autoregressions, the modified Baek and Brock test can be used to determine whether nonlinear dynamic relations exist between given time series.

The importance of testing for both linear and nonlinear Granger causality between stock prices and trading volume is illustrated by our results. The traditional linear Granger test detects unidirectional Granger causality from stock returns to trading volume. In contrast, the modified Baek and Brock test provides evidence of significant nonlinear bidirectional Granger causality between stock returns and trading volume in both sample periods. These results illustrate the promising nature of the Baek and Brock approach to causality testing as a specification tool for uncovering significant nonlinearities in the dynamic interrelationships between economic variables.

We also examine the extent to which the nonlinear predictive power of trading volume for stock returns detected by the modified Baek and Brock test can be attributed to volume serving as a proxy for the daily flow of new information into the market. According to Clark's (1973) mixture of distributions model, information flow is a latent common factor that affects both daily stock returns and trading volume. Andersen (1992), among others, notes that the common-factor model provides an explanation for the volatility persistence associated with autoregressive conditional heteroskedasticity (ARCH) in daily stock returns when Clark's i.i.d. (independent and identically distributed) assumption for information flow is relaxed. Therefore, the evidence of nonlinear Granger causality from trading volume to stock returns could be due to simple volatility effects associated with information flow. After filtering the stock returns series with exponential generalized ARCH (EGARCH) models to control for volatility persistence, the modified Baek and Brock test continues to show evidence of significant nonlinear Granger causality from trading volume to stock returns in both sample periods.

The remainder of this article proceeds as follows. In the first section, we discuss possible explanations for the presence of a causal relation between stock prices and trading volume. We also review some recent theoretical and empirical work using heterogeneous trading models that suggest that important insights about the stock market can be learned by allowing for nonlinearities between stock prices and trading volume. Section II briefly presents the notion of Granger causality and traditional linear tests for its presence. Section III presents Baek and Brock's (1992a) nonparametric approach to nonlinear Granger causality testing and the modified version of their test used here. Our application of the linear and nonlinear Granger causality tests to daily aggregate stock returns and trading volume follows in Section IV. In Section V, we examine the extent to which the detected nonlinear causality from trading volume to stock returns can be explained by simple volatility effects in stock returns. Finally, Section VI provides a summary and concludes.

I. The Stock Price-Volume Relation

A. *Explanations for a Causal Stock Price-Volume Relation*

There are several explanations for the presence of a causal relation between stock prices and trading volume. First, the sequential information arrival models of Copeland (1976) and Jennings, Starks, and Fellingham (1981) suggest a positive causal relation between stock prices and trading volume in either direction. In these asymmetric information models, new information flows into the market and is disseminated to investors one at a time. This pattern of information arrival produces a sequence of momentary equilibria consisting of various stock price-volume combinations before a final, complete information equilibrium is achieved. Due to the sequential information flow, lagged trading volume could have predictive power for current absolute stock returns and lagged absolute stock returns could have predictive power for current trading volume.

Tax- and non-tax-related motives for trading are a second explanation. Tax-related motives are associated with the optimal timing of capital gains and losses realized during the calendar year. Non-tax-related motives include window dressing, portfolio rebalancing, and contrarian strategies. Lakonishok and Smidt (1989) show that current volume can be related to past stock price changes due to tax- and non-tax-related trading motives. The dynamic relation is negative for tax-related trading motives and positive for certain non-tax-related trading motives.[1]

A third explanation involves the mixture of distributions models of Clark (1973) and Epps and Epps (1976). These models provide differing explanations for a positive relation between current stock return variance and trading volume. In the mixture model of Epps and Epps (1976), trading volume is used to measure disagreement as traders revise their reservation prices based on the arrival of new information into the market. The greater the degree of disagreement among traders, the larger the level of trading volume. Their model suggests a positive causal relation running from trading volume to absolute stock returns. On the other hand, in Clark's (1973) mixture model, trading volume is a proxy for the speed of information flow, a latent common factor that affects contemporaneous stock returns and volume. There is no true causal relation from trading volume to stock returns in Clark's common-factor model.

Noise trader models provide a fourth explanation for a causal relation between stock returns and trading volume. These models can reconcile the difference between the short- and long-run autocorrelation properties of aggregate stock returns. Aggregate stock returns are positively autocorrelated in the short run, but negatively autocorrelated in the long run. Since noise traders do not trade on the basis of economic fundamentals, they impart a transitory mispricing component to stock prices in the short run. The temporary component disappears in the long run, producing mean reversion in stock returns. A positive causal relation from volume to stock returns is consistent with the assumption made in these models that the trading strategies pursued by noise traders cause stock prices to move. A positive causal relation from stock returns to volume is consistent with the positive-feedback trading strategies of noise traders, for which the decision to trade is conditioned on past stock price movements (see DeLong, Shleifer, Summers, and Waldmann (1990)).

B. Explanations for Nonlinearities Between Stock Prices and Trading Volume

Granger (1989) argues that univariate and multivariate nonlinear models represent the proper way to model a real world that is "almost certainly nonlinear." As noted by Hsieh (1991) and Brock (1993), the recent focus on nonlinear structure in stock price movements is motivated by the richer types of asset behavior that nonlinear models provide researchers. Large stock

[1] One of the tax-related motives for trading predicts a negative lagged relation from stock price changes to volume given that investors have an incentive to realize capital losses before the end of the calendar year, since taxes are paid by most investors on a calendar-year basis. On the other hand, portfolio rebalancing, one of the non-tax-related trading motives, predicts a positive association, since investors who do not hold the market portfolio may well sell those stocks whose prices have risen to restore portfolio diversification. See Lakonishok and Smidt (1989) for further discussion.

price swings and abrupt changes in stock market volatility can only be properly modelled with nonlinear models.

Hinich and Patterson (1985), Scheinkman and LeBaron (1989), Brock, Hsieh, and LeBaron (1991), and Hsieh (1991), among others, document evidence of significant nonlinear dependence in stock returns. Hiemstra and Jones (1992) also find evidence of significant nonlinearities in aggregate trading volume. In addition to significant univariate nonlinear dependence in both series, the causal relation between stock prices and trading volume could also be nonlinear.

Recent theoretical and empirical work in finance has moved away from traditional representative-agent trading models to trading models with heterogeneous agents. This change in focus has produced models in which endogenous volume plays an important role in asset price determination. Some of the work using heterogeneous agent trading models suggests and finds evidence of nonlinear dynamics in the stock price-volume relation. For example, Brock (1993) develops a nonlinear theoretical noise trading model of stock returns and volume in which rapid stock price movements and volatility bursts are related to volume movements across different groups of traders. His model is based on trading behavior where estimation errors made by traders are correlated.

Campbell, Grossman, and Wang (1993) develop a model, with two classes of risk-averse traders, which has implications for the autocorrelation properties of stock returns as a nonlinear function of trading volume. In their model, market makers take the opposite side of liquidity-induced trades only if they are compensated with an increase in expected stock returns. There is an abnormally large increase in volume followed by stock return reversals for such trades. In contrast, for information-related trades, stock prices move to their new equilibrium levels without reversals. Campbell, Grossman, and Wang (1993) find empirical support for their model's prediction that stock return autocorrelations decline with trading volume.

LeBaron (1992) and Duffee (1992) use regression models similar to the one used by Campbell, Grossman, and Wang (1993) to document evidence of significant nonlinear interactions between stock returns and trading volume. LeBaron finds that persistence in aggregate stock returns is directly related to the current rate of change in volume. Duffee finds that the strength of the transitory component in aggregate stock prices is related to trading volume, which is used as a proxy for noise trading behavior.

II. Linear Granger Causality Testing

In this section, we discuss the definition of Granger causality and the basic approach used in previous studies to test for its presence. Because the traditional linear approach is well known, we offer only a brief discussion here. In the next section, we present a detailed discussion of new statistical techniques developed by Baek and Brock (1992a), which can be used to test for nonlinear Granger causality.

A. Definitions

As originally specified, the general formalization of Granger (1969) causality for the case of two scalar-valued, stationary, and ergodic time series $\{X_t\}$ and $\{Y_t\}$ is defined as follows. Let $F(X_t \mid \boldsymbol{I}_{t-1})$ be the conditional probability distribution of X_t given the bivariate information set $\boldsymbol{I}_{t-1}$ consisting of an Lx-length lagged vector of X_t, say $\boldsymbol{X}_{t-Lx}^{Lx} \equiv (X_{t-Lx}, X_{t-Lx+1}, \dots, X_{t-1})$, and an Ly-length lagged vector of Y_t, say $\mathbf{Y}_{t-Ly}^{Ly} \equiv (Y_{t-Ly}, Y_{t-Ly+1}, \dots, Y_{t-1})$. Given lags Lx and Ly, the time series $\{Y_t\}$ does not strictly Granger cause $\{X_t\}$ if:

$$F(X_t \mid \boldsymbol{I}_{t-1}) = F\left(X_t \mid \left(\boldsymbol{I}_{t-1} - \mathbf{Y}_{t-Ly}^{Ly}\right)\right), \qquad t = 1, 2, \dots. \tag{1}$$

If the equality in equation (1) does not hold, then knowledge of past Y values helps to predict current and future X values, and Y is said to strictly Granger cause X. Similarly, a lack of instantaneous Granger causality from Y to X occurs if:

$$F(X_t \mid \boldsymbol{I}_{t-1}) = F(X_t \mid (\boldsymbol{I}_{t-1} + Y_t)), \tag{2}$$

where the bivariate information set is modified to include the current value of Y. If the equality in equation (2) does not hold, then Y is said to instantaneously Granger cause X.

As shown in equations (1) and (2), strict Granger causality relates to the past of one time series influencing the present and future of another time series. Whereas, instantaneous causality relates to the present of one time series influencing the present of another time series. Due to problems in distinguishing between instantaneous causality and instantaneous feedback, we consider only strict Granger causality.[2]

B. Testing

To test for Granger causality in the time domain, previous studies have used the optimality of linear one-step-ahead least squares predictors to justify substituting conditional expectations into the definition in equation (1). Minimum mean square prediction error is used as the criterion to evaluate incremental predictive power. In the bivariate case, the presence of Granger causality is tested by evaluating the predictive power of one time series for another. Because linear least squares predictors are used in implementing the test, the linear approach only tests for causality in the means between economic variables (see Granger and Newbold (1986)).

We use the Granger test in our study. This is a well-known test for bivariate causality, which involves estimating a linear reduced-form vector

[2] See Geweke, Meese, and Dent (1983), Geweke (1984), and Granger and Newbold (1986) for discussion of Granger causality testing procedures and issues relating to measurement errors, aggregation bias, omitted variable bias, and distinguishing between instantaneous causality and instantaneous feedback.

autoregression (VAR):

$$\begin{aligned} X_t &= A(L)X_t + B(L)Y_t + U_{X,t}, \\ Y_t &= C(L)X_t + D(L)Y_t + U_{Y,t}, \qquad t = 1, 2, \ldots, \end{aligned} \tag{3}$$

where $A(L)$, $B(L)$, $C(L)$, and $D(L)$ are one-sided lag polynomials of orders a, b, c, and d, in the lag operator L with roots outside the unit circle and no roots in common. The regression errors, $\{U_{X,t}\}$ and $\{U_{Y,t}\}$, are assumed to be mutually independent and individually i.i.d. with zero mean and constant variance.

To test for strict Granger causality from Y to X in this linear framework, a standard joint test (F- or χ^2-test) of exclusion restrictions is used to determine whether lagged Y has significant linear predictive power for current X. The null hypothesis that Y does not strictly Granger cause X is rejected if the coefficients on the elements in $B(L)$, i.e., B_i ($i = 1, \ldots, b$), are jointly significantly different from zero. Bidirectional causality (or, feedback) exists if Granger causality runs in both directions, in which case, the coefficients on the elements in both $B(L)$ and $C(L)$ are jointly different from zero.[3]

III. Nonlinear Granger Causality Testing

One important problem with the linear approach to causality testing is that such tests can have low power detecting certain kinds of nonlinear causal relations. Brock (1991) presents a simple bivariate nonlinear model to illustrate how linear causality tests, such as the Granger test, can fail to uncover nonlinear predictive power. He uses the following model:

$$X_t = \beta Y_{t-L} \cdot X_{t-M} + \epsilon_t \tag{4}$$

where $\{Y_t\}$ and $\{\epsilon_t\}$ are mutually independent and individually i.i.d. $N(0, 1)$ time series, β denotes a parameter, and L and M denote lag lengths. Note that X_t depends on a past value of Y_t, yet linear tests would incorrectly indicate that there is no lagged dynamic relation between X_t and Y_t, since all autocorrelations and cross correlations are zero.

Baek and Brock (1992a) propose a nonparametric statistical method for uncovering nonlinear causal relations that, by construction, cannot be detected by traditional linear causality tests. Their approach uses the correlation integral, an estimator of spatial probabilities across time, to detect relations between time series. Using their method, nonlinear causal relations have been found between money and income (Baek and Brock (1992a)), aggregate stock returns and macroeconomic factors (Hiemstra and Kramer (1993)), and producer and consumer price indices (Jaditz and Jones (1993)). In this section, we describe the Baek and Brock (1992a) approach to testing for nonlinear Granger causality.

[3] Since hypothesis tests are sensitive to the truncation of the lag polynomials on the dependent and independent variables, care must be taken in choosing the lag lengths. See Thornton and Batten (1985) and Jones (1989) for a discussion and statistical comparison of alternative techniques for setting lag lengths in conducting causality tests. Besides lag length considerations, attention should also be paid to nonstationarities in the data.

A. The Baek and Brock Approach to Causality Testing

Our discussion of the Baek and Brock (1992a) approach begins with a testable implication of the definition of strict Granger noncausality in equation (1). Consider two strictly stationary and weakly dependent time series $\{X_t\}$ and $\{Y_t\}$, $t = 1, 2, \ldots$.[4] Denote the m-length lead vector of X_t by $\boldsymbol{X}_t^m$ and the Lx-length and Ly-length lag vectors of X_t and Y_t, respectively, by $\boldsymbol{X}_{t-Lx}^{Lx}$ and $\boldsymbol{Y}_{t-Ly}^{Ly}$. That is,

$$\begin{aligned} \boldsymbol{X}_t^m &\equiv (X_t, X_{t+1}, \ldots, X_{t+m-1}), \qquad m = 1, 2, \ldots, \quad t = 1, 2, \ldots, \\ \boldsymbol{X}_{t-Lx}^{Lx} &\equiv (X_{t-Lx}, X_{t-Lx+1}, \ldots, X_{t-1}), \\ &\qquad\qquad Lx = 1, 2, \ldots, \quad t = Lx+1, Lx+2, \ldots, \\ \boldsymbol{Y}_{t-Ly}^{Ly} &\equiv (Y_{t-Ly}, Y_{t-Ly+1}, \ldots, Y_{t-1}), \\ &\qquad\qquad Ly = 1, 2, \ldots, \quad t = Ly+1, Ly+2, \ldots. \end{aligned} \tag{5}$$

For given values of m, Lx, and $Ly \geq 1$ and for $e > 0$, Y does not strictly Granger cause X if:

$$\begin{aligned} &\Pr\left(\|\boldsymbol{X}_t^m - \boldsymbol{X}_s^m\| < e \,\middle|\, \|\boldsymbol{X}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx}\| < e, \|\boldsymbol{Y}_{t-Ly}^{Ly} - \boldsymbol{Y}_{s-Ly}^{Ly}\| < e\right) \\ &\qquad = \Pr\left(\|\boldsymbol{X}_t^m - \boldsymbol{X}_s^m\| < e \,\middle|\, \|\boldsymbol{X}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx}\| < e\right), \end{aligned} \tag{6}$$

where $\Pr(\cdot)$ denotes probability and $\|\cdot\|$ denotes the maximum norm.[5] The probability on the LHS of equation (6) is the conditional probability that two arbitrary m-length lead vectors of $\{X_t\}$ are within a distance e of each other, given that the corresponding Lx-length lag vectors of $\{X_t\}$ and Ly-length lag vectors of $\{Y_t\}$ are within e of each other. The probability on the RHS of equation (6) is the conditional probability that two arbitrary m-length lead vectors of $\{X_t\}$ are within a distance e of each other, given that their corresponding Lx-length lag vectors are within a distance e of each other.

B. A Nonparametric Statistical Testing Procedure

To implement a test based on equation (6), it is useful to express the conditional probabilities in terms of the corresponding ratios of joint probabil-

[4] The Baek and Brock approach to nonlinear Granger causality testing uses correlation-integral estimators of spatial probabilities for vector time series. For certain strictly stationary and weakly dependent processes, Denker and Keller (1983) show that correlation-integral estimators are consistent. By definition, weakly dependent processes display short-term temporal dependence, which decays at a sufficiently fast rate. See Denker and Keller (1983, pp. 505–507) for formal discussions of weakly dependent processes and the conditions under which their consistency results hold.

[5] The maximum norm for $\boldsymbol{Z} \equiv (Z_1, Z_2, \ldots, Z_K) \in \mathbb{R}^K$ is defined as $\max(Z_i)$, $i = 1, 2, \ldots, K$. A more general version of the test can be devised by using different scale parameter values, e, corresponding to the lead and lag vectors. The test can also be easily generalized beyond the bivariate case considered here. As will be discussed in Section III.C, when applied to the null errors of vector autoregressions, equation (6) is a condition for nonlinear strict Granger noncausality. In this case, equation (6) holds for all m, Lx, and $Ly \geq 1$ and for all $e > 0$.

ities. Let $C1(m + Lx, Ly, e)/C2(Lx, Ly, e)$ and $C3(m + Lx, e)/C4(Lx, e)$ denote the ratios of joint probabilities corresponding to the LHS and RHS of equation (6). These joint probabilities are defined as,[6]

$$\begin{aligned} C1(m + Lx, Ly, e) &\equiv \Pr\left(\|\boldsymbol{X}_{t-Lx}^{m+Lx} - \boldsymbol{X}_{s-Lx}^{m+Lx}\| < e, \|\boldsymbol{Y}_{t-Ly}^{Ly} - \boldsymbol{Y}_{s-Ly}^{Ly}\| < e\right), \\ C2(Lx, Ly, e) &\equiv \Pr\left(\|\boldsymbol{X}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx}\| < e, \|\boldsymbol{Y}_{t-Ly}^{Ly} - \boldsymbol{Y}_{s-Ly}^{Ly}\| < e\right), \\ C3(m + Lx, e) &\equiv \Pr(\|\boldsymbol{X}_{t-Lx}^{m+Lx} - \boldsymbol{X}_{s-Lx}^{m+Lx}\| < e), \\ C4(Lx, e) &\equiv \Pr(\|\boldsymbol{X}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx}\| < e). \end{aligned} \tag{7}$$

The strict Granger noncausality condition in equation (6) can then be expressed as

$$\frac{C1(m + Lx, Ly, e)}{C2(Lx, Ly, e)} = \frac{C3(m + Lx, e)}{C4(Lx, e)}, \tag{8}$$

for given values of m, Lx, and $Ly \geq 1$ and $e > 0$.

Correlation-integral estimators of the joint probabilities in equation (7) are used to test the condition in equation (8). For the time series of realizations on X and Y, say $\{x_t\}$ and $\{y_t\}$ for $t = 1, 2, \ldots, T$, let $\{\boldsymbol{x}_t^m\}$, $\{\boldsymbol{x}_{t-Lx}^{Lx}\}$, and $\{\boldsymbol{y}_{t-Ly}^{Ly}\}$ denote the m-length lead and Lx-length lag vectors of $\{x_t\}$ and the Ly-length lag vector of $\{y_t\}$ as defined in equation (5). Also, let $I(\boldsymbol{Z}_1, \boldsymbol{Z}_2, e)$ denote a kernel that equals 1 when two conformable vectors $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ are within the maximum-norm distance e of each other and 0 otherwise. Correlation-integral estimators of the joint probabilities in equation (7) can then be written as

$$\begin{aligned} C1(m + Lx, Ly, e, n) &\equiv \frac{2}{n(n-1)} \sum\sum_{t<s} I(\boldsymbol{x}_{t-Lx}^{m+Lx}, \boldsymbol{x}_{s-Lx}^{m+Lx}, e) \\ &\quad \cdot I\left(\boldsymbol{y}_{t-Ly}^{Ly}, \boldsymbol{y}_{s-Ly}^{Ly}, e\right), \\ C2(Lx, Ly, e, n) &\equiv \frac{2}{n(n-1)} \sum\sum_{t<s} I(\boldsymbol{x}_{t-Lx}^{Lx}, \boldsymbol{x}_{s-Lx}^{Lx}, e) \\ &\quad \cdot I\left(\boldsymbol{y}_{t-Ly}^{Ly}, \boldsymbol{y}_{s-Ly}^{Ly}, e\right), \\ C3(m + Lx, e, n) &\equiv \frac{2}{n(n-1)} \sum\sum_{t<s} I(\boldsymbol{x}_{t-Lx}^{m+Lx}, \boldsymbol{x}_{s-Lx}^{m+Lx}, e), \\ C4(Lx, e, n) &\equiv \frac{2}{n(n-1)} \sum\sum_{t<s} I(\boldsymbol{x}_{t-Lx}^{Lx}, \boldsymbol{x}_{s-Lx}^{Lx}, e), \end{aligned} \tag{9}$$

$$t, s = \max(Lx, Ly) + 1, \ldots, T - m + 1, \quad n = T + 1 - m - \max(Lx, Ly).$$

[6] By definition, the conditional probability $\Pr(A \mid B)$ can be expressed as the ratio $\Pr(A \cap B)/\Pr(B)$. Note that the maximum norm allows us to write $\Pr(\|\boldsymbol{X}_t^m - \boldsymbol{X}_s^m\| < e, \|\boldsymbol{X}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx}\| < e)$ as $\Pr(\|\boldsymbol{X}_{t-Lx}^{m+Lx} - \boldsymbol{X}_{s-Lx}^{m+Lx}\| < e)$.

Using the joint probability estimators in equation (9), the strict Granger noncausality condition in equation (6) can be tested. For given values of m, Lx, and $Ly \geq 1$ and $e > 0$, under the assumptions that $\{X_t\}$ and $\{Y_t\}$ are strictly stationary, weakly dependent, and satisfy the mixing conditions of Denker and Keller (1983), if $\{Y_t\}$ does not strictly Granger cause $\{X_t\}$ then,

$$\sqrt{n}\left(\frac{C1(m+Lx, Ly, e, n)}{C2(Lx, Ly, e, n)} - \frac{C3(m+Lx, e, n)}{C4(Lx, e, n)}\right) \overset{a}{\sim} N(0, \sigma^2(m, Lx, Ly, e)), \tag{10}$$

where $\sigma^2(m, Lx, Ly, e)$ and an estimator for it are given in the Appendix.[7]

C. Testing for Nonlinear Granger Causality

To test for nonlinear Granger causality between $\{X_t\}$ and $\{Y_t\}$, the test in equation (10) is applied to the two estimated residual series from the VAR model in equation (3), $\{\hat{U}_{X,t}\}$ and $\{\hat{U}_{Y,t}\}$. In this case, the null hypothesis is that $\{Y_t\}$ does not nonlinearly strictly Granger cause $\{X_t\}$, and equation (10) holds for all m, Lx, and $Ly \geq 1$ and for all $e > 0$. By removing linear predictive power with a linear VAR model, any remaining incremental predictive power of one residual series for another can be considered nonlinear predictive power (see Baek and Brock (1992a)).

Two issues related to the statistical properties of the test should also be discussed before proceeding. One issue concerns the asymptotic distribution of the test when it is applied to the residuals of VAR models. Baek and Brock (1992b) show that the asymptotic distribution of their variant of the test in equation (10) is the same when it is applied to consistently estimated residuals as when it is applied to the mutually independent and individually i.i.d. errors of the maintained VAR model.[8] Their version of the test is said to be nuisance-parameter-free (NPF) for such models. A similar NPF result has yet to be produced for the modified Baek and Brock test used here. Nonetheless, Hiemstra and Jones (1993) present Monte Carlo evidence that suggests that the modified test is robust to nuisance-parameter problems. They find a close correspondence between the asymptotic and finite-sample statistical properties of the modified test when it is applied to consistently estimated errors corresponding to a given VAR model.

[7] A significantly positive test statistic in equation (10) suggests that lagged values of Y help to predict X, whereas a significant negative value suggests that knowledge of the lagged values of Y confounds the prediction of X. For this reason, we argue that the test statistic in equation (10) should be evaluated with right-tailed critical values when testing for the presence of Granger causality.

[8] Baek and Brock's version of the test is based on the assumption of mutually independent and individually i.i.d. for the errors of the maintained VAR model. The modified test holds under the more general case where the errors are allowed to be weakly dependent. The fundamental difference between the two versions of the test occurs in the estimators for $\sigma^2(m, Lx, Ly, e)$ in equation (10).

Another issue relates to the finite-sample size and power properties of the Baek and Brock testing procedure. Using Monte Carlo simulations, Hiemstra and Jones (1993) find that the modified Baek and Brock test has remarkably good finite-sample size and power properties against a variety of linear and nonlinear Granger causal and noncausal relations.[9] On the other hand, Baek and Brock (1992a) and Hiemstra and Jones (1993) find that the finite-sample size of Baek and Brock's version of the test, which assumes that the series to which the test is applied are mutually independent and individually i.i.d., can be considerably larger than its nominal size. For this reason, we recommend that applications of the test should use either our modified version of the test or Baek and Brock's version of the test using bootstrapped critical values.

IV. Granger Causality Between Aggregate Stock Prices and Trading Volume

In this section, we present our application of the linear and modified Baek and Brock causality tests to aggregate daily stock prices and trading volume. We first discuss the data and sample periods used in the tests. Then we report the results of the tests and offer several caveats in interpreting them.

A. Data and Sample Periods

We compute stock returns from daily closing prices for the Dow Jones Price Index. For the period 1915 to 1940, stock returns are based on the Dow Jones Industrial Average. For the period 1941 to 1990, stock returns are based on the Dow Jones 65 Composite Index.[10] As in Campbell, Grossman, and Wang (1993) and Gallant, Rossi, and Tauchen (1992), the trading volume series is total daily trading volume on the NYSE. The daily stock returns are continuous rates of return, computed as 100 times the first difference of the natural logarithm of the daily stock price, P_t, in successive time periods; that is, $100 \cdot \ln(P_t/P_{t-1})$.[11]

Because causality tests can be sensitive to nonstationarities associated with structural breaks, it is important to study periods when the univariate

[9] The Monte Carlo results are for a variety of bivariate time-series processes and are based on sample sizes of 500 and 1000 observations, a lead length of $m = 1$, common lag lengths of $Lx = Ly = 1, 2, \ldots, 5$, and scale parameter values of $e = 1.5$, 1.0, and 0.5, corresponding to standardized series for X and Y. The Monte Carlo experiments relating to the finite-sample size of the test suggest among other things that the test is robust to the presence of structural breaks in time series and contemporaneous correlation in VAR errors.

[10] For the 1915 to 1918, 1919 to 1928, and the 1929 to 1940 periods, stock returns are based on the Dow Jones 12, 20, and 30 Industrials indices, respectively. The authors thank Harold Mulherin and Mason Gerety for providing the daily stock price and trading volume data for the 1915 to 1987 period. One of the authors updated the data through the end of 1990. For the entire 1915 to 1990 period, the data come from the *Wall Street Journal* and *Barron's*.

[11] The stock return is not a total market return since dividends are not included. In their work on the S & P 500 Index, Gallant, Rossi, and Tauchen (1992) find that their conclusions are not affected by including or excluding dividends in the calculation of stock returns.

and bivariate stochastic processes generating stock prices and trading volume can be considered stationary. To avoid disruptions from the closing of the NYSE at the end of 1914, we omit pre-1915 data. Kim, Nelson, and Startz (1991) document a structural break at the end of 1946 in the stochastic process generating aggregate stock returns. Stock returns over the period up to 1946 display mean reversion, while for the post-1946 period stock returns display persistence.[12] In other work, Jones, Mulherin, and Titman (1992) find that certain measures of noise trading behavior only have significant predictive power for stock market volatility during a pre-1947 period. Based on these results, we examine the dynamic relation between stock returns and trading volume for the 1915 to 1946 and 1947 to 1990 periods.[13]

Although not presented here, the autocorrelation functions for the natural logarithm of daily trading volume in both the 1915 to 1946 and 1947 to 1990 periods display the slow decay characteristic of integrated time series. This autocorrelation pattern suggests that differencing might be the appropriate transformation to make volume stationary.[14] We conduct augmented Dickey-Fuller tests, which indicate an autoregressive unit root in the logarithm of volume for both sample periods. The augmented Dickey-Fuller regressions are estimated with a constant and linear time trend and the data-dependent procedures discussed in Hall (1993) are used to determine the number of augmentation terms.[15] Based on the augmented Dickey-Fuller test results, trading volume is expressed as a percentage change; that is, $100 \cdot \ln(V_t/V_{t-1})$.

We remove systematic day-of-the-week and month-of-the-year calendar effects from stock returns and percentage volume changes using a two-step procedure similar to the one used in Gallant, Rossi, and Tauchen (1992).[16]

[12] See also Fama and French (1988) and Poterba and Summers (1988). Richardson (1993) argues that what appears to be mean reversion in these studies is actually consistent with a random walk model of stock returns.

[13] In addition, Saturday trading ended in the latter part of 1952, minimum brokerage commission rates were deregulated in 1975, and early 1982 witnessed the introduction of stock index futures and index options. These changes also suggest the possibility of a different dynamic relation between stock returns and trading volume between the two periods.

[14] Gallant, Rossi, and Tauchen (1992) detrend the logarithm of volume by regressing it on a quadratic time trend for the period 1928 to 1987 to adjust for the U-shaped pattern of volume over this period. LeBaron (1992) and Campbell, Grossman, and Wang (1993) use a long, moving-average adjustment to detrend volume, expressing it as a deviation from a 100-day moving average of volume.

[15] To determine the number of augmentation terms, we use a general-to-specific data-dependent procedure in Hall (1993). In this procedure, the maximum lag space searched over is set at some arbitrarily large upper bound a priori and a t- or F-test is used to eliminate insignificant augmentation terms. The computed Dickey-Fuller t-statistics for the 1915 to 1946 and 1947 to 1990 periods are -3.36 and -3.68, indicating a failure to reject the unit root null hypothesis at the 95 and 99 percent confidence levels, respectively. The critical values for the test are taken from Fuller (1976, p. 373).

[16] No adjustment is made to account for the Bank Holiday of 1933. Unlike Gallant, Rossi, and Tauchen (1992), we make no adjustment for intramonth effects or gaps between trading days due to weekends and holidays. To account for the possibility of different calendar effects across the two sample periods, we estimate the calendar-adjustment regressions in equations (11) and (12) separately for the two sample periods.

We adjust both the mean and variance of the stock return and volume series for both of these effects. For the returns series the two-step adjustment procedure involves estimating the regression equations,

$$R_t = \boldsymbol{D}_t \beta_R + \epsilon_t \quad \text{(Mean Equation)} \tag{11}$$

$$\ln(\hat{\epsilon}_t^2) = \boldsymbol{D}_t \gamma_R + \nu_t \quad \text{(Variance Equation)} \tag{12}$$

where $\boldsymbol{D}_t$ denotes a vector of daily, monthly, and World War II dummy variables, β_R and γ_R denote conformable parameter vectors, ϵ_t and ν_t denote error terms, and $\hat{\epsilon}_t$ denotes the ordinary least squares (OLS) estimated error in equation (11). Analogous regressions are estimated for percentage volume changes.

For each series, the variance equation (12) is used to standardize the residuals from the mean equation (11). For example, the calendar-adjusted, standardized stock returns are computed as,

$$R_t^{\dagger} = \frac{\hat{\epsilon}_t}{\exp(\boldsymbol{D}_t \hat{\gamma}_R / 2)}, \tag{13}$$

where $\hat{\gamma}_R$ denotes the OLS estimate of γ_R. We use the calendar-adjusted, standardized stock returns, $\{R_t^{\dagger}\}$, and similarly adjusted percentage volume changes, $\{V_t^{\dagger}\}$, in our analysis.

B. Linear Granger Test Results

To implement the linear Granger test, we estimate the VAR model specified in equation (3) with calendar-adjusted stock returns and percentage volume changes. The model can be expressed as,

$$\begin{aligned} R_t^{\dagger} &= A(L)R_t^{\dagger} + B(L)V_t^{\dagger} + U_{R,t}, \\ V_t^{\dagger} &= C(L)V_t^{\dagger} + D(L)R_t^{\dagger} + U_{V,t}, \qquad t = 1, 2, \ldots, T. \end{aligned} \tag{14}$$

We estimate the parameters in each equation with OLS and calculate White (1980) heteroskedasticity-consistent standard errors. To determine appropriate lag lengths for the lag polynomials, we use Akaike's (1974) information criterion.[17] Although not reported, residual diagnostics based on Durbin's (1969) cumulative periodogram test show that the lag lengths on the dependent variable eliminate serial correlation in the residuals.

Table I reports the results of the Granger test. Lag lengths on the dependent and independent variables and computed χ^2-statistics with their marginal significance levels are also reported. Focusing on rejections of the

[17] To determine the optimal univariate lag lengths, we use Akaike's (1974) information criterion to search over a maximum lag space of 40 lags. To determine the optimal bivariate lag lengths, we conduct a search over a maximum lag of 20 lags for the independent variable. The substantial difference in the univariate lag structure for both stock returns and trading volume in the post-1946 period is consistent with a structural break in the stochastic processes generating both series at the end of 1946.

Table I

Linear Granger Causality Test Results

This table reports the results of the linear Granger causality test. *Lr* and *Lv* denote the number of lags on the calendar-adjusted stock returns and percentage volume change series. Both lag lengths are set with the Akaike (1974) information criterion. Sig denotes the marginal significance level of the computed χ^2-statistic used to test the zero restrictions implied by the null hypothesis of Granger noncausality.

H_o: Volume Changes Do Not Cause Stock Returns				H_o: Stock Returns Do Not Cause Volume Changes			
Lr	*Lv*	χ^2	Sig	*Lv*	*Lr*	χ^2	Sig
Panel A: January 1915–December 1946 (No. of Observations = 9,526)							
11	7	13.7	0.06	9	12	71.4	0.00
Panel B: January 1947–December 1990 (No. of Observations = 11,238)							
34	2	2.8	0.25	39	20	91.3	0.00

null hypothesis of Granger noncausality at the 5 percent nominal significance level, the Granger test shows evidence of unidirectional causality from stock returns to percentage volume changes for both the 1915 to 1946 and 1947 to 1990 periods. On the other hand, Granger noncausality from volume changes to stock returns cannot be rejected at 5 percent significance in either period. (The null can be rejected, however, at the 6 percent significance level for the 1915 to 1946 period.) These results contrast sharply with those of the modified Baek and Brock test, which are reported next.

C. Modified Baek and Brock Test Results

To conduct the modified Baek and Brock test, values for the lead length m, the lag lengths Lx and Ly, and the scale parameter e must be chosen. Unlike linear causality testing, there is no literature on the appropriate way to specify optimal values for lag lengths and the scale parameter. On the basis of the Monte Carlo results in Hiemstra and Jones (1993), for all cases, we set the lead length at $m = 1$, and set $Lx = Ly$, using common lag lengths of 1 to 8 lags. In addition, for all cases, the test is applied to standardized series using a common scale parameter of $e = 1.5\sigma$, where $\sigma = 1$ denotes the standard deviation of the standardized time series.[18]

Table II presents the results of the modified Baek and Brock test applied to the estimated VAR residuals, $\{\hat{U}_{R,t}\}$ and $\{\hat{U}_{V,t}\}$, corresponding to stock returns and percentage volume changes in equation (14). There is evidence of bidirec-

[18] To implement the test, each series is standardized so that the two series share a common standard deviation, i.e., $\sigma = 1$, and thereby share a common scale parameter. We also use scale parameter values of 1.0σ and 0.5σ in conducting the test. These results are not reported since they are similar to the results reported in Table II. The complete set of results is available from the authors on request.

Table II

Nonlinear Granger Causality Test Results

This table reports the results of the modified Baek and Brock nonlinear Granger causality test applied to the VAR residuals corresponding to the calendar-adjusted stock returns and percentage volume change series. $Lx = Ly$ denotes the number of lags on the residuals series used in the test. In all cases, the tests are applied to unconditionally standardized series, the lead length, m, is set to unity, and the length scale, e, is set to 1.5. CS and TVAL, respectively, denote the difference between the two conditional probabilities in equation (8) and the standardized test statistic in equation (10). Under the null hypothesis of nonlinear Granger noncausality, the test statistic is asymptotically distributed $N(0, 1)$.

	H_o: Stock Returns Do Not Cause Volume Changes		H_o: Volume Changes Do Not Cause Stock Returns	
$Lx = Ly$	CS	TVAL	CS	TVAL
Panel A: January 1915–December 1946 (No. of Observations = 9,526)				
1	0.0084	7.208	0.0078	6.249
2	0.0170	9.659	0.0127	7.426
3	0.0187	9.448	0.0129	6.662
4	0.0207	9.453	0.0135	6.368
5	0.0214	9.156	0.0144	6.286
6	0.0217	8.723	0.0149	6.112
7	0.0217	8.232	0.0147	5.717
8	0.0225	7.964	0.0140	5.097
Panel B: January 1947–December 1990 (No. of Observations = 11,238)				
1	0.0078	7.124	0.0049	4.600
2	0.0141	9.109	0.0092	6.105
3	0.0165	9.298	0.0111	6.209
4	0.0185	9.112	0.0139	6.761
5	0.0199	9.025	0.0147	6.463
6	0.0228	9.692	0.0164	6.705
7	0.0251	9.767	0.0155	5.948
8	0.0253	9.018	0.0140	5.118

tional nonlinear Granger causality between stock returns and trading volume in both the 1915 to 1946 and 1947 to 1990 periods. This result holds for all the common lag lengths used in conducting the test. None of the standardized test statistics is smaller than 4.60, seemingly strong evidence of nonlinear Granger causality between stock returns and trading volume in both directions.

D. Discussion

It is beyond the scope of our study is determine which of the four general explanations for the presence of a causal relation between stock prices and trading volume described in Section I.*A* are supported by the causality tests. The results can be viewed as being consistent with the predictions of more than one of the competing explanations. For example, both non-tax-related trading models and noise trading models predict a significant causal relation from stock prices to volume. Another example is that causality from trading volume to stock returns is consistent with sequential information arrival models and the mixture of distributions model of Epps and Epps (1976).

The use of the nonlinear causality test is complicated further by the fact that each of the explanations predicts a particular signed causal relation. It is not possible to determine whether significant nonlinear predictive power detected by the modified Baek and Brock test is evidence of positive or negative nonlinear causality. Nonetheless, the substantial difference in the results between the linear and nonlinear Granger causality tests demonstrates the importance of testing for both linear and nonlinear predictive power between economic variables. Moreover, these results illustrate the value of the Baek and Brock statistical methodology as a specification tool in modeling dynamic relationships.

It is interesting to compare the results of the modified Baek and Brock test with those of Gallant, Rossi, and Tauchen (1993). Gallant, Rossi, and Tauchen (1993) use nonlinear impulse response functions to examine the joint dynamics between daily Standard and Poor's 500 Index stock returns and NYSE trading volume over the 1928 to 1987 period. They too find evidence of strong nonlinear impacts from lagged stock returns to current and future trading volume. In contrast to our results, however, only weak evidence of a nonlinear impact from lagged volume to current and future stock returns is detected using their modeling technique. They argue that their results suggest that stock returns can be viewed as being nearly Granger causally prior to trading volume. Our results, showing strong nonlinear Granger causality between stock returns and trading volume in both directions, do not lend themselves to such an interpretation.

V. Volatility Persistence and Nonlinear Causality from Volume to Returns

In this section, we investigate whether the modified Baek and Brock test is influenced by a latent-variable effect associated with information flow that can account for volatility persistence in stock returns. In particular, we examine the extent to which the nonlinear predictive power of trading volume for stock returns can be explained by volume serving as a proxy for daily information flow in the stochastic process generating stock return variance as suggested by Clark's (1973) latent common-factor model.[19]

A. Clark's Common-Factor Model

Letting R_t, V_t, and S_t denote the daily stock return, the level of trading volume, and the latent speed of information flow into the market at time t,

[19] We thank the referee for suggesting that we address this issue and for indicating how to conduct the analysis.

the common factor model can be expressed as

$$\begin{aligned} R_t &= G(S_t) \cdot \epsilon_t \\ V_t &= F(S_t), \end{aligned} \tag{15}$$

where ϵ_t denotes an i.i.d. noise term and $G(\cdot)$ and $F(\cdot)$ denote certain functions. Note in equation (15) that information flow affects the stock return variance, i.e., $\mathrm{var}(R_t) = G^2(S_t) \cdot \mathrm{var}(\epsilon_t)$.

Andersen (1992) notes that the common-factor model in equation (15) provides an explanation for the volatility persistence associated with ARCH dependence in daily stock returns when Clark's i.i.d. assumption for information flow is relaxed. Since Hsieh (1991) finds that much of the nonlinear structure in daily stock returns is related to ARCH dependence, it is possible that the nonlinear Granger causality from trading volume to stock returns could be due to simple volatility effects associated with information flow. As such, if lagged volume captures temporal dependence in the latent speed of information flow, the modified Baek and Brock test could merely be detecting spurious causality from lagged volume to current stock return variance.[20]

We fit one of the family of ARCH models to the aggregate stock returns series to control for volatility persistence. Because our interest lies only in testing the null hypothesis of nonlinear Granger noncausality from volume to stock returns after controlling stock returns for conditional heteroskedasticity, we apply the modified Baek and Brock test to the new VAR residual series for ARCH-filtered stock returns and the residual series for volume from equation (14) used previously.

B. Results of the Adjustment with EGARCH Filtering of Stock Returns

The EGARCH model developed by Nelson (1991) is used to control for ARCH dependence in calendar-adjusted stock returns. The EGARCH(p, q) model is given by,

$$\begin{aligned} R_t^{\dagger} &= \epsilon_t,\ \epsilon_t \mid \boldsymbol{I}_{t-1} \sim \boldsymbol{N}(0, \sigma_t^2), \\ \ln(\sigma_t^2) &= \alpha_0 + \alpha_1 \ln(\sigma_{t-1}^2) + \cdots + \alpha_p \ln\left(\sigma_{t-p}^2\right) \\ &\quad + \beta_1\left[\phi\left(\epsilon_{t-1}/\sqrt{\sigma_{t-1}^2}\right) + \gamma\left(|\epsilon_{t-1}/\sqrt{\sigma_{t-1}^2}| - \sqrt{(2/\pi)}\right)\right] \\ &\quad + \cdots + \beta_q\left[\phi\left(\epsilon_{t-q}/\sqrt{\sigma_{t-q}^2}\right) + \gamma\left(|\epsilon_{t-q}/\sqrt{\sigma_{t-q}^2}| - \sqrt{(2/\pi)}\right)\right], \\ &\qquad t = 1, 2, \ldots, \end{aligned} \tag{16}$$

where $\alpha_0, \ldots, \alpha_p$, $\beta_1, \ldots, \beta_q$, ϕ, and γ denote parameters. In such a model, GARCH effects are parameterized by the α_i parameters, and the asymmetric

[20] Lamoureux and Lastrapes (1990) find that volatility persistence in individual common stock returns can be explained by current trading volume as suggested by Clark's (1973) model. In contrast, Richardson and Smith (1994) do not find much support for Clark's model in daily stock returns using generalized method of moments estimation procedures.

sign- and magnitude-of-error ARCH effects on the conditional variance are respectively modelled by the $\beta_j\phi(\epsilon_{t-j}/\sqrt{\sigma_{t-j}^2})$ and $\beta_j\gamma(|\epsilon_{t-j}/\sqrt{\sigma_{t-j}^2}| - \sqrt{(2/\pi)})$ terms. The EGARCH specification allows negative shocks to affect the conditional variance differently than positive shocks, and therefore is consistent with the conditional skewness displayed by stock returns.

We use the EGARCH model for two reasons. First, Nelson (1991) finds that an EGARCH model adequately captures volatility persistence in daily aggregate stock returns for the 1962 to 1987 period. Second, Nelson (1990) demonstrates that EGARCH models have lognormal conditional variances in continuous time. An implication of this is that, as the sampling interval becomes shorter in discrete time, the distribution of innovations (i.e., squared returns) is a normal-lognormal mixture of distributions as in Clark's model. For these reasons, the EGARCH model seems to be an appropriate ARCH model to use in testing for causality from volume to stock returns in the context of Clark's common-factor model.

We use the Akaike (1974) criterion to determine the lag truncation lengths associated with the GARCH effects. In comparison to Nelson (1991), larger values of p are required to control for persistence in the conditional variance of the stock returns series used here. Based on our specification testing, we fit EGARCH(4,1) and EGARCH(5,1) models to calendar-adjusted stock returns in the 1915 to 1946 and 1947 to 1990 periods, respectively.[21] Table III reports the maximum-likelihood parameter estimates and their asymptotic t-statistics for the two EGARCH models.[22] The table also reports test statistics associated with Engle's (1982) Lagrange multiplier test for remaining ARCH dependence in the standardized estimated residuals. In both sample periods, there is evidence of significant GARCH and sign- and magnitude-of-error effects in daily stock returns. And, the null hypothesis of i.i.d. standardized errors for both EGARCH models is not rejected at the 5 percent nominal significance level by the Engle test.

We estimate the following modified version of the linear regression for stock returns in equation (14) to produce a new residual series for stock returns, namely

$$\left(\epsilon_t/\sqrt{\hat{\sigma}_t^2}\right) = A(L)\left(\epsilon_t/\sqrt{\hat{\sigma}_t^2}\right) + B(L)V_t^{\dagger} + U_{\epsilon,t}, \qquad t = 1, 2, \ldots, T, \quad (17)$$

where $\{\epsilon_t/\sqrt{\hat{\sigma}_t^2}\} = \{R_t^{\dagger}/\sqrt{\hat{\sigma}_t^2}\}$ denotes the conditionally standardized calendar-adjusted stock returns and $\{U_{\epsilon,t}\}$ denotes the VAR regression error.[23] We test for nonlinear Granger causality from volume to stock returns using the

[21] In the specification testing, we use the Akaike (1974) criterion to search over values of p that range from 1 to 6, with $q = 1$.

[22] We use the BFGS algorithm (see Press *et al.* (1988), pp. 324–327) to estimate the parameters.

[23] We use the Akaike (1974) criterion to determine appropriate lag lengths for the lag polynomials $A(L)$ and $B(L)$. For the 1915 to 1946 period, lag lengths of 11 and 1 for the EGARCH(4,1)-filtered stock returns and trading volume are used. For the 1947 to 1990 period, lag lengths of 3 and 2 for the EGARCH(5,1)-filtered stock returns and trading volume are used.

Table III

Estimated EGARCH Models for Stock Returns

This table reports the maximum-likelihood parameter estimates and Berndt *et al.* (1974) estimated asymptotic t-statistics ($\hat{\alpha}$, AT) for EGARCH(4, 1) and EGARCH(5, 1) models given by

$$R_t^{\dagger} = \epsilon_t,\ \epsilon_t \mid \boldsymbol{I}_{t-1} \sim \boldsymbol{N}(0, \sigma_t^2),$$

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \ln(\sigma_{t-1}^2) + \alpha_2 \ln(\sigma_{t-2}^2) + \alpha_3 \ln(\sigma_{t-3}^2) + \alpha_4 \ln(\sigma_{t-4}^2)$$
$$+ \alpha_5 \ln(\sigma_{t-5}^2) + \alpha_6\left(\epsilon_{t-1}/\sqrt{\sigma_{t-1}^2}\right) + \alpha_7\left(|\epsilon_{t-1}/\sqrt{\sigma_{t-1}^2}| - \sqrt{(2/\pi)}\right),$$
$$t = 1, 2, \ldots.$$

for the calendar-adjusted daily stock returns in the 1915 to 1946 and 1947 to 1990 sample periods. Also reported are Engle's (1982) Lagrange multiplier test statistics (LM) for ARCH dependence in the conditionally standardized estimated residuals. The Engle test is based on 12 autocovariances. Under the i.i.d. null hypothesis, the test is asymptotically distributed $\chi^2(12)$. Critical values for the test at the 5 and 1 percent nominal significance levels are 21.03 and 26.22, respectively.

Parameter i	$\hat{\alpha}_i$	AT_i
Panel A: January 1915–December 1946 EGARCH(4, 1) Estimates (No. of Observations = 9,526)		
0	0.0011	0.098
1	0.9059	24.064
2	−0.1240	−2.147
3	0.0201	0.301
4	0.1701	3.925
5	—	—
6	−0.1228	−14.579
7	0.2413	21.471
LM	19.36	
Panel B: January 1947–December 1990 EGARCH(5, 1) Estimates (No. of Observations = 11,238)		
0	0.0031	0.314
1	0.4249	12.840
2	0.2437	6.303
3	0.1713	4.255
4	0.0384	0.843
5	0.0933	2.117
6	−0.1200	−16.945
7	0.2391	27.839
LM	16.02	

modified Baek and Brock test applied to the estimated VAR residuals for conditionally standardized stock returns, $\{\hat{U}_{\epsilon,t}\}$, corresponding to equation (17), and the estimated VAR residuals for volume, $\{\hat{U}_{V,t}\}$, in equation (14).

Table IV reports the results of the nonlinear Granger causality test applied to the EGARCH-filtered stock returns. At 5 percent nominal significance, the modified Baek and Brock test rejects the null hypothesis of strict nonlinear Granger noncausality from trading volume to the EGARCH-filtered stock

Table IV

Nonlinear Granger Causality Test Results with EGARCH-Filtered Stock Returns

This table reports the results of the modified Baek and Brock nonlinear Granger causality test applied to the EGARCH(4, 1)-filtered and EGARCH(5, 1)-filtered stock returns in the 1915 to 1946 and 1947 to 1990 sample periods, respectively. The test is used to test the null hypothesis that volume changes do not nonlinearly Granger cause stock returns using residuals from the VAR specifications with EGARCH-filtered stock returns and volume changes. $Lx = Ly$ denotes the number of lags on the residuals series used in the test. In all cases, the tests are applied to unconditionally standardized series, the lead length, m, is set to unity, and the length scale, e, is set to 1.5. CS and TVAL, respectively, denote the difference between the two conditional probabilities in equation (8) and the standardized test statistic in equation (10). Under the null hypothesis of nonlinear Granger noncausality, the test statistic is asymptotically distributed $N(0, 1)$.

$Lx = Ly$	CS	TVAL
Panel A: January 1915–December 1946 (No. of Observations = 9,526)		
1	0.0016	1.581
2	0.0031	2.054*
3	0.0025	1.394
4	0.0030	1.312
5	0.0047	1.779*
6	0.0057	1.855*
7	0.0066	1.974*
8	0.0047	1.277
Panel B: January 1947–December 1990 (No. of Observations = 11,238)		
1	0.0010	1.087
2	0.0030	2.198*
3	0.0035	2.066*
4	0.0052	2.564**
5	0.0051	2.144*
6	0.0067	2.530**
7	0.0054	1.825*
8	0.0027	0.817

*Significance at the 5 percent nominal level for a one-sided test.
**Significance at the 1 percent nominal level for a one-sided test.

returns in both the 1915 to 1946 and 1947 to 1990 periods for many of the common lag lengths used.[24] However, the EGARCH-filtering of stock returns yields test statistics that are substantially smaller in both magnitude and statistical significance than those corresponding to the case where adjust-

[24] To examine whether the results are sensitive to the order in which the estimation is conducted, we also apply the nonlinear causality test to the estimated standardized residuals from EGARCH models that are fitted to the VAR residuals corresponding to the calendar-adjusted stock returns from equation (14). Using this approach, we find that EGARCH(4,1) models adequately capture volatility persistence in the VAR residual series for stock returns in both sample periods. Since the results of the modified Baek and Brock test produced by this estimation approach are similar to those reported in Table IV, we do not report them here.

ments for ARCH dependence are not made. Note in Table II that the test statistics are significant at the 1 percent level for all the common lag lengths used. In contrast, Table IV shows evidence of nonlinear Granger causality from volume to stock returns that is statistically weaker in both sample periods. The substantial difference in the statistical significance of the two sets of results indicates that the nonlinear causality from calendar-adjusted volume to stock returns detected by the modified Baek and Brock is in large part, but not completely, due to simple volatility effects.

The results reported in Table IV should be interpreted subject to the following caveats. First, the asymptotic distribution of the modified Baek and Brock test statistic might be affected by the use of estimated EGARCH residuals. We are unaware of any NPF theorems relating to applications of the test to estimated EGARCH residuals.[25] Second, the use of an EGARCH model to remove volatility persistence in stock returns might affect underlying structure unrelated to ARCH dependence. As noted by Brock, Hsieh, and LeBaron (1991), the use of misspecified ARCH models can garble structure present in a time series. And finally, Hsieh (1991) argues that more general forms of conditional heteroskedasticity than that associated with the family of ARCH models may better explain volatility persistence in stock returns.

In sum, we argue that the results in this section can be interpreted as evidence that volume has significant nonlinear explanatory power for stock returns over and above that due to simple volatility effects. As such, the bidirectional nonlinear causal relation between stock returns and volume detected by the modified Baek and Brock test in Section IV.*C* cannot be wholly explained by a latent-variable effect associated with information flow. It may be worthwhile for future research to focus on what may account for nonlinear causality from volume to stock returns that is unrelated to volatility persistence in stock returns.

VI. Summary and Conclusion

This article uses linear and nonlinear Granger causality tests to examine the dynamic relation between aggregate daily stock prices and trading volume. We apply the tests to daily Dow Jones stock returns and percentage changes in NYSE trading volume over the 1915 to 1946 and 1947 to 1990 periods. The modified Baek and Brock test provides evidence of significant bidirectional nonlinear Granger causality between stock returns and trading

[25] Brock and Potter (1992) and de Lima (1994) provide NPF results for correlation-integral-based tests applied to GARCH residuals, although their results do not strictly apply to the modified Baek and Brock test. A study of the effects of EGARCH parameter estimation error on the modified Baek and Brock test is beyond the scope of this article. However, Hiemstra and Jones (1993) find from Monte Carlo simulations that the finite-sample rejection rates of the test when applied to the estimated standardized residuals of certain independent EGARCH processes are very close to their corresponding nominal sizes.

volume in both sample periods. We also examine whether the nonlinear causality from volume to stock returns detected by the modified Baek and Brock test could be due to volume serving as a proxy for daily information flow in the stochastic process generating stock return variance. After controlling for simple volatility effects, the modified Baek and Brock test continues to provide evidence of significant nonlinear Granger causality from trading volume to stock returns.

Our results contribute to the empirical literature on the stock price-volume relation by indicating the presence of bidirectional nonlinear Granger causality between aggregate daily stock prices and trading volume. This finding may prove useful to future theoretical and empirical research on the stock market. It suggests that researchers should consider nonlinear theoretical mechanisms and empirical regularities when devising and evaluating models of the joint dynamics of stock prices and trading volume.

Although the nonlinear approach to causality testing presented here can detect nonlinear causal dependence with high power, it provides no guidance regarding the source of the nonlinear dependence. Such guidance must be left to theory, which may suggest specific parameterized structural models. Nonetheless, our results demonstrate the promising nature of the Baek and Brock approach to causality testing as a specification tool for uncovering significant nonlinearities in the dynamic interrelations between time series.

Appendix: The Variance of the Modified Baek and Brock Test

To describe the variance of the modified Baek and Brock test and a consistent estimator for it, we begin by defining the joint probabilities $h1_{C1}(\boldsymbol{x}_{t-Lx}^{Lx+m}, \boldsymbol{y}_{t-Ly}^{Ly}, e)$, $h1_{C2}(\boldsymbol{x}_{t-Lx}^{Lx}, \boldsymbol{y}_{t-Ly}^{Ly}, e)$, $h1_{C3}(\boldsymbol{x}_{t-Lx}^{m+Lx}, e)$, and $h1_{C4}(\boldsymbol{x}_{t-Lx}^{Lx}, e)$, which are conditioned on combinations of the realizations $\boldsymbol{x}_t^m$, $\boldsymbol{x}_{t-Lx}^{Lx}$, and $\boldsymbol{y}_{t-Ly}^{Ly}$, as[26]

$$
\begin{aligned}
h1_{C1}\left(\boldsymbol{x}_{t-Lx}^{m+Lx}, \boldsymbol{y}_{t-Ly}^{Ly}, e\right) &\equiv \Pr\left(\|\boldsymbol{x}_{t-Lx}^{m+Lx} - \boldsymbol{X}_{s-Lx}^{m+Lx}\| < e, \|\boldsymbol{y}_{t-Ly}^{Ly} - \boldsymbol{Y}_{s-Ly}^{Ly}\| < e\right),\\
h1_{C2}\left(\boldsymbol{x}_{t-Lx}^{Lx}, \boldsymbol{y}_{t-Ly}^{Ly}, e\right) &\equiv \Pr\left(\|\boldsymbol{x}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx}\| < e, \|\boldsymbol{y}_{t-Ly}^{Ly} - \boldsymbol{Y}_{s-Ly}^{Ly}\| < e\right),\\
h1_{C3}(\boldsymbol{x}_{t-Lx}^{m+Lx}, e) &\equiv \Pr(\|\boldsymbol{x}_{t-Lx}^{m+Lx} - \boldsymbol{X}_{s-Lx}^{m+Lx}\| < e),\\
h1_{C4}(\boldsymbol{x}_{t-Lx}^{Lx}, e) &\equiv \Pr(\|\boldsymbol{x}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx}\| < e).
\end{aligned} \tag{A1}
$$

[26] As previously noted, we follow the standard convention of denoting random variables in the upper case and their realizations in the lower case. These joint probabilities relate to the probability that arbitrarily selected triplets $(\boldsymbol{X}_s^m, \boldsymbol{X}_{s-Lx}^{Lx}, \boldsymbol{Y}_{s-Ly}^{Ly})$ defined in equation (5) are close to the realized triplets $(\boldsymbol{x}_t^m, \boldsymbol{x}_{t-Lx}^{Lx}, \boldsymbol{y}_{t-Ly}^{Ly})$.

Using equation (A1) and the delta method (Serfling (1980, pp. 122–125)), under the assumption that the underlying series are strictly stationary, weakly dependent, and satisfy the mixing conditions of Denker and Keller (1983), an expression for the variance of the Baek and Brock test in equation (10) is given by

$$\sigma^2(m, Lx, Ly, e) = \boldsymbol{d}\Sigma\boldsymbol{d}', \tag{A2}$$

where

$$\begin{aligned}
\boldsymbol{d} &= [d_i], i = 1, \ldots, 4 \\
&= [1/C2(Lx, Ly, e), -C1(m + Lx, Ly, e)/C2^2(Lx, Ly, e), \\
&\quad -1/C4(Lx, e), C3(m + Lx, e)/C4^2(Lx, e)], \qquad \text{(A3)}\\
\Sigma &= [\Sigma_{i,j}], i, j = 1, \ldots, 4 \\
&= \left[4 \cdot \sum_{k \geq 1} \omega_k E(A_{i,t} \cdot A_{j,t+k-1})\right], \quad \omega_k = \begin{cases} 1, & \text{if } k = 1 \\ 2, & \text{otherwise,} \end{cases} \qquad \text{(A4)}
\end{aligned}$$

$$\begin{aligned}
A_{1,t} &= h1_{C1}\left(\boldsymbol{x}_{t-Lx}^{m+Lx}, \boldsymbol{y}_{t-Ly}^{Ly}, e\right) - C1(m + Lx, Ly, e), \\
A_{2,t} &= h1_{C2}\left(\boldsymbol{x}_{t-Lx}^{Lx}, \boldsymbol{y}_{t-Ly}^{Ly}, e\right) - C2(Lx, Ly, e), \\
A_{3,t} &= h1_{C3}(\boldsymbol{x}_{t-Lx}^{m+Lx}, e) - C3(m + Lx, e), \\
A_{4,t} &= h1_{C4}(\boldsymbol{x}_{t-Lx}^{Lx}, e) - C4(Lx, e),
\end{aligned} \tag{A5}$$

and where E in equation (A4) denotes expected value and the $Ci(\cdot)$ terms are defined in equation (7).

Using the results of Denker and Keller (1983) and Newey and West (1987), a consistent estimator of the $\Sigma_{i,j}$ elements in equation (A2) is given by

$$\begin{aligned}
\hat{\Sigma}_{i,j}(n) &= 4 \cdot \sum_{k=1}^{K(n)} \omega_k(n) \left[\frac{1}{2(n-k+1)} \sum_t \left(\hat{A}_{i,t}(n) \cdot \hat{A}_{j,t-k+1}(n) \right.\right. \\
&\quad \left.\left. + \hat{A}_{i,t-k+1}(n) \cdot \hat{A}_{j,t}(n)\right)\right], \\
t &= \max(Lx, Ly) + k, \ldots, T - m + 1, \\
n &= T + 1 - m - \max(Lx, Ly), \\
K(n) &= (\text{int})n^{1/4}, \\
\omega_k(n) &= \begin{cases} 1, & \text{if } k = 1 \\ 2(1 - [(k-1)/K(n)]), & \text{otherwise,} \end{cases}
\end{aligned} \tag{A6}$$

where

$$\hat{A}_{1,t}(n) = \frac{1}{n-1}\left(\sum_{s\neq t} I(\boldsymbol{x}_{t-Lx}^{m+Lx}, \boldsymbol{x}_{s-Lx}^{m+Lx}, e)\cdot I\left(\boldsymbol{y}_{t-Ly}^{Ly}, \boldsymbol{y}_{s-Ly}^{Ly}, e\right)\right) - C1(m+Lx, Ly, e, n),$$

$$\hat{A}_{2,t}(n) = \frac{1}{n-1}\left(\sum_{s\neq t} I(\boldsymbol{x}_{t-Lx}^{Lx}, \boldsymbol{x}_{s-Lx}^{Lx}, e)\cdot I\left(\boldsymbol{y}_{t-Ly}^{Ly}, \boldsymbol{y}_{s-Ly}^{Ly}, e\right)\right) - C2(Lx, Ly, e, n), \tag{A7}$$

$$\hat{A}_{3,t}(n) = \frac{1}{n-1}\left(\sum_{s\neq t} I(\boldsymbol{x}_{t-Lx}^{m+Lx}, \boldsymbol{x}_{s-Lx}^{m+Lx}, e)\right) - C3(m+Lx, e, n),$$

$$\hat{A}_{4,t}(n) = \frac{1}{n-1}\left(\sum_{s\neq t} I(\boldsymbol{x}_{t-Lx}^{Lx}, \boldsymbol{x}_{s-Lx}^{Lx}, e)\right) - C4(Lx, e, n),$$

$$t, s = \max(Lx, Ly) + 1, \ldots, T - m + 1,$$

and where the $Ci(\cdot, n)$ correlation integrals are defined in equation (9) and the $I(\cdot)$ indicators are described in Section III.B.[27] The $Ci(\cdot, n)$ correlation integrals provide a consistent estimator of $\boldsymbol{d}$ in equation (A3), namely,

$$\hat{\boldsymbol{d}}(n) = [1/C2(Lx, Ly, e, n), -C1(m+Lx, Ly, e, n)/C2^2(Lx, Ly, e, n), -1/C4(Lx, e, n), C3(m+Lx, e, n)/C4^2(Lx, e, n)]. \tag{A8}$$

A consistent estimator for $\sigma^2(m, Lx, Ly, e)$ in equation (10) can then be expressed as

$$\hat{\sigma}^2(m, Lx, Ly, e, n) = \hat{\boldsymbol{d}}(n)\hat{\Sigma}(n)\hat{\boldsymbol{d}}(n)'. \tag{A9}$$

The test statistics reported in Tables II and IV use this variance estimator. See Hiemstra and Jones (1993) for a more detailed discussion of the modified Baek and Brock test and its finite-sample size and power properties.

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