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Chaos in an emerging capital market? The case of the Athens Stock Exchange

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This paper investigates the existence of a deterministic nonlinear structure in the stock returns of the Athens Stock Exchange (Greece), an emerging capital market. The analysis utilizes the concepts of correlation dimension and Kolmogorov entropy, and it also includes a forecasting experiment. Application of the BDS statistical test to raw and filtered returns series suggests the presence of nonlinearities. The findings provide very weak, at best, evidence in support of a nonlinear deterministic data generating process.

I. INTRODUCTION

Numerous studies have investigated the stochastic properties of stock returns of major national stock markets. The obtained empirical evidence suggests that stock returns are not normally distributed but leptokurtic (i.e, fat tailed) and exhibit dependence in the second moments. To account for the non-Gaussianity of stock returns, past research suggested distributions such as stable Paretian and mixtures of normals as a model of speculative prices (Mandelbrot, 1963; Fama, 1965, 1976; Blattberg and Gonedes, 1974; Hsu, 1982; Kon, 1984). The autoregressive conditional heteroscedastic (ARCH) model suggested by Engle (1982) and its various extensions have been proposed as models of conditional volatility of asset returns. The presence of time-dependent conditional heteroscedasticity in returns implies that the empirical distribution of returns is unconditionally leptokurtic. The ARCH family models and their applications are surveyed in Bollerslev *et al.* (1992). Semiparametric forms of conditional volatility have also gained acceptance in recent years (Gallant, 1981; Hamilton, 1989; Hornik *et al.*, 1989a, b; and Kamstra, 1991a, b).

Other researchers have argued that stock returns may not follow a stochastic process but they might be generated by deterministic chaos. Chaos refers to bounded steady-state behaviour that is not an equilibrium point, not periodic, and not quasi-periodic. Certain parameterizations of nonlinear difference equations or systems of at least three nonlinear differential equations can produce chaotic behaviour. The distinctive features of chaotic dynamical systems is a sensitive dependence on initial conditions. This property means that nearby points become exponentially separated in finite time, which makes the evolution of those systems very complex and essentially random by standard statistical tests like the autocorrelation function and spectral density analysis. Combined with measurement limitations of the current state, sensitive dependence places an upper bound on the ability to forecast chaotic systems, even if the model is known with certainty. Only short-term predictability is possible. If a deterministic structure is shown to exist in asset prices, the empirical validity of the weak form market efficiency would be questioned. According to the market efficiency hypothesis in its weak form, asset prices incorporate all relevant information, rendering asset returns unpredictable. The price of an asset determined in an efficient market should follow a martingle process in which each price change is unaffected by its predecessor and has no memory. The presence of deterministic chaos would indicate the possibility of improved short-term, though not long-term predictability. Concerning the US, Scheinkman and LeBaron (1989) reported some evidence of chaos for daily and weekly stock returns. Mayfield and Mizrach (1992) did not, however, find evidence supportive of chaos in real-time stock data. Using nonlinearity tests from chaos theory, Phillipatos *et al.* (1993) could not infer evidence of low-dimensional structure in the stock markets across major economic regions.

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In contrast to major stock markets, the question of a nonlinear structure in smaller markets has received little attention. Outside the world's developed economies, there is a host of emerging capital markets (hereafter ECM) in several developing economies that, in recent years, have attracted a great deal of attention by international corporations and investors that seek to diversify further their assets. The nature of dynamics of stock returns in ECMs is therefore of great interest. It must be noted that ECMs are very likely to exhibit characteristics different from those observed in developed capital markets. Biases due to market thinness and non-synchronous trading should be expected to be more severe in the case of ECMs. Also, in contrast to developed capital markets, which are highly efficient in terms of the speed of information reaching all traders, investors in new capital markets tend to react slowly and gradually to new information. One such emerging capital market is the Greek stock market.

The Greek stock market is represented by the Athens Stock Exchange (hereafter ASE), which has about 220 listings for common and preferred equities as of the end of 1990. Until the beginning of 1987, interest in the ASE was limited to Greek nationals. Then the government freed the capital movement for securities investments which helped the market to take off due to the interest shown by the European Union (EU) and third country investors. This movement was further helped by the government stabilization programme of 1985–1987, which positively affected company profits and created much optimism about their future growth. The market rallied during the first nine months of 1987 resulting in an increase of 1068.27% in the stock index.[1] This rally was interrupted by the October 1987 international stock market crisis. Stock prices fell sharply but, despite the stock prices falling sharply in the last three months of 1987, the stock index enjoyed its highest annual return of over 250% during 1987. The market did not however, manage to overcome the negative effect of the October stock market crisis and, for the next year and a half, foreign investors left the Greek market (the stock index decreased 18.04% in that period). In mid-1989, due to the impressive positive developments that occurred in western economies, especially in EU countries, and to the expectations that the conservative party would return to power, foreign investors returned to Greece and a new rally began. In 1990, the return of a conservative government into power and the expectation towards a more free economy together with the intention of the government to privatize many public companies provided a boost to the market and brought stock prices and trade volume up to record levels. From July 1989 to the beginning of July 1990 the stock index recorded an increase of 613.20%. The rally ended in July 1990 as the market reacted negatively to the Middle East crisis (the Iraqi invasion of Kuwait) and, later on, to the loss of the bid for the government to host the Olympic games in 1996. From July 1990 until the end of 1990 the stock index recorded a decrease of 41.68%.

The Greek authorities are committed to modernizing and liberalizing the ASE in order to increase its efficiency and make it more accessible to international investors. The new reforms that were introduced by the new stock exchange law (L. 1806/88) are expected to affect the market positively and lead to the expansion of its activities. The introduction of new financial instruments, like warrants, options, commercial paper, etc. are currently under way. There is no capital gains tax in Greece.

There is scant evidence of stock market behaviour in the ASE. Previous studies have primarily focused on efficiency and conditional heteroscedasticity issues using standard statistical tests. More specifically, Papaioannou (1982, 1984) reports price dependencies in stock returns for a period of at least six days. Panas (1990) provides evidence of weak-form efficiency for ten large Greek firms. Koutmos *et al.* (1993) find that an exponential generalized ARCH model is an adequate representation of volatility in weekly Greek stock returns. The intertemporal relation between the US and Greek stock markets is analysed in Theodossiou *et al.* (1993).

In this paper, we investigate the possibility of a deterministic nonlinear structure in Greek stock returns which has not been addressed in previous studies. To our knowledge, this is the first attempt to examine the presence of chaotic dynamics in any ECM. We employ the concepts of correlation dimension and Kolmogorov entropy to search for a chaotic structure in the ASE. To do so, we make use of high-frequency (daily) data on a carefully constructed stock index over a ten-year period. We also perform a forecasting experiment.

The plan of the paper is as follows. Section II describes deterministic chaos and its diagnostics, namely, the correlation dimension and Kolmogorov entropy. In Section III we describe the data and present estimates of the diagnostics for chaos using raw and filtered data. A forecasting experiment is performed in Section IV. We conclude in Section V with a summary of our results.

II. DETERMINISTIC CHAOS

Following Mayfield and Mizrach (1992), we study discrete-time autonomous dynamical systems of the form

$$x_t = F(x_{t-1}),\ x \in R^n \qquad (1)$$

where $F: U \rightarrow R^n$ with U an open subset of R^n. A closed invariant set $A \subset U$ is the attracting set for (1) if there exists

[1] See Section III for description of the stock index used here.

an open neighbourhood V of A such that for all $x \in V$ the limit set of iterates of (1) as $t \rightarrow \infty$ is A. A great deal of empirical research has focused on determining the dimension of the attracting set for (1).

In the present application, the vector x_t is thought of as the market. What we get to observe is a scalar signal of the market in the form of a univariate time series of the ASE stock price index

$$y_t = g(x_t) \quad (2)$$

where $g: R^n \rightarrow R$ is an observer function of the market, which is assumed to be continuously differentiable. We therefore observe the time series of stock prices, which is the output of a dynamical system including a certain number of variables and obeying certain dynamical laws. The question is how to recover the market dynamics by analysing the time series y_t. This is accomplished through the Taken's (1980, 1983) embedding theorem. Define an m-dimensional vector constructed from the observed time series

$$y_t^m = (y_t, \ldots, y_{t+m-1}) = (g(x_t), \ldots, g(F^{m-1}(x_t))) \equiv I_m \quad (3)$$

where F^{m-1} is the composition of F with itself $m - 1$ times. Given that the true system that generated the time series is n-dimensional, Taken's embedding theorem states that for smooth pairs (g, F) the map $I_m: R^n \rightarrow R^m$ will be an embedding for $m \geqslant 2n + 1$.

Taken's theorem essentially guarantees that if the embedding dimension m is sufficiently large with respect to the dimension of the manifold on which the attractor lies, the m-dimensional image of the attractor provides a correct topological picture of its dynamics (for example, its dimension and entropy).

Correlation dimension

One important characteristic of a chaotic attractor is its dimension, which is a lower bound on the number of state variables (degrees of freedom) needed to describe the steady-state behaviour. The dimension of a non-chaotic attractor is an integer, and the dimension of a chaotic attractor is almost always a non-integer. Almost all strange attractors are fractals, that is, they possess a non-integer dimension. To estimate the dimension of the reconstructed attractor we use the Grassberger and Procaccia (GP: 1983, 1984) algorithm, which makes use of the idea of the correlation integral.[2] Define a sequence of m-histories of the ASE index,

$$y_t^m = (y_j, \ldots, y_{j+m-1}) \quad (4)$$

that is, the m-dimensional vectors obtained by putting m consecutive observations together. The correlation integral measures the number of a vectors within an ε distance from one another and is given by

$$C_m(\varepsilon) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \times \# \ \{(j, k) \| \ y_j^m - y_k^m \| < \varepsilon\}, \quad m = 2, 3, \ldots, \quad (5)$$

where $\{\Lambda\}$, $\|\cdot\|$, N, and m denote the cardinality of the set Λ, some norm, the number of m histories and the embedding dimension, respectively. As GP showed with $\varepsilon \rightarrow 0$, $C_m(\varepsilon) \sim \varepsilon^v$, where v is the correlation exponent. Therefore for small ε,

$$\ln_2 C_m(\varepsilon) = \ln_2 S + v \ln_2 \varepsilon \quad (6)$$

where S is a constant. The estimate of v as $m \rightarrow \infty$ provides the correlation dimension estimate of the dynamical system. For $m \geqslant 2n + 1$, Brock (1986) showed that the correlation exponent is independent of both the norm used and the embedding m.

The dimension of the dynamical system is determined by first estimating the slope of the regression line of $\ln_2 C_m(\varepsilon)$ on $\ln_2 \varepsilon$ and an intercept for each embedding dimension m. If the data are purely stochastic, the correlation dimension will equal m for all m (the data are space filling). If the data are deterministic, the slope estimates will 'saturate' at some m, not rising any more as m is further increased. This saturation value of the slope is the correlation dimension estimate for the unobserved structure generating the data.

The data for which the correlation dimension method is applied must be stationary. If the data are nearly non-stationary, in phase space the reconstructed attractor will be stretched along a ray, resulting in an underestimation of the true dimension. Since the ASE30 stock prices are non-stationary, we perform the analysis on the rates of return.

To ensure that we truly capture nonlinear structure in the data, we employ the residual and shuffle diagnostic tests. To address the problem of correlated data, Brock (1986) suggested a residual diagnostic test. Consider the model (1), (3) with F possessing a strange attractor. Brock's residual test theorem states that the residuals from a finite-dimensional autoregressive (AR) processes fit to y_t will have the same dimension as y_t. Consequently, we calculate the correlation dimension for the rates of return as well as for AR- and AR-ARCH- transformed rates of return.

Scheinkman and LeBaron (1989) proposal another diagnostic tool – shuffling the data. A shuffling of the original series results in a series without temporal dependence. For a purely random process, shuffling will not affect the dimension, since the shuffled series will also be a purely random process. For a chaotic process of low dimensionality, however, the loss of structure due to shuffling will result in no

[2] Besides correlation dimension, other types of fractal dimension exist: the capacity or Housdorff dimension, information dimension, kth nearest neighbour dimension, and Lyapunov dimension.

saturation of the slope estimates. Therefore if we observe that an actual dimension estimate is less than that of every shuffled series, that would provide evidence in support of a nonlinear deterministic process underlying the data.

Kolmogorov entropy

A distinguishing feature of chaotic processes is sensitive dependence on initial conditions (SDIC): given two distinct initial conditions arbitrarily close to one another, the trajectories emanating from these initial conditions diverge, at a rate characteristic of the system, until for all practical purposes, they become uncorrelated. Besides the dimension, we estimate the Kolmogorov entropy K for the system, which measures the mean rate of creation of information.[3]

The Kolmogorov entropy is a measure of how fast a pair of states become distinguishable to a measuring apparatus with fixed precision under forward iteration (Eckmann and Ruelle, 1985, p. 637). In other words, it quantifies the rate at which indistinguishable paths become distinguishable when the system is observed with only some finite level of accuracy. The extraction of the Kolmogorov entropy from an experimental system is important since this quantity also quantifies 'how chaotic' a system is. A regular trajectory has $K = 0$. A purely random process has $K = -\infty$. A deterministic chaotic process is characterized by a finite K. Grassberger and Procaccia (1984) showed that the vertical change in the position of the invariant portion of the correlation integral over the scaling region in ε is a lower-bound estimate of Kolmogorov entropy. They defined

$$K_{2,m}(\varepsilon) = \frac{1}{\tau} \ln \frac{C_m(\varepsilon)}{C_{m+1}(\varepsilon)} \tag{7}$$

where $C_m(\varepsilon)$ is defined as before and τ is the delay time between observations. GP showed that

$$\lim_{m\to\infty,\, \varepsilon\to 0} K_{2,m}(\varepsilon) \sim K_2 \tag{8}$$

where K_2 is order-2 Renyi entropy, which is a lower-bound estimate of Kolmogorov entropy ($K_2 \leqslant K$).

III. DATA

The data used are Greek daily stock returns based on the closing prices of a value-weighted index comprising the 30 most marketable stocks (during the period 1988–1990) in the Athens Stock Exchange, referred to as ASE30 henceforth. The sample period is 1 January 1981 to 31 December 1990. A main feature of the data set is that prices of individual stocks have been adjusted to reflect any distribution of cash and/or securities, such as cash dividends, stock dividends, etc. Also, stock prices are adjusted for any changes in the firm's capital accounts which cause artificial changes in the associated stock prices. This data set is of relatively high quality, especially compared with the Athens Stock Exchange composite index. The latter index is very prone to biases due to market thinness. High quality data are of great importance in our analysis and we have made every effort to secure them. Figures 1 and 2 provide a graph of the ASE30 index and the corresponding series of returns over the sample period.

The period under analysis is of major importance because it was associated with major changes in the political and economic environment in Greece. First, in the political arena the ruling conservative political party was replaced in government by a socialist party which, in turn, was replaced by the conservative one. Second, during this period Greece became a full member of the EU and undertook many institutional changes in the money and capital markets. These changes affected the investment opportunities of investors and, consequently, the securities risk-return characteristics.

Before we proceed with formal statistical analysis we provide some more evidence regarding the performance of the Greek stock market. 100 drachmas invested on 31 December 1980 in the portfolio of stocks contained in our stock index grows to 6005 drachmas on 31 December 1990 resulting in an average – in terms of geometric mean – annual rate of return of 50.61%. Investors of stocks were subjected to a large standard deviation (88.75%) of the annual rate of return. For comparison purposes the associated geometric mean (standard deviation) for common stocks in the US over the past decade was 13.93% (13.32%). That is, over the period 1981–1990 the average annual rate of stock returns in the ASE was about four times larger than in the US market, while the total risk was about seven times larger. Figure 3 presents the annual stock returns for every year in the sample period. The highest annual return was over 250% in 1987 and the lowest return reached about −35% in 1983.

Table 1 reports the summary statistics for ASE30 returns. The sample mean return is positive and statistically significant at the 1% level. There are significant departures from normality as the series is positively skewed and leptokurtic. Table 2 reports the autocorrelation structure of the returns series and its absolute values at various lag orders. There is evidence of linear dependence as the autocorrelation

[3] A common tool to diagnose the presence of SDIC in a dynamic system is the algorithm by Wolf *et al.* (1985), which calculates the largest Lyapunov exponent of the system (it must be positive for the system to be chaotic). Due to the judgemental nature of the algorithm, we use instead an approximation to Kolmogorov entropy to examine the orbital instability of the system. Recently, Nychka *et al.* (1992) and Gencay and Dechert (1992) proposed alternative algorithms to estimate the Lyapunov exponent(s) of the dynamical system based on the indirect or Jacobian method.

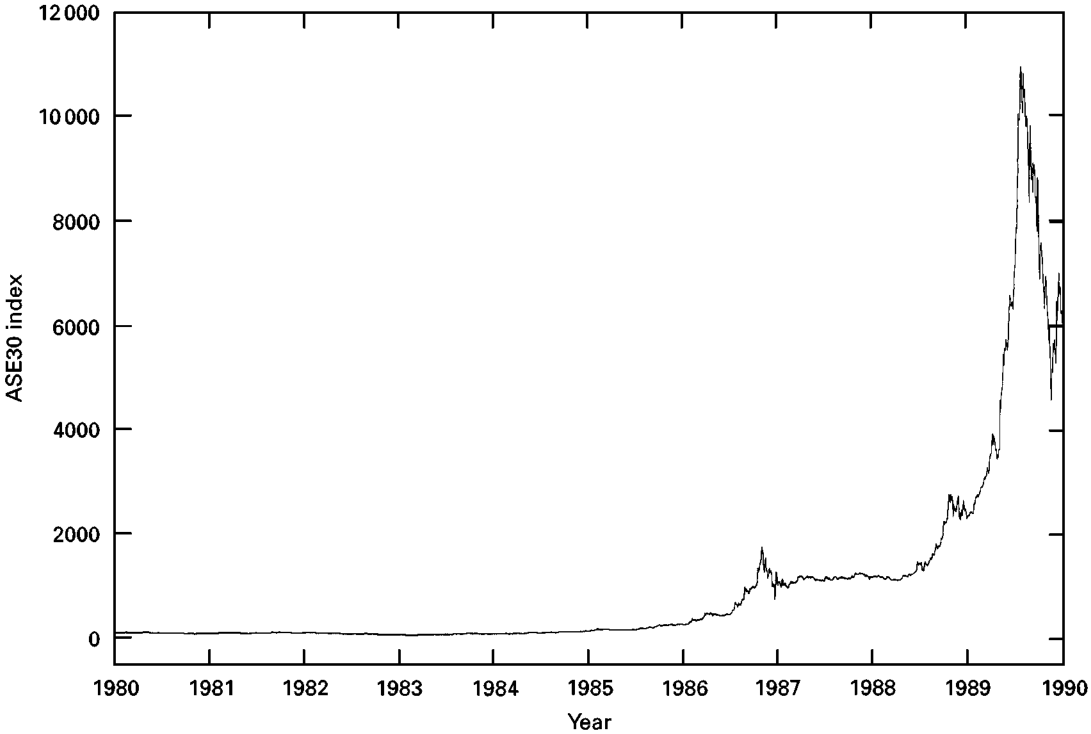

Fig. 1. *ASE30 stock index*

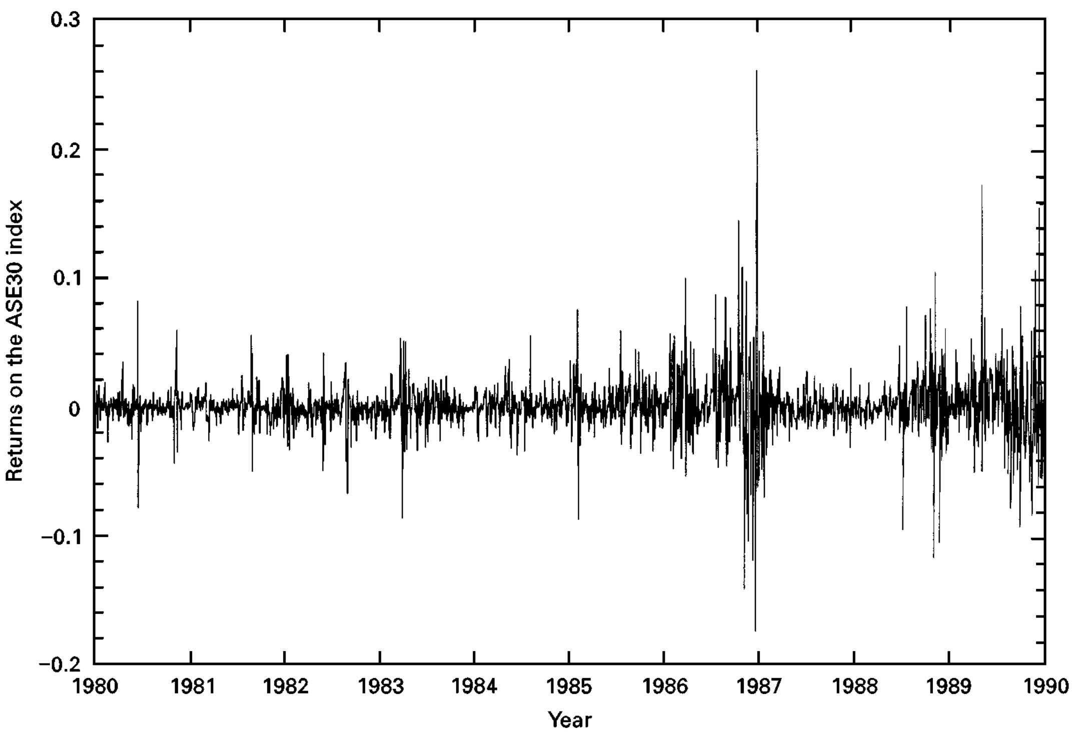

Fig. 2. *Returns series on the ASE30 stock index*

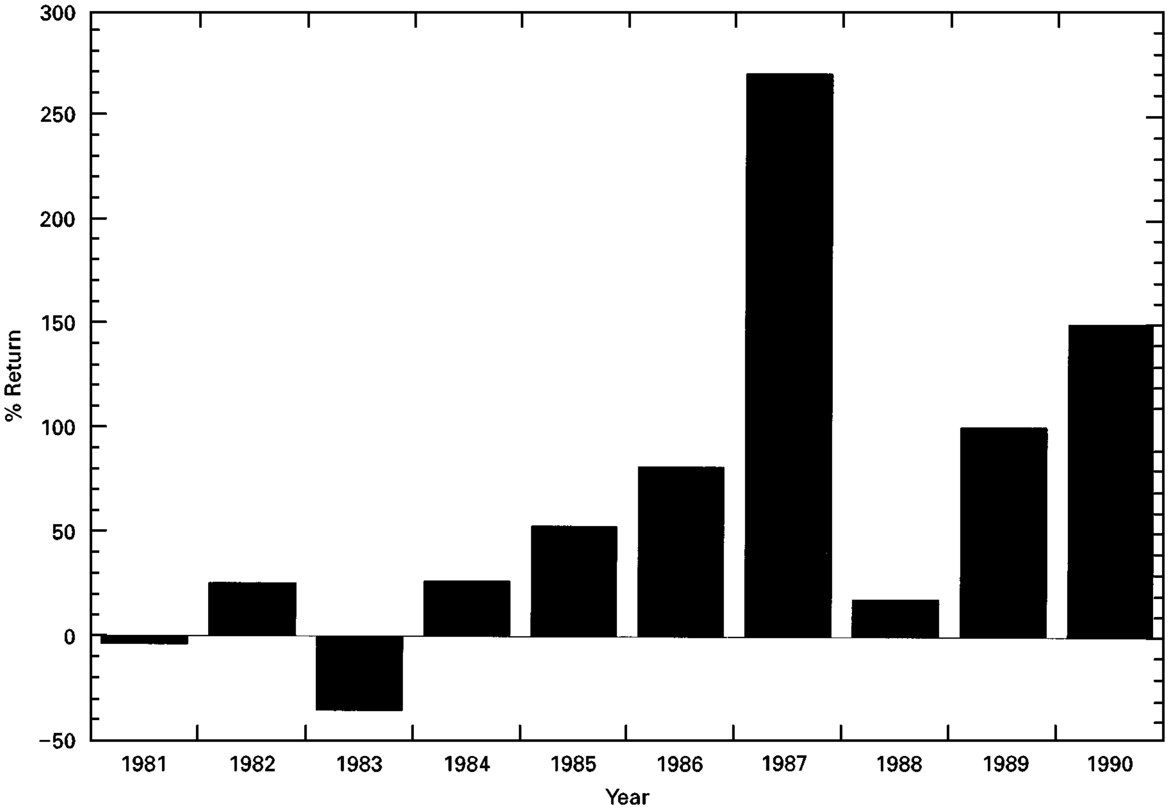

Fig. 3. *Yearly returns on the ASE30 stock index*

Table 1. *Summary statistics of ASE30 returns (1981–1990)*

Statistic	ASE30
Mean	0.00188***
Median	0.000755
Standard deviation	0.02156
Skewness	1.0439***
Kurtosis	19.112***
Minimum	− 0.1725
Maximum	0.2629

***indicates statistical significance at the 1% level.

coefficients at lags one and three are statistically significant (0.281 and − 0.089, respectively). Also, the autocorrelation structure of the absolute values of the returns series reveals the presence of nonlinear dependence, which is consistent with conditional heteroscedasticity and other types of nonlinearity.[4]

Based on these findings and following Brock (1986), we prewhiten the returns series by means of autoregressive (AR) and autoregressive conditionally heteroscedastic (ARCH) models. The return series are prefiltered by the following autoregression:

$$y_t = \beta_0 + \sum_{i=1}^{11} \beta_i y_{t-i} + \beta_M D_{M,t} + \beta_T D_{T,t} + \beta_W D_{W,t} + \beta_R D_{R,t} + \beta_H HOL_t + u_t \tag{9}$$

where $D_{M,t}$, $D_{T,t}$, $D_{W,t}$ and $D_{R,t}$ are dummy variables for Monday, Tuesday, Wednesday, and Thursday, respectively, and HOL_t is the number of holidays (excluding weekends) between two successive trading days.[5] We next model the dependence in the second moment of the return series. An

[4] Application of Engle's (1982) Lagrange Multiplier test for autoregressive conditional heteroscedasticity to the linearly filtered returns series (Equation (9) below) suggests the presence of significant time variation in the second moment of the returns series. These results are not reported here to conserve space but are available upon request from the authors.

[5] Based on the Schwarz information criterion (SIC) we obtained an AR(3) model; however, the residuals from fitting this model were serially correlated. We kept increasing the lag order of the AR model until the estimated residual series did not exhibit any serial dependence for at least 24 lags (corresponding approximately to a one-month period). This process resulted in an AR(11) specification for the returns series; the Q-test statistics (marginal significance levels) for the corresponding residual vector series for orders 15 and 30 are 7.09 (0.95) and 39.25 (0.12), respectively.

Table 2. *Autocorrelations for ASE30 Returns (1981–1990)*

Autocorrelations	Original returns series	Absolute returns series
$\rho(1)$	0.281 (0.051)***	0.515 (0.074)***
$\rho(2)$	− 0.028 (0.052)	0.400 (0.076)***
$\rho(3)$	− 0.089 (0.39)**	0.339 (0.055)***
$\rho(4)$	− 0.076 (0.051)	0.342 (0.075)***
$\rho(5)$	− 0.037 (0.037)	0.295 (0.052)***
$\rho(6)$	0.036 (0.38)	0.325 (0.053)***
$\rho(7)$	0.061 (0.36)	0.291 (0.050)***
$\rho(8)$	0.036 (0.030)	0.232 (0.040)***
$\rho(9)$	0.052 (0.036)	0.265 (0.049)***
$\rho(10)$	0.030 (0.040)	0.282 (0.057)***
$\rho(20)$	0.042 (0.031)	0.238 (0.042)***
$\rho(30)$	0.026 (0.032)	0.190 (0.042)***
$\rho(40)$	0.034 (0.026)	0.153 (0.032)***
$\rho(50)$	− 0.047 (0.027)	0.151 (0.035)***
$Q(20)$	38.32 [0.007]	601.7 [0.00]
$Q(50)$	62.42 [0.111]	1171.9 [0.000]

Statistic $\rho(k)$ is the autocorrelation coefficient at lag k, $Q(k)$ is the heteroscedasticity-adjusted Box–Pierce Q-test statistic for autocorrelation of order k (Diebold, 1986). Heteroscedasticity-consistent standard errors are given in parentheses. Marginal significance levels are given in brackets. ***(**) indicates statistical significance at the 1% (5%) level.

ARCH(14) model is fit to account primarily for the excess kurtosis, and the data are filtered again. The conditional- mean equation for the AR(11)-ARCH(14) model has the same form as in Equation (9), where u_t (conditional on past data) is normally distributed with zero mean and variance h_t, such that

$$h_t = \gamma_0 + \sum_{i=1}^{14} \gamma_i u_{t-i}^2 + \gamma_M D_{M,t} + \gamma_T D_{T,t} + \gamma_W D_{W,t} + \gamma_R D_{R,t} + \gamma_H HOL_t \quad (10)$$

After estimation we standardize the obtained residuals by their estimated conditional standard deviations, that is,

$$z_t = \frac{\hat{u}_t}{\sqrt{\hat{h}_t}} \quad (11)$$

where $\hat{u}_t$ is the residual from the conditional-mean equation and $\hat{h}_t$ is its estimated conditional variance.[6] The Brock residual theorem applies to this series as well.

Our subsequent analysis will therefore be based on the following three series: (i) the original series of returns on the ASE30 index; (ii) the residuals from passing the return series through an AR(11) filter according to Equation (9); and (iii) the standardized AR(11)–ARCH(14) residuals in Equation (11).

To obtain additional evidence regarding the presence of nonlinearities and further motivate our testing for chaotic structure in the ASE, we perform the test suggested by Brock, Dechert and Scheinkman (BDS, 1987) to the returns series and two prefiltered series. The BDS test checks the null hypothesis of independent and identical distribution (i.i.d.) in the data against an unspecified departure from i.i.d. A rejection of the i.i.d. null hypothesis in the BDS test is consistent with some type of dependence in the data, which would result from a linear stochastic system, a nonlinear stochastic system, or a nonlinear deterministic system. Under the null hypothesis, the BDS test statistic asymptotically converges to a standard normal variate. However, the asymptotic distribution is not appropriate when applied to the standardized residuals of ARCH models.[7]

Table 3 reports the BDS test statistics for three sets of data: the returns series and two prewhitened versions created with the autoregressive and autoregressive conditionally heteroscedastic (ARCH) filters in Equations (9) and (10), respectively. We applied the BDS test to these three sets of series for embedding dimensions of $m = 2, 3, 4$ and 5. For each m, ε is set to 0.5, 1.0, 1.5 and 2.0 standard deviations (σ) of the data. The i.i.d. null hypothesis is overwhelmingly rejected in all cases for the returns series. When the BDS test is applied to the AR-filtered series, we still obtain strong rejections of the i.i.d. null hypothesis suggesting that linear dependence in the first moment does not fully account for rejection of i.i.d in the returns series. Strong rejections of the i.i.d. null are also obtained when the BDS test is applied to the standardized residuals from the AR-ARCH model in (10) thus suggesting

[6] More parsimonious ARCH specifications (including specifications with declining weighting schemes) as well as generalized ARCH (GARCH) specifications were also estimated. The ARCH(14) specification was chosen over alternative parameterizations for the conditional-variance equation on the basis of its superior performance on diagnostic tests for serial correlation in the standardized and squared standardized residuals. More specifically, the Q(15) and Q(30) test statistics (marginal significance levels) for the standardized residuals are 22.75 (0.11) and 42.40 (0.06), respectively. The Q(15) and Q(30) test statistics (marginal significance levels) for the squared standardized residuals are 16.36 (0.35) and 20.19 (0.91), respectively. The sum of the ARCH coefficients is 0.95 and the restriction that the ARCH coefficients sum up to unity cannot be rejected: the χ^2 test statistic for this restriction takes the values of 0.01 corresponding to a marginal significant level of 0.91.

[7] Brock's (1986) residual theorem, stating that the asymptotic distribution of the BDS test is not altered by using residuals instead of raw data in linear models, extends to some nonlinear models but not to ARCH models.

Table 3. *BDS test results*

		ASE30 returns series	ASE30 AR-filtered returns series	ASE30 AR-ARCH filtered returns series	Quantiles of the BDS distribution	
Dimension	ε/σ				2.5%	97.5%
$m = 2$	0.5	31.49**	25.49**	2.59*	− 1.84	1.80
	1.0	30.39**	25.14**	2.19**	− 1.56	1.55
	1.5	27.73**	24.07**	1.91§		
	2.0	25.27**	22.75**	1.68§		
$m = 3$	0.5	40.45**	32.95**	3.28**	− 1.72	1.79
	1.0	34.57**	29.63**	2.61**	− 1.31	1.31
	1.5	29.53**	26.60**	2.07§		
	2.0	26.04**	24.27**	1.52§		
$m = 4$	0.5	49.89**	40.36**	3.94**	− 1.80	1.92
	1.0	38.08**	53.04**	2.90**	− 1.19	1.17
	1.5	30.82**	28.27**	2.05§		
	2.0	26.00**	24.75**	1.31§		
$m = 5$	0.5	62.63**	49.63**	5.56**	− 2.05	2.19
	1.0	41.92**	36.72**	3.48**	− 1.07	1.10
	1.5	32.10**	29.84**	2.22§		
	2.0	26.20**	25.34**	1.25§		

The BDS(m, ε) tests for i.i.d., where m is the embedding dimension and ε is distance, set in terms of the standard deviation of the data (σ) to 0.5, 1.0, 1.5 and 2.0 standard deviations. The AR-filtered series is the residual series obtained acording to Equation (9). The AR-ARCH filtered series is the residual series obtained according to Equation (10). The critical values for the BDS test applied to linear series (raw returns and AR-filtered returns series) are the 2.5% and 97.5% quantiles of the standard normal distribution, − 1.96 and 1.96, respectively. The critical values for the BDS test applied to AR-ARCH standardized residuals in the case of $\varepsilon/\sigma = 1.0$ they are approximated by the 2.5% and 97.5% quantiles reported by Brock *et al.* (1991 Table F3, p. 278) on GARCH(1, 1) standardized residuals for 1000 observations; in the case of $\varepsilon/\sigma = 1.0$ they are approximated by the 2.5% and 97.5% quantiles reported by Brock *et al.* (1991, Table F4, p. 279) on GARCH(1, 1) standardized residuals for 2500 observations.** indicates statistical significance at the 5% level. § indicates that the corresponding critical values for the BDS test statistic are not available and no hypothesis testing has therefore been performed. However, given the behaviour of the critical values across various values of ε/σ for each m, rejection of the i.i.d. null hypothesis is almost certain in those cases as well.

the presence of an unspecified omitted structure. The evidence clearly suggests that these data are not simple AR-ARCH processes and the forecastable structure remains even after accounting for dependence in the first and second moments. A potential source of the neglected nonlinearity is the deterministic nonlinear structure and we next test for its presence using the concepts of correlation dimension and Kolmogorov entropy.

IV. EMPIRICAL ESTIMATES

Dimension calculations

We calculate estimates of the correlation dimension over the range of embedding dimensions $m = 1, 2, \ldots, 15$. Results are reported in Table 4. Columns (1) and (2) report the estimates for the ordered time series and shuffled time series, respectively. The average correlation dimension estimates of 20 random draws from the raw and filtered series are reported in column (2). We use a uniform pseudo-random number generator to create our shuffled series, which are constructed by random draws without replacement from the associated original series. The results indicate that the correlation dimension estimates increase very slowly with embedding dimensions for the three ordered series and are well below the theoretical values for a completely random process. However, the levels of dimension estimates do not reach a plateau even though their rate of change with respect to embedding dimension is much less than one and decreasing with embedding dimension, suggesting

Table 4. *Correlation dimension estimates for original and shuffled series*

	ASE30		ARMA		ARCH	
m	(1)	(2)	(1)	(2)	(1)	(2)
1	0.816	0.540	0.757	0.585	0.646	0.675
2	1.478	1.083	1.383	1.171	1.280	1.352
3	2.054	1.622	1.938	1.756	1.901	2.029
4	2.580	2.159	2.459	2.339	2.511	2.707
5	3.063	2.695	2.966	2.922	3.090	3.386
6	3.500	3.231	3.448	3.510	3.643	4.065
7	3.878	3.767	3.885	4.094	4.189	4.749
8	4.226	4.298	4.283	4.679	4.733	5.433
9	3.565	4.827	4.683	5.257	5.291	6.111
10	4.896	5.353	5.060	5.844	5.873	6.790
11	5.164	5.876	5.413	6.430	6.413	7.464
12	5.364	6.402	5.747	7.024	6.934	8.146
13	5.552	6.928	6.086	7.630	7.440	8.837
14	5.711	7.458	6.392	8.235	7.937	9.507
15	5.842	7.967	6.686	8.826	8.429	10.126

m is the embedding dimension. Column (1) reports correlation dimension estimates for the original series. Column (2) reports the average correlation dimension estimates to 20 random draws from the original series.

saturation in that respect.[8] The correlation dimension estimates increase for the AR series and substantially so for the AR-ARCH series. This may suggest that: (i) the series fails to pass Brock's residual test; and (ii) the strange attractor, if it exists, is of relatively high complexity. It must be stressed however, that the noticeable increase in the correlation dimension estimates especially for the AR-ARCH filtered series may be inevitable due to the tremendous filtering the ASE returns series is subjected to, given our small sample.

The data appear to pass the shuffle diagnostic for chaos. For the larger embedding dimensions, the average correlation dimension estimates are higher for the shuffled series. For embedding dimensions greater than seven, the correlation dimension estimate for the ordered series is never greater than the minimum correlation dimension estimate of the 20 shuffled series. The evidence in much stronger for the AR-ARCH series where the shuffling procedure results overwhelmingly in higher dimension estimates for the shuffled series at all embedding dimensions. This suggests that some deterministic nonlinear structure may exist in the data that are lost when the data are randomly reordered.[9]

Based on this evidence, no definite conclusions can be drawn regarding the dimensionality of the system, especially in light of the small sample size. In interpreting the evidence it should be kept in mind that the GP algorithm may produce dimension estimates with substantial upward bias for attractors and with downward bias for random noise (Ramesy and Yuan, 1990; and Ramsey *et al.* 1990). The evidence in support of a strange attractor in the ASE returns is very weak at best. The strange attractor, if it exists, is not of low dimensionality. For comparison purposes, Scheinkman and LeBaron (1989) estimate the correlation dimension for US stock returns to be between 6 and 7.

Kolmogorov entropy estimate

Another way to investigate whether a process is chaotic (rather than multiperiodic or random) is to measure its Kolmogorov entropy. Actually, even if a system possesses low dimensionality, it does not imply that it has a strange attractor; it must also be shown that it has positive entropy. Kolmogorov entropy is a measure of the amount of the disorder in the system, or alternatively, of the information necessary to specify the state of the system. It is zero for

[8] If the time series is a realization of a random process, the slope estimates should increase monotonically with the dimensionality of the space within which the points are contained. In finite data sets, however, stochastic data may give slope estimates which are substantially lower than the embedding dimension *m* and which rise slowly with *m* while chaotic data may not give complete saturation. Consequently, declaring saturation in finite samples can be quite judgemental.

[9] It must be noted however that in finite samples and in the presence of a non-chaotic series with nonlinear structure, randomizing would also cause the series to behave as if it were i.i.d. or more space filling.

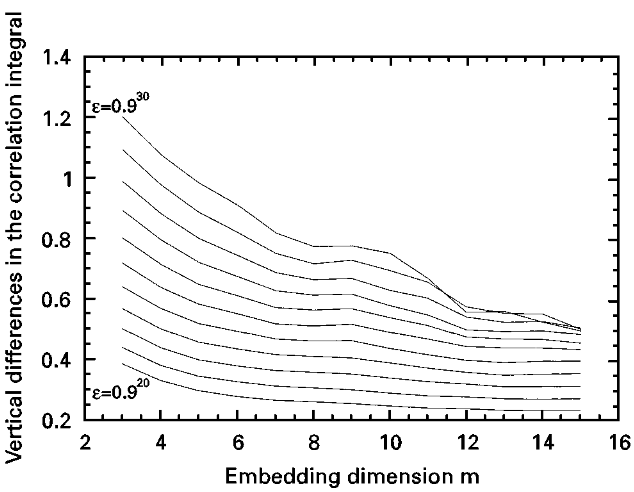

Fig. 4. *Approximation to Kolmogorov entropy*

a stable process, infinite for a completely random process, and finite for the chaotic process. $K_2 > 0$ is a sufficient condition for chaos.

As previously indicated, we calculate the Grassberger and Procaccia (1984) approximation to the Kolmogorov entropy, denoted by K_2. Figure 4 shows a plot of $[\ln_2 C_m(\varepsilon) - \ln_2 C_{m+1}(\varepsilon)]$ versus m over the scaling region for ε ($\varepsilon = 0.9^{20}$ to $\varepsilon = 0.9^{30}$). These quantities settle down to a roughly constant value at low embedding dimensions. As we expected the curves are decreasing, and indeed as m increases and for small values of ε these curves tend to a common value $K_2 \cong 0.50$. Its interpretation is that the rate at which the system processes create or destroy information is 0.50 bits per day. This estimate is a lower bound on the metric entropy and is consistent with the chaos interpretation of the returns series.

For comparison purposes, Mayfield and Mizrach (1992) estimate the entropy for real-time stock data at 0.33 bits per minute. Frank and Stengos (1989) report estimates for K_2 between 0.15 and 0.24 for daily and weekly gold and silver rates of return.

Taken the evidence from all diagnostic tools for chaos together, nonlinear determinism can only very weakly be supported as a representation of the data generating process for the Greek stock returns. The evidence from the correlation dimension technique is very shaky at best, but more encouraging results are provided by the entropy estimate. The dynamical behaviour of the Greek stock returns series is also likely to be the result of an underlying nonlinear stochastic dynamical path, possibly subjected to sporadic shocks.

V. FORECASTING EXPERIMENT

If Greek stock returns are consistent with a deterministic time path, then the potential for short-term predictability arises. As Farmer and Sidorowich (1987) argue 'Ultimately the ability to forecast successfully with deterministic methods may be the strongest test of whether or not low-dimensionality chaos is present'. They suggest a nearest-neighbour forecasting method, which is basically the locally weighted regression (LWR), to predict low-to-moderate dimensionality time series. Hsieh (1991) employed the LWR method to forecast weekly US stock returns. He also argues that 'If stock returns are governed by low complexity chaos, we should be able to use locally weighted regression to forecast returns much better than simple models, such as the random walk. Both Farmer and Sidorowich and Hsieh performed simulation experiments and showed the effectiveness of LWR to forecast in the case of a number of known chaotic maps.

In this section we make an attempt to explore the potential for increased forecasting performance by comparing the in-sample and out-of-sample predictive ability of LWR relative to simple linear models: a random walk with drift (RW) and an AR(11) model. The RW model is the benchmark model. The AR(11) model was earlier found to account adequately for linear dependence in the first moment of the returns series. The LWR estimated here is a nonlinear autoregression of order 9.[10,11] The last 674 observations (one-third of the sample) are reserved for forecasting purposes, the out-of-sample forecasting horizon is one step ahead, and the criteria for forecasting performance are the root mean squared error (RMSE) and mean absolute deviation (MAD).

Tables 5 and 6 report the in-sample and out-of-sample predictive performance of the alternative models of ASE30 returns, respectively. Comparing the performance between the linear models first, we observe that the AR(11) model outperforms the RW model in-sample on both prediction criteria, but only on the MAD criterion out-of-sample.[12] The LWR model significantly outperforms its linear counterparts in-sample and maintains its superior performance out-of-sample on the basis of both RMSE and MAD. The superior performance of the LWR model holds true for the vast majority of window sizes, thus providing evidence of robustness. Not surprisingly, the out-of-sample forecasting improvements of the non-parametric fit are smaller than those achieved in-sample. This evidence compares favourably with the disappointing forecasting performance of the

[10] Experimentation with alternative lag structures for the LWR model produced similar results.
[11] For details on the method of LWR, see Cleveland and Devlin (1988).
[12] This suggests that the correlation structure did not remain constant through time.

Table 5. *In-sample forecasting performance from alternative models for predicting ASE30 returns*

Window size	LWR	AR(11)/LWR	RW/LWR
0.10	15.5015	1.1991	1.2990
	9.6232	1.1146	1.1886
0.20	16.2035	1.1471	1.2427
	9.9652	1.0764	1.1478
0.30	16.6406	1.1170	1.2100
	10.1021	1.0618	1.1322
0.40	16.8896	1.1005	1.1922
	10.1523	1.0565	1.1266
0.50	17.0242	1.0918	1.1828
	10.1844	1.0532	1.1231
0.60	17.2016	1.0806	1.1706
	10.2448	1.0470	1.1165
0.70	17.3759	1.0697	1.1588
	10.3059	1.0408	1.1098
0.80	17.6386	1.0538	1.1416
	10.3692	1.0344	1.1031
0.90	17.9401	1.0361	1.1224
	10.4045	1.0309	1.0993
AR(11)	18.5885		
	10.7268		
RW	20.1365		
	11.4385		

Window size refers to the percentage of total observations which are chosen as nearest neighbours. The first entry of each cell is the root mean squared error (RMSE), while the second is the mean absolute deviation (MAD). LWR stands for locally weighted regression with weights given by $w_{it} = 1 - u$, where $u \equiv \| y_{it} - y_t^* \| / \sum_{i=1}^{q} \| y_{it} - y_t^* \|$. We also tried the tricube weighting function suggested by Cleveland and Devlin (1988) but the above weighting function proved superior empirically. The LWR model is a nonlinear autoregression of order 9. AR stands for the autoregression model. RW stands for random walk (with drift). The smallest RMSE and MAD are underlined. AR(11) [RW]/LWR is the ratio of the forecasting criteria values (RMSE and MAD), obtained from the AR(11) [RW] model, to the ones obtained from the LWR model.

Table 6. *Out-of-sample forecasting performance from alternative models for predicting ASE30 returns*

Windows size	LWR	AR(11)/LWR	RW/LWR
0.10	25.3521	0.9928	0.9877
	15.9590	0.9737	1.0100
0.20	24.7629	1.0164	1.0112
	15.5636	0.9984	1.0356
0.30	24.5873	1.0237	1.0184
	15.3806	1.0103	1.0479
0.40	24.5701	1.0244	1.0191
	15.3324	1.0135	1.0512
0.50	24.6058	1.0229	1.0176
	15.2826	1.0167	1.0546
0.60	24.2989	1.0273	1.0221
	15.2237	1.0207	1.0587
0.70	24.7118	1.0185	1.0133
	15.3008	1.0156	1.0534
0.80	24.8285	1.0137	1.0085
	15.2698	1.0176	1.0555
0.90	24.9728	1.0079	1.0027
	15.2710	1.0176	1.0555
AR(11)	25.1701		
	15.5399		
RW	25.0412		
	16.1187		

See the notes in Table 5 for an explanation of the table.

LWR model in the case of the US stock returns (Hsieh 1991; LeBaron, 1988) or exchange rates (Meese and Rose, 1990, 1991; Mizrach, 1992). The evidence provided by this forecasting experiment may be consistent with a chaos interpretation of the Greek stock returns, but it is not conclusive by any means.

VI. CONCLUSIONS

We apply nonlinear dynamic analysis to a stock price index in an emerging capital market, namely, the Athens Stock Exchange in Greece. Application of the BDS test detects remaining unspecified hidden structure in the Greek stock returns after accounting for dependencies in the first and second moments. To test for chaos we rely on the diagnostic tools of correlation dimension and Kolmogorov entropy, which capture different aspects of deterministic nonlinear behaviour. Consistent with similar studies on major stock markets, we do not find strong evidence in support of a chaotic structure in the Athens Stock Exchange.

Several explanations can be put forward in interpreting the obtained evidence. First, the lack of strong convergence in the correlation dimension estimate is consistent with the presence of stochastic nonlinearities in the data generating process of the Greek stock returns. Also, the potential downward bias in the correlation dimension estimates in small samples for random noise could further provide evidence against a chaos interpretation of the evidence. Second, the evidence could be consistent with the presence of a higher-dimensional strange attractor. Such an eventuality would, however, be of little help as only low-dimensional strange attractors (with dimension estimates less than 5 or 6) could be useful in practice. Finally, an under-appreciated explanation of the evidence obtained here and in other related studies is the one offered by DeCoster and Mitchell (1991). Their simulation study demonstrated the lack of power of the correlation dimension technique to detect the presence of deterministic structure in sample sizes similar to ours when slightly complicated structures are considered.

Consequently, negative evidence on empirical data leaves open the possibility of slightly complex, yet simple enough (relative to the real world), deterministic structures. Overall, the behaviour of Greek stock returns may be consistent with a nonlinear stochastic process and, unless stronger evidence in support of chaos is obtained, we deem the present evidence as being only very weakly supportive of a chaos interpretation.

We suggest that a similar analysis be implemented for other ECMs. Despite temporary setbacks, ECMs will continue to be important conduits for diversification for international investors and corporations. For the development of investment strategies, a complete characterization of stock returns in ECMs is warranted and nonlinear dynamics should be part of it. If the behaviour of ECMs differs, this should be in the investor conditioning information set.

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