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Mean Reversion in Equilibrium Asset Prices

By STEPHEN G. CECCHETTI, POK-SANG LAM, AND NELSON C. MARK*

This paper demonstrates that negative serial correlation in long horizon stock returns is consistent with an equilibrium model of asset pricing. When investors display only a moderate desire to smooth their consumption, commonly used measures of mean reversion in stock prices calculated from historical returns data nearly always lie within a 60 percent confidence interval of the median of the Monte Carlo distributions implied by our equilibrium model. From this evidence, we conclude that the degree of serial correlation in the data could plausibly have been generated by our model. (JEL 313)

Recent research into the behavior of the stock market reports evidence that returns are negatively serially correlated. James Poterba and Lawrence Summers (1988) find that variance ratio tests reject the hypothesis that stock prices follow a random walk, and Eugene Fama and Kenneth French (1988) show that there is significant negative autocorrelation in long-horizon returns.[1] It is well known (see Stephen Leroy, 1973; Robert Lucas, 1978; and Ronald Michener, 1982) that serial correlation of returns does not in itself imply a violation of market efficiency.[2] Nevertheless, there is a tendency to conclude that evidence of mean reversion in stock prices constitutes a rejection of equilibrium models of rational asset pricing. Fama and French suggest this interpretation as a logical possibility, while Poterba and Summers argue that the serial correlation in returns should be attributed to "price fads." In this paper we demonstrate that the serial correlation in returns that is computed from stock market data is consistent with an equilibrium model of asset pricing.

Our approach is to combine the methods of model calibration and statistical inference to critically evaluate the conclusions that can be drawn from the available data. We begin by specifying both an economic model of asset pricing and a stochastic model for the exogenous forcing process driving the economic fundamentals. The forcing process is then calibrated to actual data from the U.S. economy over a long historical period. From this structure, we compute the Monte Carlo distributions of the statistics that previous investigators have used, under the null hypothesis that our rational equilibrium model is true. Finally, we can state the likelihood that those statistics, computed with historical returns, were actually generated from a model in the class that we consider.

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[1]Poterba and Summers find negative serial correlation for stock returns over long horizons using monthly and annual data. Interestingly, Andrew Lo and A. Craig MacKinlay (1988) find that stock returns are positively correlated, using weekly observations.

[2]Sanford Grossman and Robert Shiller (1981) make this same point in showing that the "excess" volatility implied by variance bounds tests can be partly explained by risk aversion in a consumption beta model. More recently, Fischer Black (1988) has discussed the relation between mean reversion and consumption smoothing.

Specifically, we begin with a variant of the Lucas (1978) model of an exchange economy in which the parametric representations for preferences and the stochastic process governing the exogenous forcing variable (i.e., the endowment stream) admit a closed form solution to the asset pricing problem. We assume that the period utility function belongs to the constant relative risk aversion family. For these preferences, the coefficient of relative risk aversion is also the inverse of the elasticity of intertemporal substitution in consumption so that it is not possible to separate agents' tolerance for risk from their desire to have smooth consumption. Since our focus is on the serial correlation in asset returns implied by a model where agents confront an intertemporal consumption/investment decision, we believe that it makes more sense to interpret the concavity of the utility function in terms of the consumption smoothing motive.

The theory provides little guidance as to which time-series (i.e., consumption, output, or dividends) should serve as the endowment and from which to calibrate the model. That is because in the Lucas model, equilibrium consumption equals output, which also equals dividends. Since none of these time-series seem to be more appropriate than the others a priori, we examine each of the three series separately. The stochastic process followed by the growth rate of the endowment is assumed to follow James Hamilton's (1989) Markov switching model. This characterization of the forcing process has two important attributes. First, it allows us to model both the negative skewness and the excess kurtosis that is present in the growth rates of the raw data we employ.[3] And second, the Markov switching model admits a closed form solution to the intertemporal asset pricing problem. Neither of these objectives can be accomplished with standard linear ARIMA models.

The parameters of the Markov switching model are estimated by maximum likelihood employing annual observations on each of the series. Using these estimates, the empirical soundness of the Markov switching model is demonstrated by showing that it matches all three time-series well in the dimension of the mean, variance, skewness, kurtosis, and first-order serial correlation. Furthermore, comparisons of k-step ahead in-sample forecast errors of the Markov switching model with autoregressions reveal roughly similar predictive capabilities.

We then study the measures of mean reversion that have appeared in the literature. These are the variance ratio statistics used by Poterba and Summers and the long-horizon regression coefficients calculated by Fama and French.[4] First we calculate these statistics from historical data on returns drawn from the Standard and Poors' index. The asset pricing model is calibrated by setting the parameters of the endowment process equal to the maximum likelihood estimates. Using the calibrated equilibrium model, we construct Monte Carlo distributions for these statistics. Inferences regarding the model can then be drawn using classical hypothesis testing procedures and the Monte Carlo distributions as the null. We are primarily interested in two hypotheses. The first is the random walk model of stock prices, which is an implication of the Lucas model when agents have linear utility. The second hypothesis is that observed asset prices are determined in equilibrium but agents attempt to smooth their consumption. In this setting, asset returns can be negatively serially correlated even though they rationally reflect market fundamentals.

To summarize our results at the outset, we find that for all return horizons both the variance ratio statistics and the long-horizon regression coefficients calculated from the actual Standard and Poors' returns lie near

[3] The negative skewness of growth rates for many macroeconomic time-series, and hence their asymmetric behavior over the cycle, is also discussed in Salih Neftci (1984).

[4] We might also have examined variance bounds tests. But as John Campbell and Shiller (1988) point out, there is an equivalence between variance ratio tests of the type in Poterba and Summers and variance bounds tests pioneered by Shiller (1981).

the 60 percent confidence band of the median of the Monte Carlo distribution generated under the linear utility (random walk) model. When investors display only a moderate desire to smooth their consumption, these same statistics calculated from the data lie at or near the median of the Monte Carlo distribution. When we test the null hypothesis against a diffuse alternative, we cannot reject the random walk model at the standard 5 percent significance level, but the *p*-values of the test are much higher when the null distribution is generated assuming the utility function is concave. We conclude that much of the serial correlation in historical stock returns can be attributed to small sample bias. However, the serial correlation of returns found in the data better resembles that of the model when the utility function is concave.[5]

The remainder of the paper consists of three sections. Section I presents the solution to the equilibrium asset pricing problem of the Lucas model when agents have constant relative risk aversion preferences, and the endowment process is assumed to follow the Markov switching model. We include in this section the maximum likelihood estimates of the stochastic model, along with an evaluation of the model's performance. Section II describes the Monte Carlo experiments and reports the main results of the paper. The final section offers some conclusions.

I. The Equilibrium Model

A. *A Case of the Lucas Model*

Consider the economy studied by Lucas (1978) in which there are a large number of infinitely lived and identical agents and a fixed number of assets that exogenously produce units of the same nonstoreable consumption good. Let there be K agents and N productive units. Each asset has a single perfectly divisible claim outstanding on it, and these claims are traded in a competitive equity market. The first-order necessary conditions for a typical agents' optimization problem are

$$(1)\quad P_{j,t}U'(C_t) = \beta E_t U'(C_{t+1})\left[P_{j,t+1} + D_{j,t+1}\right], \qquad j = 1,2,\ldots,N,$$

where P_j = the real price of asset j in terms of the consumption good.
$U'(C)$ = marginal utility of consumption, C, for a typical consumer/investor.
β = a subjective discount factor, $0 < \beta < 1$.
D_j = the payoff or dividend from the jth productive unit.
E_t = the mathematical expectation conditioned on information available at time t.

In equilibrium, per capita ownership of asset j is $1/K$. It follows that equilibrium per capita consumption, C, is the per capita claim to the total endowment in that period, $(1/K)\Sigma_{j=1}^{N} D_j$. Now, make this substitution in equation (1) and sum over j to obtain an equilibrium condition involving economy-wide or market prices and quantities on a per capita basis. That is,

$$(2)\quad P_t U'(D_t) = \beta E_t U'(D_{t+1})[P_{t+1} + D_{t+1}],$$

where $P_t \equiv (1/K)\Sigma P_{j,t}$ is the share of the market's value owned by a typical agent and $D_t = (1/K)\Sigma D_{j,t}$. Since each productive unit has only a single share outstanding and the number of productive units are fixed, these are the theoretical value-weighted market indices adjusted for population.

Let preferences be given by constant relative risk aversion utility:

$$U(C) = (1+\gamma)^{-1} C^{(1+\gamma)},$$

where $-\infty < \gamma \leq 0$ is the coefficient of relative risk aversion. Now (2) simplifies to a

[5] Myung Kim, Charles Nelson, and Richard Startz (1988) and Matthew Richardson (1988) have recently examined the issue of small sample bias in these tests of serial correlation in stock returns.

stochastic difference equation that is linear in PD^{γ}. That is,

$$(3)\quad P_t D_t^{\gamma} = \beta E_t P_{t+1} D_{t+1}^{\gamma} + \beta E_t D_{t+1}^{(1+\gamma)}.$$

Iterating (3) forward, the current market value, P_t, can be expressed as a nonlinear function of current and expected future payoffs,

$$(4)\qquad P_t = D_t^{-\gamma} \sum_{k=1}^{\infty} \beta^k E_t D_{t+k}^{\{1+\gamma\}}.$$

To obtain a closed form solution, we must specify the stochastic process governing (D_t), and this is done in Section I, Part C. We will refer to the exogenous forcing variable as dividends in the next two subsections. We do this because it helps to clarify the exposition, not because we restrict our attention to dividends when assessing the performance of the model. In fact, we consider alternatives as well.

B. *Characteristics of the Data*

The theory provides little guidance regarding the appropriate empirical counterpart to the exogenous forcing variable D. Because equilibrium consumption equals output, which also equals dividends, there are three natural variables to serve this role. We consider all three candidate time-series in real, per capita terms:[6] dividends, consumption, and GNP.

[6]The standard procedure in the literature has been to calibrate the endowment process to consumption (for example, Rajnish Mehra and Edward Prescott, 1985; Thomas Reitz, 1988; Shmuel Kandel and Robert Stambaugh, 1988; and George Constantinides, 1988, to name a few). It turns out that our results are robust to the particular time-series to which the endowment process is calibrated, whether it be consumption, dividends, or GNP. Because the variability of consumption, dividends, and GNP are very different from each other, the choice of the time-series to which the model is calibrated will have different implications for other aspects of the model such as the implied size and volatility of returns and the risk premium. The aim of this paper is quite modest, however, in that it seeks only to examine conclusions that can be drawn from serial correlation in returns. We make no claims that our model can match every dimension of the data (see Kandel and Stambaugh, 1988, who undertake a more ambitious project).

To choose an appropriate time-series model for the endowment process, it is useful to know some of the details of the data. Table 1 reports various summary statistics computed for the growth rates of dividends, GNP, and consumption. The data sources are described in the appendix. The following observations emerge from the table. Relative to a normal distribution, growth rates of the raw data are negatively skewed and have excess kurtosis. The coefficient of skewness is a measure of asymmetry, while the coefficient of kurtosis in excess of 3 implies that the distribution of the data is "fat tailed." The negative skewness indicates that, relative to a normal distribution, the data contain too many large negative values or "crashes." As Reitz (1988) suggests in his examination of the equity premium puzzle, these crashes are potentially important for studying the dynamics of asset returns.

The negative skewness is statistically significant at the 5 percent level for consumption and at the 1 percent level for dividends and GNP. Consequently, conventional time-series models, such as linear ARIMA models with Gaussian innovations, will be inappropriate. That is, ARIMA models with normal error terms can never explain nonzero third moments in the distribution of the raw data.

Excess kurtosis is also statistically significant at the 5 percent level for consumption and at the 1 percent level for dividends and GNP. Thus standard linear models will not capture this important characteristic of the data either. While conditionally normal but heteroskedastic models such as Robert Engle's (1982) ARCH model can give rise to fat tails, they cannot model asymmetry.

Our objective is to find a model that captures these important features of the data and at the same time admits a solution to the intertemporal asset pricing problem set forth in Section 1, Part A. Hamilton's (1989) Markov switching model meets both of these criteria.

C. *Hamilton's Markov Switching Model*

Hamilton (1989) has suggested modeling the trends in nonstationary time-series as Markov processes, and has applied this ap-

TABLE 1—SUMMARY STATISTICS FOR GROWTH RATES IN SAMPLE

	Consumption	Dividends	GNP
Mean	0.0184	−0.0038	0.0183
Std. Dev.	0.0379	0.1359	0.0547
Skewness	−0.4097[a]	−0.5979[b]	−0.7574[b]
Coefficient	(0.247)	(0.227)	(0.225)
Kurtosis	3.8750[a]	5.8048[b]	7.6630[b]
Coefficient	(0.495)	(0.455)	(0.451)
Minimum	−0.1044	−0.4673	−0.2667
Maximum	0.0989	0.4056	0.1662
First Autocorrelation	−0.067	0.134	0.390[c]
	(0.101)	(0.093)	(0.092)

Source: See data appendix.
Notes: Standard errors for the skewness and kurtosis coefficients are reported in parentheses and are computed under the null hypothesis that growth rates of the data are distributed as i.i.d. normal. Significance tests are based on E. S. Pearson and H. O. Hartley (1976), Tables 34.B and 34.C.
[a]Significantly different from normal at the 5 percent level.
[b]Significantly different from normal at the 1 percent level.
[c]Greater than two standard errors from zero.

proach to the study of post-World War II real GNP. One of the attractive features of this approach is its ability to model the asymmetry and the leptokurtosis reported in Table 1. Let d_t denote the logarithm of the endowment, D_t. The Markov switching model can be written as

$$(5) \qquad d_t = d_{t-1} + \varepsilon_t + \alpha_0 + \alpha_1 S_{t-1},$$

where $\{\varepsilon_t\}$ is a sequence of independent and identically distributed normal variates with zero mean and variance σ^2, and $\{S_t\}$ is a sequence of Markov random variables that take on values of 0 or 1 with transition probabilities,

$$(6) \qquad \Pr[S_t = 1 | S_{t-1} = 1] = p,$$

$$\Pr[S_t = 0 | S_{t-1} = 1] = 1 - p,$$

$$\Pr[S_t = 0 | S_{t-1} = 0] = q,$$

and $$\Pr[S_t = 1 | S_{t-1} = 0] = 1 - q.$$

The endowment process thus follows a random walk in logarithms ($d_t = d_{t-1} + \varepsilon_t$) with stochastic drift ($\alpha_0 + \alpha_1 S_{t-1}$). As a normalization we restrict α_1 to be negative. The economy is said to be in a high-growth state or boom when $S = 0$, and in a low-growth state or depression when $S = 1$. The probability of a boom next period given that the economy currently enjoys a boom is q, while the probability of a depression next period given a current depression state is p. The probabilities of transition from boom to depression and depression to boom are then 1-q and 1-p, respectively. The endowment grows at the rate α_0 during a boom, and $\alpha_0 + \alpha_1$ during a depression. The process $\{S_t\}$ can be represented as a first-order autoregression with an autocorrelation coefficient of $(p + q - 1)$ that can be interpreted as a measure of persistence in the forcing process.

It is also useful to think of the process loosely within the following context. The theory relates dividends to asset prices. In actual economies, future nominal dividend payments are announced in advance so a good deal of next period's dividend growth is currently known. This is captured by the timing of the state in the Markov trend, and in the next subsection agents in the artificial economy will be assumed to observe the current state of the economy. From (5), the forecastable part of dividend growth during period $t-1$ is $\alpha_0 + \alpha_1 S_{t-1}$, which is revealed at $t-1$. The unforecastable part of real divi-

TABLE 2—MAXIMUM LIKELIHOOD ESTIMATES OF THE MARKOV TREND MODEL

$$y_{t+1} = y_t + \alpha_0 + \alpha_1 s_t + \varepsilon_{t+1}$$
$$\text{Prob}[s_{t+1}=1|s_t=1]=p, \text{Prob}[s_{t+1}=0|s_t=1]=1-p,$$
$$\text{Prob}[s_{t+1}=0|s_t=0]=q, \text{Prob}[s_{t+1}=1|s_t=0]=1-q,$$
$$\varepsilon \text{ i.i.d.} \sim N(0,\sigma^2).$$

Parameter	Consumption	Dividends	GNP
p	0.5279	0.1748	0.5096
	(1.985)	(0.832)	(2.034)
q	0.9761	0.9508	0.9817
	(46.525)	(40.785)	(76.705)
σ	0.0320	0.1050	0.0433
	(12.297)	(13.682)	(14.932)
α_0	0.0228	0.0171	0.0246
	(6.467)	(1.579)	(5.950)
α_1	−0.0926	−0.3700	−0.1760
	(−4.894)	(−6.548)	(−7.116)
$\Pr(S_t=1)$	0.0482	0.0563	0.0360

Note: Asymptotic t-ratios in parentheses.

dend growth, ε_t, might be thought of as a combination of unanticipated inflation and productivity shocks.

We note at this point that it is the data and not the discretion of the investigator that will choose the regime. That is, when we calculate the Monte Carlo distributions implied by the model, the parameters ($\alpha_0, \alpha_1, p, q, \sigma$) of the forcing process will be set equal to maximum likelihood estimates obtained from the data.

D. *Maximum Likelihood Estimates of the Markov Switching Model*

This section reports maximum likelihood estimates of the Markov switching model described above for annual dividends, GNP, and consumption. The model is nonlinear in the sense that the current minimum mean square error predictor of future values is a nonlinear function of current and lagged observations. Even though the state, S, is unobservable to the econometrician, given the normality assumption on the ε's, the parameters of the process, ($p, q, \alpha_0, \alpha_1, \sigma$) can be estimated by maximum likelihood. The interested reader is directed to Hamilton (1989) for details on estimation or Pok-sang Lam (1988), who generalizes the Hamilton model.

The estimation results and some summary statistics are reported in Table 2. For the most part, the parameters are accurately estimated. When the economy is in a boom this year, the estimated probability that it continues in a boom next year is q. This is estimated to be 0.95 for dividends, and 0.98 for GNP and consumption. The estimates of growth during a boom, α_0, are 0.017, 0.025, and 0.023 for dividends, GNP, and consumption, respectively. When in a boom, the estimated probability of a transition to a negative growth state next period, $1-q$, is 0.05 for dividends, and 0.02 for GNP and consumption. While in a depression state, expected growth, $\alpha_0+\alpha_1$, is -0.35 for dividends, -0.15 for GNP and -0.07 for consumption.

The table also reports the unconditional probability of observing a depression state, $\Pr(S_t=1)$. These are 0.056 for dividends, 0.036 for GNP, and 0.048 for consumption. In other words, we expect real dividends, the most volatile of the three series, to crash by one-third in roughly 7 of the 116 years of the sample. While this may seem surprising, it is consistent with the historical experience. For example, the dividend model estimates imply that, if the economy is currently in the bad state ($S_t=1$), the 95 percent confidence in-

terval for growth in per capita real dividends is $(-0.14, -0.56)$. The same confidence interval given that the economy is in the good state ($S_t = 0$) is $(0.23, -0.19)$. Consequently, if dividends fall by 20 percent or more, we can be fairly certain that $S_t = 1$. Of the 116 years in the sample, 8 meet this criterion.[7]

Once the economy finds itself in a depression, the probability that it will be in a depression the following year, p, is estimated to be 0.1748 for dividends, 0.5096 for GNP, and 0.5279 for consumption.[8]

E. *Evaluating the Markov Switching Model*

To assess the quality of the Markov switching model, we now compare it with some popular alternatives. The upper panel of Table 3 reports the results of two diagnostic tests. The first is a test of the Markov switching model against the simple nested null hypothesis that the data follow a geometric random walk with i.i.d. innovations. Because the Markov switching model is not identified under the null of the geometric random walk, the likelihood ratio statistic does not have the standard chi-squared distribution. Therefore, we have tabulated the distribution of the pseudo-likelihood ratio statistic in order to perform this test. Our Monte Carlo experiment involved 1000 replications where we first drew samples of 116 for growth rates of GNP and dividends, and 96 for growth rates of consumption under the null of a normal distribution with variance set to values computed from the data. Next, we fitted the Markov switching model to this artificial data, and finally, we computed a standard likelihood ratio statistic as twice the difference in the maximized log likelihood values of the null and alternative models. As reported in the table, the weakest case is for consumption, where we reject the random walk at the 0.8 percent level. For both GNP and dividends, we observed fewer than two cases in 1000 where the LR statistic in the Monte Carlo experiment exceed the value obtained in the sample. Given the results in Table 1, where the data clearly reject the hypothesis that growth rates were drawn from a normal distribution, it is perhaps not surprising that a formal statistical test rejects the random walk model for the log-levels.

The second test reported in Table 3 is for symmetry of the Markov transition matrix, which implies symmetry of the unconditional distribution of the growth rates. This test examines the maintained hypothesis that $p = q$ against the alternative that $p < q$.[9] The table reports statistics that are asymptotically standard normal under the null. We can reject the hypothesis of symmetry at the 5 percent level in all three cases.

The lower panel of Table 3 reports the distributional characteristics for the Markov switching process implied by the estimates in Table 2. We report the population values of the mean, standard deviation, coefficient of skewness, coefficient of kurtosis, and first-order autocorrelation computed from the point estimates of the Markov switching model. The values implied by the model generally lie within two standard deviations of the sample values reported in Table 1.[10] The lone exception is the coefficient of kurtosis for GNP. We conclude that the Markov switching model can produce both the degree of negative skewness and the amount of kurtosis that are found in the data.[11]

[7]Real per capita dividends fell by more than 30 percent during 4 years, from 20 percent to 30 percent during four years, and by 10 percent to 20 percent during nine years of the sample.

[8]We note that the likelihood function is fairly flat for variations in p, particularly in the estimation of the dividend process. This is not surprising given the asymmetric behavior of dividends over the business cycle. That is, downturns have generally been short lived, lasting between four and six quarters. This makes it difficult to obtain a good estimate of p using annual observations. Hamilton does not encounter this problem since he estimates his model using *quarterly* GNP data.

[9]This is a one-sided test of symmetry against the alternative of negative skewness.

[10]It is worth noting that distribution of growth rates implied by the Markov switching model is always leptokurtic. Consequently, it is a natural candidate for modeling data that exhibit fat tails.

[11]The results in Table 3 demonstrate that the two-state model we employ is capable of matching the first four central moments of the data and the first-order

TABLE 3—EVALUATING THE MARKOV SWITCHING MODEL

	Data Set		
	Consumption	Dividends	GNP
L.R. Statistic[a]	11.39	17.27	27.87
p-value	(0.008)	(0.001)	(0.000)
Symmetry test[b]	1.72	3.71	1.89
p-value	(0.04)	(0.00)	(0.03)

Distributional Characteristics of the Process Implied by the Estimates

	Consumption	Dividends	GNP
Mean	0.0184	−0.0038	0.0183
Std. Dev.	0.038	0.135	0.054
Skewness Coefficient	−0.617	−0.965	−1.096
Kurtosis Coefficient	4.22	5.03	6.030
First Autocorrelation	0.140	0.05	0.179

[a] Based on a Monte Carlo experiment with 1000 replications.
[b] This is the test that $p = q$. The alternative hypothesis is $p < q$.

As a final test of the Markov switching model, we have compared its ability to forecast growth rates in the three series with that of first- and second-order autoregressions. Table 4 reports the in-sample root mean square forecast error at horizons from one to ten years for an AR(1), and AR(2), and the Markov switching model.[12] The results show that for dividends, the root mean squared forecast error of the Markov model is one-half that of the other models at all horizons. For consumption and GNP, the predictive ability of the three models is roughly the same. The Markov model marginally outperforms the autoregressions for consumption but marginally underperforms an AR(2) for GNP.

The purpose of this section has been to demonstrate that the Markov switching model is a reasonable alternative to obvious linear models in standard use. In addition to its ability to capture certain prominent features of the data that linear models cannot, the added attractiveness of the Markov switching model for our purposes is its analytical tractability. We conclude that while there is no obvious and clear winner, a credible case can be made for the Markov switching model. While a better model of the data (especially for GNP) might incorporate both regime switching and AR components, we have been unsuccessful in our attempt to solve the asset pricing problem when the endowment follows such a process. Our choice of the simple Markov switching model thus embodies a tradeoff between a model that completely matches the data and one that is tractable.

F. *Equilibrium Asset Prices*

Assume that the process driving the endowment is given by the Markov switching model of equations (5) and (6). We now obtain a solution to the asset pricing problem stated in Section I, Part A, using the

autocorrelation. By contrast, linear ARIMA models can match higher-order serial correlation, but not the higher-order moments. A three-state Markov switching model would have ten parameters and would in principle be able to match the first through sixth autocorrelation of the data in addition to the first four central moments. We have explored the three-state model but have thus far been unable to obtain sensible MLE parameter estimates with the annual data series studied here.

[12] Since these models are not nested, there is no natural statistical procedure for model selection.

TABLE 4—ROOT MEAN SQUARE FORECAST ERROR COMPARISONS USING LEVELS[a]

A. Consumption					Horizon					
Model	1	2	3	4	5	6	7	8	9	10
AR1	0.136	0.205	0.254	0.287	0.313*	0.340*	0.366	0.393*	0.415*	0.428*
AR2	0.136	0.205	0.257	0.288	0.314	0.341	0.367	0.396	0.417	0.431
MARKOV	0.131*	0.196*	0.249*	0.286*	0.315	0.342	0.366*	0.396	0.419	0.432
B. GNP					Horizon					
Model	1	2	3	4	5	6	7	8	9	10
AR1	0.324	0.594	0.808	0.962	1.055	1.093	1.109	1.126	1.186	1.241
AR2	0.318*	0.578*	0.790*	0.936	1.023	1.059	1.079	1.095*	1.154*	1.202*
MARKOV	0.351	0.608	0.796	0.936*	1.020*	1.054*	1.074*	1.102	1.167	1.220
C. Dividends					Horizon					
Model	1	2	3	4	5	6	7	8	9	10
AR1	0.017	0.021	0.025	0.029	0.033	0.038	0.042	0.047	0.052	0.057
AR2	0.031	0.033	0.034	0.037	0.040	0.044	0.047	0.051	0.055	0.060
MARKOV	0.009*	0.012*	0.014*	0.017*	0.020*	0.023*	0.026*	0.029*	0.032*	0.035*

[a]A * indicates lowest root mean square forecast error. Some entries appear the same due to rounding.

method of undetermined coefficients.[13] Conjecture the following solution:

$$(7) \qquad P_t = \rho(S_t) D_t.$$

The problem is to verify that (7) solves (3) and to find the function $\rho(S_t)$. To do this, substitute (7) into (3) to obtain

$$(8) \quad \rho(S_t) D_t^{\gamma} = \beta E_t D_{t+1}^{\gamma+1} [\rho(S_{t+1}) + 1].$$

Next, write (5) in levels,

$$(9) \qquad D_{t+1} = D_t e^{(\alpha_0 + \alpha_1 S_t + \varepsilon_{t+1})}.$$

Now substitute (9) into (8) and note that ε is i.i.d. normal with variance σ^2 to obtain

$$(10) \quad \rho(S_t) = \beta e^{[\alpha_0(1+\gamma)+(1+\gamma)^2\sigma^2/2]} \times e^{[\alpha_1(1+\gamma)S_t]} E_t[\rho(S_{t+1}) + 1].$$

Because S_t can take on only two values, 0 or 1, (10) is a system of two linear equations in $\rho(0)$ and $\rho(1)$. The solution is given by

$$(11) \quad \rho(0) = \tilde{\beta}[1 - \tilde{\beta}\tilde{\alpha}_1(p+q-1)]/\Delta,$$

$$(12) \quad \rho(1) = \tilde{\beta}\tilde{\alpha}_1[1 - \tilde{\beta}(p+q-1)]/\Delta,$$

where $\tilde{\beta} = \beta e^{[\alpha_0(1+\gamma)+(1+\gamma)^2\sigma^2/2]}$, $\tilde{\alpha}_1 = e^{\alpha_1(1+\gamma)}$, and $\Delta = 1 - \tilde{\beta}(p\tilde{\alpha}_1 + q) + \tilde{\beta}^2\tilde{\alpha}_1(p+q-1)$. This establishes that (7) is the solution to (3).[14]

A number of interesting features of the equilibrium price function emerge. First, asset prices are proportional to the endowment.[15] Second, the factor of proportionality depends on the inverse of the elasticity of intertemporal substitution and whether the

[13]Using formulas derived by Hamilton (1989), one can also evaluate the series given in equation (4) directly. The appendix to the working paper version of this paper shows that both methods yield the same answer.

[14]It is possible to show that as long as both $\rho(0) > 0$ and $\rho(1) > 0$, the transversality condition is met and the power series (4) converges. In addition, this solution technique can easily be generalized to the case of n-states in the mean and the variance.

[15]In the simple model studied here this implies that the price dividend ratio takes on one of two values, $\rho(0)$ or $\rho(1)$. This is a consequence of assuming that agents observe S_t. In the more realistic case in which S_t is unobserved and must be estimated, the price dividend ratio would be a continuous variable fluctuating between the two bounds of $\rho(0)$ and $\rho(1)$.

economy is currently in the high-growth state or low-growth state according to

$$\rho(0) \gtreqless \rho(1) \text{ as } \gamma \gtreqless -1.$$

The interpretation of this is straightforward. For a given level of the current endowment, suppose that the economy is known to be in a high-growth state ($S_t = 0$). By (6), this implies that the economy is likely to remain in a high-growth state into the future, and agents anticipate high future levels of the endowment. This has two effects on asset prices that work in opposite directions. First, there is an intertemporal relative price effect in which the higher expected future endowment implies a lower relative price of future goods. This induces agents to want to increase saving and to increase their demand for assets. The increased asset demand arising from this intertemporal relative price effect works to raise current asset prices. Working in the opposite direction is a substitution effect arising from agents' attempts to smooth their consumption. When the expected future endowment is high, the consumption smoothing motive leads agents to want to increase current consumption in anticipation of higher future investment income. To finance higher current consumption, they attempt to sell off part of their asset holdings, which in equilibrium results in falling asset prices.

Log utility ($\gamma = -1$) is a borderline case in which the intertemporal relative price effect and the consumption smoothing effect exactly cancel out. This can be seen, perhaps, more clearly from (4), in which the solution for $\gamma = -1$ is $P_t = (\beta/[1-\beta])D_t$. In this case, the factor of proportionality relating prices to dividends is a constant. When the utility function is less concave than it is in the log case, the intertemporal relative price effect assumes greater importance, so that $\rho(0) > \rho(1)$. In the limiting case of linear utility ($\gamma = 0$), the intertemporal relative price effect is all that matters since agents have no desire to smooth consumption. Conversely, when the utility function is more concave than is implied by log utility, the intertemporal consumption smoothing effect dominates the intertemporal relative price effect causing $\rho(1) > \rho(0)$.

From (5) and (7), equilibrium gross returns are computed as

$$(13) \quad R_t = (P_t + D_t)/P_{t-1} = \{[\rho(S_t)+1]/\rho(S_{t-1})\} \times \exp[\alpha_0 + \alpha_1 S_{t-1} + \varepsilon_t].$$

Notice that because the gross return depends on ε_t, it is a continuous random variable on $[0, \infty)$ and not a two-point process.

II. The Serial Correlation of Equilibrium and Historical Returns

In this section, returns obtained from the equilibrium model of Section I are used to generate Monte Carlo distributions of the variance ratio statistics used by Poterba and Summers and the regression coefficients calculated by Fama and French. These distributions are generated both for the case of linear utility and for a case in which the utility function is concave. They are then used to draw inference about the equilibrium model and the model driving the exogenous forcing variable. For a given value of the elasticity of intertemporal substitution, the model is calibrated to the estimated dividend, consumption, and GNP processes reported in Table 1. That is, the parameters of the forcing process, $(p, q, \sigma, \alpha_0, \alpha_1)$ are set to the values in the columns of Table 2, and each case is considered in turn. The subjective discount factor β is assumed to be 0.97 throughout.

The procedure is as follows: First, given p and q, we generate a sequence of 116 S_t's according to (6). Second, given σ, 116 independent draws from a normal distribution with zero mean and variance σ^2 are taken to form a sequence of ε_t's. Third, given α_0, α_1, β, γ, $\{s_t\}$, and $\{\varepsilon_t\}$, we generate a sample of 116 returns according to equation (13). For each sample of returns, the variance ratio and regression coefficients are calculated for horizons 1 through 10. This experiment is repeated 10,000 times. The tabulation of these calculations is the Monte Carlo distri-

bution of the statistic from which we draw inference. The sample size of 116 is chosen to correspond to the 116 annual observations available in the actual Standard and Poors' returns. We calculate the median, 60 percent confidence intervals about the median of the distribution, and the p-value for the statistic under investigation. We refer to these as the "small" sample results. A median is also calculated from 10,000 time-series samples of 1160 returns each, to get an idea of the rate of convergence of the variance ratio or regression coefficient statistic to its true population value. We refer to this as the "large" sample median. The results we obtain when the model is calibrated to consumption is representative of the results for dividend and GNP models. To facilitate the exposition, the results for the consumption model are also displayed in figures. Each figure displays the small sample medians, the 60 percent confidence intervals about the median, and the point estimates calculated from the historical Standard and Poors' returns.

From the Monte Carlo distributions of the variance ratio statistic and the autocorrelation coefficient on returns, we can determine the likelihood that the estimates obtained from the historical data were drawn from the Monte Carlo distribution implied by equilibrium returns.

A. *Variance Ratios*

Let R_t be the one period real rate of return, and R_t^k be the simple k-period return. That is, $R_t^k = \sum_{j=0}^{k-1} R_{t-j}$. Poterba and Summers define the variance ratio for returns at the kth horizon as

$$VR(k) = \frac{\mathrm{Var}(R_t^k)}{k\,\mathrm{Var}(R_t)}. \tag{14}$$

It is easy to show that the variance ratio can be expressed in terms of the return's autocorrelations. That is,

$$VR(k) = 1 + \frac{2}{k}\sum_{j=1}^{k-1}(k-j)\rho_j, \tag{15}$$

where ρ_j is the jth autocorrelation of annual returns. When returns are serially uncorrelated, the variance ratio is equal to one for all k in large samples.[16] This is usually taken as the null hypothesis in tests of "market efficiency," corresponds to the case where stock prices follow a random walk, and is true in the equilibrium model of Section I only when investors have linear utility. Stock prices are said to be "mean reverting" if returns are negatively serially correlated and evidence of mean reversion is inferred from variance ratios that lie below unity. This is the finding of Poterba and Summers.

We consider the case of linear utility first. Figure 1 displays the results under linear utility for the model calibrated to the consumption process. Since these returns are uncorrelated by construction, all of the deviation of the median of the variance ratio's distribution from unity is due to small sample bias.[17] In the large sample ($T=1160$), most of the bias has disappeared. It is also seen that the variance ratios calculated from the Standard and Poors' data fall within the 60 percent confidence interval of the Monte Carlo distribution.[18] The serial correlation of returns, and hence their predictability, is only apparent.

This result can be viewed in the same light as the business cycle in which recessions occur with random periodicity. Although real GNP may appear to be mean reverting, this does not imply that business cycle turning points are predictable. In the equilibrium model of asset prices, the exogenous forcing

[16] In small samples, as Poterba and Summers point out, the sample autocorrelations of returns are biased so $E[VR(k)] < 1$ even when returns are independent.

[17] It bears mentioning that even in the case of linear utility and the geometric random walk, our empirical distributions differ from those reported in Poterba and Summers and Fama and French. The reason is that we assume a probability model for the endowment process and study the dynamics of the returns implied by an equilibrium model, whereas these authors assume a probability model for the returns. Since the return is a nonlinear function of the endowment in our setup, these two approaches need not yield the same small sample results.

[18] These estimates of the variance ratios are smaller than those reported by Poterba and Summers because they make a bias correction assuming a null hypothesis of a homoscedastic random walk for asset prices. The bias correction is irrelevant for our purposes.

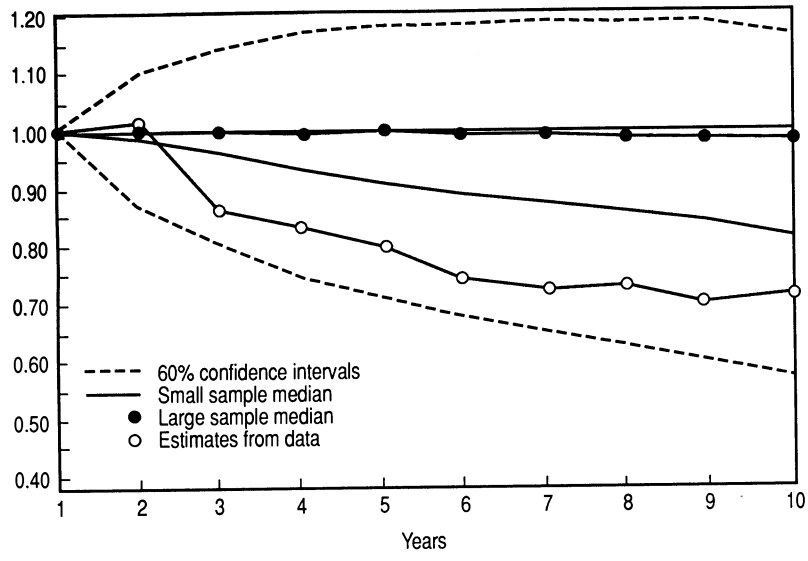

FIGURE 1. VARIANCE RATIOS. EQUILIBRIUM RETURNS GENERATED BY LINEAR UTILITY USING CONSUMPTION

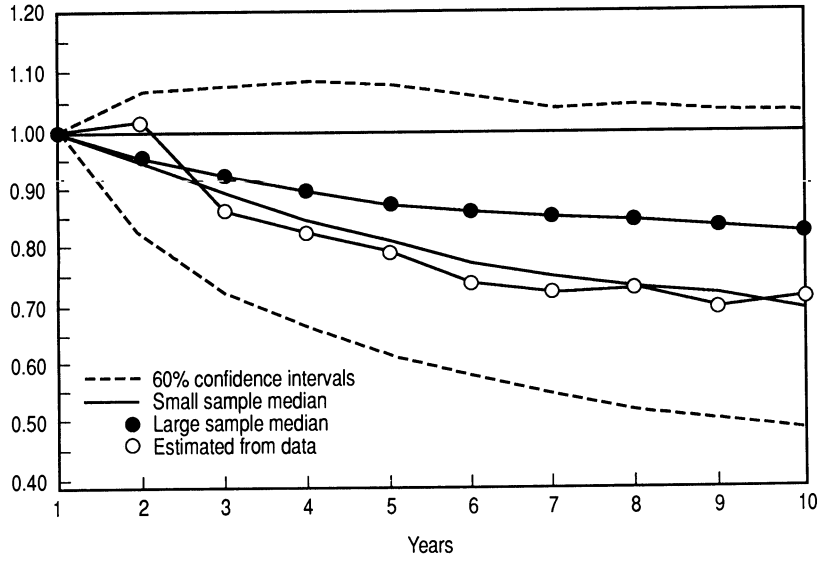

FIGURE 2. VARIANCE RATIOS. EQUILIBRIUM RETURNS GENERATED BY CONCAVE UTILITY USING CONSUMPTION

variable has a business cycle interpretation. The stochastic process of the forcing variable implies that a boom will eventually be followed by a recession, which will quickly be followed by a boom. Since equilibrium asset prices are proportional to the forcing variable, the appearance of mean reversion of asset prices is produced, but this does not mean that returns are predictable.

When agents' utility function is concave, the results are even more favorable to the model. Figure 2 reports the results of the

TABLE 5—VARIANCE RATIOS FOR HISTORICAL AND EQUILIBRIUM RETURNS

		Linear Utility			Concave Utility			
		$T=116$		$T=1160$	$T=116$		$T=1160$	
k	Actual	Median	p-Value	Median	Median	p-Value	Median	Population
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
			Endowment Calibrated to Consumption:					
2	1.0137	0.9865	0.5779	1.0004	0.9462	0.6811	0.9523	0.9524
3	0.8664	0.9669	0.3049	1.0005	0.8941	0.4482	0.9212	0.9206
4	0.8351	0.9385	0.3301	0.9977	0.8511	0.4742	0.9006	0.8987
5	0.7978	0.9115	0.3227	0.9989	0.8181	0.4671	0.8812	0.8831
6	0.7459	0.8926	0.2903	0.9950	0.7782	0.4515	0.8707	0.8716
7	0.7259	0.8811	0.2877	0.9953	0.7576	0.4534	0.8613	0.8630
8	0.7363	0.8678	0.3307	0.9912	0.7380	0.4965	0.8528	0.8564
9	0.7102	0.8527	0.3238	0.9876	0.7264	0.4796	0.8474	0.8511
10	0.7242	0.8268	0.3737	0.9869	0.7083	0.5216	0.8391	0.8469
			Endowment Calibrated to Dividends:					
2	1.0137	0.9835	0.6151	0.9959	0.8721	0.8986	0.8786	0.8828
3	0.8664	0.9683	0.2374	0.9934	0.8166	0.6350	0.8266	0.8340
4	0.8351	0.9541	0.2420	0.9912	0.7746	0.6425	0.8021	0.8086
5	0.7978	0.9367	0.2401	0.9906	0.7478	0.6069	0.7848	0.7933
6	0.7459	0.9230	0.2042	0.9891	0.7286	0.5355	0.7742	0.7831
7	0.7259	0.9058	0.2168	0.9876	0.7103	0.5277	0.7654	0.7758
8	0.7363	0.8906	0.2582	0.9875	0.6957	0.5759	0.7595	0.7704
9	0.7102	0.8851	0.2490	0.9838	0.6821	0.5492	0.7547	0.7661
10	0.7242	0.8682	0.3000	0.9840	0.6718	0.5858	0.7496	0.7627
			Endowment Calibrated to GNP:					
2	1.0137	0.9782	0.5877	0.9966	0.9401	0.6632	0.9463	0.9490
3	0.8664	0.9403	0.3722	0.9936	0.8760	0.4868	0.9102	0.9153
4	0.8351	0.9075	0.3837	0.9911	0.8296	0.5074	0.8839	0.8923
5	0.7978	0.8806	0.3757	0.9859	0.7858	0.5168	0.8658	0.8761
6	0.7459	0.8590	0.3451	0.9850	0.7516	0.4911	0.8492	0.8643
7	0.7259	0.8385	0.3541	0.9778	0.7424	0.4800	0.8390	0.8554
8	0.7363	0.8252	0.3909	0.9773	0.7136	0.5309	0.8285	0.8486
9	0.7102	0.8120	0.3763	0.9770	0.6933	0.5229	0.8204	0.8432
10	0.7242	0.7857	0.4227	0.9730	0.6833	0.5487	0.8172	0.8389

Notes: Under linear utility, $\gamma = 0$. Under concave utility, γ is set to -1.7 for the consumption model, -1.4 for the dividend model, and -1.6 for the GNP model. $\beta = 0.97$ throughout. Column 1: Horizon, in years. Column 2: Variance ratios of historical Standard and Poors' returns. Column 3: Median of Monte Carlo distribution of variance ratios for 116 equilibrium returns generated with linear utility. Column 4: Percentage of Monte Carlo distribution having values less than the value in column 3. Column 5: Median of Monte Carlo distribution for variance ratios of 1160 equilibrium returns generated with linear utility. Column 6: Median of Monte Carlo distribution of variance ratios of 116 equilibrium returns generated with concave utility. Column 7: Percentage of Monte Carlo distribution with values less than the value in column 6. Column 8: Median of Monte Carlo distribution for variance ratios of 1160 equilibrium returns generated with concave utility. Column 9: Population variance ratio of equilibrium returns.

above calculations assuming concave utility with $\gamma = -1.7$ and the forcing process matched to consumption. Now the median of both the small and large sample distributions of the variance ratio statistics are well below 1.0 at every horizon. The median of the small sample ($T = 116$) distribution closely matches the variance ratios calculated from the annual returns on the Standard and Poors'.[19]

Table 5 reports results for the model calibrated to consumption, dividends, and GNP,

[19]When $\gamma = -2$, the model yields much more mean reversion than is in the data. The entire 60 percent confidence band lies below the sample values.

from which we make the following observations. First, the results for variance ratios are fairly robust to the choice of the time-series to which the model is calibrated. *P*-values between 0.2 and 0.8 imply that the sample variance ratios lie within the 60 percent confidence interval of the Monte Carlo distribution median. Thus, it can be seen that even under linear utility, the model cannot be rejected at conventional significance levels. From column 4, the smallest p-value is obtained at the six-year horizon in the dividend model (p-value = 0.204). When the sample size is increased tenfold, ($T=1160$), most of the small sample bias disappears. When the utility function is concave, the median of the distribution matches up well with the values implied by the data as the p-values in column 7 are generally in the neighborhood of 0.5. A sizable small sample bias remains present, but as in the linear utility case, most of this bias vanishes if the sample is made ten times longer.

B. *Regression Coefficients on Returns of Varying Horizons*

Consider estimating the first-order serial correlation coefficient on τ-year returns by running the following regression:

$$(16)\quad R_{t,t+\tau} = a_\tau + b_\tau R_{t-\tau,t} + u_{t,t+\tau},$$

$$\tau = 1,2,\ldots,10 \text{ years},$$

where $R_{t,t+\tau}$ is the continuously compounded real stock return from t to $t+\tau$. It is easy to show that the relation between the autocorrelations of one-period returns and the autocorrelation of the τ-period return is

$$b_\tau = \frac{\rho_1 + 2\rho_2 + \cdots + \tau\rho_\tau + (\tau-1)\rho_{\tau+1}}{\tau + 2(\tau-1)\rho_1 + 2(\tau-2)\rho_2} \times \frac{+\cdots+2\rho_{2\tau-2}+\rho_{2\tau-1}}{+\cdots+2\rho_{\tau-1}}.$$

Using monthly returns on the CRSP index, Fama and French find that the slope coefficient b_τ is negative for τ greater than one year. From this they infer that stock prices are mean reverting. We examine their result by computing the empirical distribution of these regression coefficients implied by the model in Section I.

We begin with the linear utility ($\gamma = 0$) case. Figure 3 displays results for the model calibrated to consumption, the small sample median, and 60 percent confidence intervals of the Monte Carlo distribution of the regression coefficient b_τ, the population values implied by the model, and the estimates obtained from the Standard and Poors' returns. Again, the deviation of the median of the small sample ($T=116$) distribution from zero is due to small sample bias. This bias increases as τ gets larger, because the effective sample size, as measured by the number of independent pieces of information (nonoverlapping observations), decreases with τ. For example, at the ten-year horizon, there are only ten nonoverlapping observations available in the Standard and Poors' data, and six nonoverlapping observations available in the CRSP returns!

The median of the large sample distribution ($T=1160$), on the other hand, is reasonably close to the true value of zero. The regression coefficients calculated from the Standard and Poors' data uniformly lie below the median of the small sample Monte Carlo distribution in the consumption model.

Figure 4 displays the details of the Monte Carlo distributions of the regression coefficients obtained from the equilibrium returns when $\gamma = -1.7$ in the consumption model. As in Figure 3, the regression coefficient uniformly lies within the 60 percent confidence interval of the median. The distance between the small sample medians and the actual estimates tend to be smaller here than when agents have linear utility.

Table 6 reports results for the model calibrated to consumption, dividends, and GNP, from which we make the following observations. As we found with the variance ratios, the results on the regression coefficients are robust to the series to which the model is calibrated. Under linear utility, the strongest evidence against the model comes when the model is calibrated to dividends at the two-year horizon (p-value = 0.1099). Most of the small sample bias vanishes when $T=1160$.

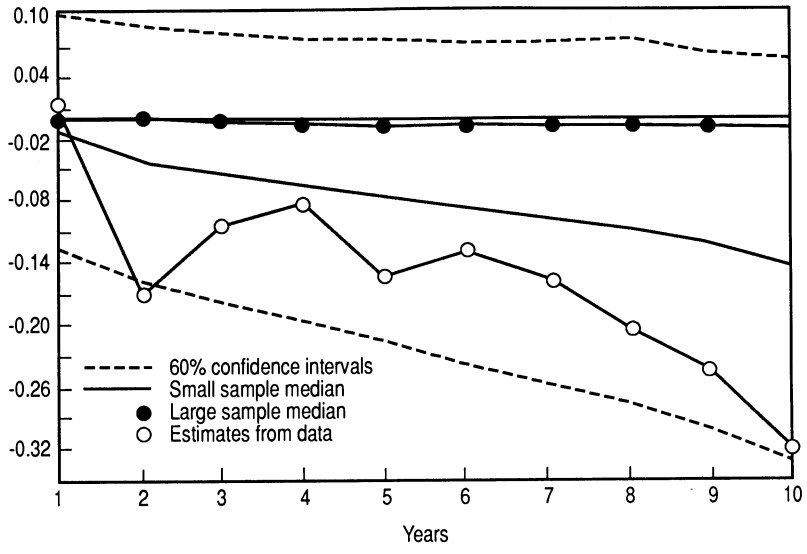

FIGURE 3. RETURN REGRESSION COEFFICIENTS. EQUILIBRIUM RETURNS GENERATED BY LINEAR UTILITY USING CONSUMPTION

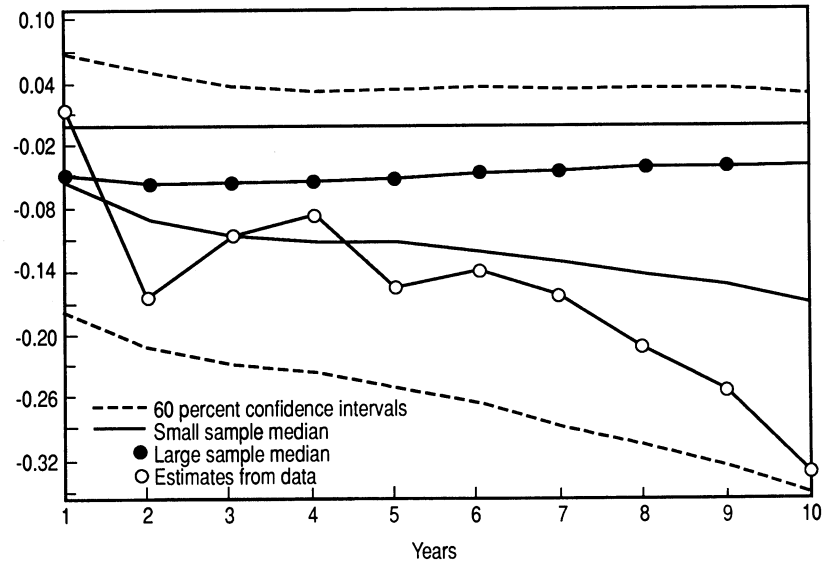

FIGURE 4. RETURN REGRESSION COEFFICIENTS. EQUILIBRIUM RETURNS GENERATED BY CONCAVE UTILITY USING CONSUMPTION

TABLE 6—REGRESSION COEFFICIENTS FOR HISTORICAL AND EQUILIBRIUM RETURNS

		Linear Utility			Concave Utility			
		$T=116$		$T=1160$	$T=116$		$T=1160$	
k	Actual	Median	p-Value	Median	Median	p-Value	Median	Population
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				Endowment Calibrated to Consumption:				
1	0.0147	−0.0108	0.5747	0.0002	−0.0541	0.6788	−0.0466	−0.0475
2	−0.1671	−0.0416	0.1761	−0.0006	−0.0905	0.3039	−0.0555	−0.0564
3	−0.1059	−0.0543	0.3654	−0.0024	−0.1052	0.4989	−0.0532	−0.0531
4	−0.0836	−0.0658	0.4565	−0.0039	−0.1109	0.5650	−0.0510	−0.0470
5	−0.1530	−0.0758	0.3273	−0.0067	−0.1092	0.3940	−0.0474	−0.0409
6	−0.1345	−0.0866	0.3952	−0.0076	−0.1175	0.4629	−0.0430	−0.0357
7	−0.1601	−0.1000	0.3816	−0.0096	−0.1283	0.4335	−0.0405	−0.0314
8	−0.2098	−0.1108	0.3148	−0.0099	−0.1400	0.3586	−0.0365	−0.0279
9	−0.2488	−0.1267	0.2883	−0.0127	−0.1482	0.3119	−0.0362	−0.0251
10	−0.3246	−0.1472	0.2158	−0.0125	−0.1643	0.2277	−0.0330	−0.0227
				Endowment Calibrated to Dividends:				
1	0.0147	−0.0129	0.6047	−0.0035	−0.1259	0.8992	−0.1215	−0.1171
2	−0.1671	−0.0254	0.1099	−0.0039	−0.1058	0.2943	−0.0865	−0.0840
3	−0.1059	−0.0383	0.3095	−0.0031	−0.0948	0.4681	−0.0645	−0.0609
4	−0.0836	−0.0472	0.4055	−0.0045	−0.0985	0.5379	−0.0511	−0.0473
5	−0.1530	−0.0587	0.2914	−0.0051	−0.1030	0.3827	−0.0434	−0.0386
6	−0.1345	−0.0756	0.3813	−0.0064	−0.1050	0.4322	−0.0393	−0.0325
7	−0.1601	−0.0919	0.3617	−0.0069	−0.1141	0.4052	−0.0356	−0.0282
8	−0.2098	−0.1078	0.3150	−0.0078	−0.1269	0.3451	−0.0342	−0.0248
9	−0.2488	−0.1275	0.2896	−0.0101	−0.1387	0.3124	−0.0311	−0.0222
10	−0.3246	−0.1410	0.2080	−0.0108	−0.1589	0.2321	−0.0289	−0.0200
				Endowment Calibrated to GNP:				
1	0.0147	−0.0187	0.5862	−0.0022	−0.0602	0.6659	−0.0538	−0.0509
2	−0.1671	−0.0577	0.2209	−0.0070	−0.1094	0.3584	−0.0656	−0.0597
3	−0.1059	−0.0667	0.3988	−0.0084	−0.1181	0.5299	−0.0631	−0.0557
4	−0.0836	−0.0731	0.4734	−0.0117	−0.1170	0.5814	−0.0594	−0.0489
5	−0.1530	−0.0822	0.3337	−0.0121	−0.1188	0.4202	−0.0544	−0.0424
6	−0.1345	−0.0951	0.4132	−0.0126	−0.1227	0.4739	−0.0506	−0.0369
7	−0.1601	−0.1041	0.3840	−0.0128	−0.1322	0.4413	−0.0454	−0.0324
8	−0.2098	−0.1160	0.3213	−0.0133	−0.1409	0.3665	−0.0424	−0.0288
9	−0.2488	−0.1269	0.2819	−0.0154	−0.1458	0.3133	−0.0406	−0.0258
10	−0.3246	−0.1433	0.2082	−0.0136	−0.1657	0.2262	−0.0403	−0.0234

Notes: Under linear utility, $\gamma = 0$. Under concave utility, γ is set to -1.7 for the consumption model, -1.4 for the dividend model, and -1.6 for the GNP model. $\beta = 0.97$ throughout. Column 1: Horizon, in years. Column 2: Regression coefficients of historical Standard and Poors' returns. Column 3: Median of Monte Carlo distribution of regression coefficients for 116 equilibrium returns generated with linear utility. Column 4: Percentage of Monte Carlo distribution having values less than the value in column 3. Column 5: Median of Monte Carlo distribution for regression coefficients of 1160 equilibrium returns generated with linear utility. Column 6: Median of Monte Carlo distribution of regression coefficients of 116 equilibrium returns generated with concave utility. Column 7: Percentage of Monte Carlo distribution with values less than the value in column 6. Column 8: Median of Monte Carlo distribution for regression coefficients of 1160 equilibrium returns generated with concave utility. Column 9: Population regression coefficients of equilibrium returns.

The model matches the data more closely when the utility function is concave. From column 7 it can be seen that the regression coefficient for one-year returns for the dividend model lies near the 95 percent confidence bound. At the remaining horizons in the dividend model and at all horizons for the consumption and GNP models, the regression coefficients computed with the data uniformly lie within a 60 percent confidence interval of the median of the Monte Carlo distribution. The distance between the small

TABLE 7—THE SOURCE OF THE NEGATIVE SERIAL CORRELATION, MEDIAN OF DISTRIBUTION OF VARIANCE RATIOS OF RETURNS FOR MODEL CALIBRATED TO CONSUMPTION

k	Actual	GRW $\gamma=0$ $T=116$	MSM $\gamma=0$ $T=116$	GRW $\gamma=0$ $T=1160$	MSM $\gamma=0$ $T=1160$	GRW $\gamma=-1.7$ $T=116$	MSM $\gamma=-1.7$ $T=116$	GRW $\gamma=-1.7$ $T=1160$	MSM $\gamma=-1.7$ $T-1160$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
2	1.0137	0.9913	0.9865	0.9983	1.0004	0.9907	0.9462	0.9991	0.9523
		(0.50)	(0.58)	(0.70)	(0.62)	(0.60)	(0.68)	(0.69)	(0.90)
3	0.8664	0.9732	0.9669	0.9953	1.0005	0.9758	0.8941	0.9964	0.9212
		(0.21)	(0.30)	(0.00)	(0.02)	(0.21)	(0.45)	(0.00)	(0.22)
4	0.8351	0.9605	0.9385	0.9944	0.9977	0.9587	0.8511	0.9955	0.9006
		(0.22)	(0.33)	(0.00)	(0.02)	(0.22)	(0.47)	(0.00)	(0.22)
5	0.7978	0.9458	0.9115	0.9937	0.9989	0.9474	0.8181	0.9927	0.8812
		(0.21)	(0.32)	(0.00)	(0.01)	(0.22)	(0.47)	(0.00)	(0.18)
6	0.7459	0.9366	0.8926	0.9904	0.9950	0.9302	0.7782	0.9921	0.8707
		(0.17)	(0.29)	(0.00)	(0.00)	(0.18)	(0.45)	(0.00)	(0.10)
7	0.7259	0.9208	0.8811	0.9922	0.9953	0.9227	0.7576	0.9912	0.8613
		(0.20)	(0.29)	(0.00)	(0.00)	(0.18)	(0.45)	(0.00)	(0.09)
8	0.7363	0.9062	0.8678	0.9892	0.9912	0.9077	0.7380	0.9908	0.8528
		(0.25)	(0.33)	(0.00)	(0.01)	(0.18)	(0.50)	(0.00)	(0.15)
9	0.7102	0.8887	0.8527	0.9884	0.9876	0.8938	0.7264	0.9874	0.8474
		(0.24)	(0.32)	(0.00)	(0.01)	(0.24)	(0.48)	(0.00)	(0.12)
10	0.7242	0.8817	0.8268	0.9858	0.9869	0.8718	0.7083	0.9874	0.8391
		(0.29)	(0.37)	(0.00)	(0.01)	(0.29)	(0.52)	(0.00)	(0.17)

Note: GRW is the geometric random walk model. MSM is the Markov switching model.

sample medians and the actual estimates tend to be smaller when agents have concave utility.

C. *Mean Reversion, Small Sample Bias, and Consumption Smoothing*

There are three features of our model that contribute to apparent mean reverting asset prices. They are (1) the nonlinearity of the endowment process (i.e., the presence of S_t), (2) pure small sample bias ($T=116$), and (3) consumption smoothing ($\gamma<0$). The role of consumption smoothing can be deduced from a comparison of columns 4 and 7 in Tables 5 and 6. For the values of γ that we work with, introducing concavity to the utility function shifts roughly an additional 15 percent to 20 percent of the variance ratio distribution and 1 percent to 10 percent of the regression coefficient distribution below the data. Table 7 reports additional results that allow us to separate these three influences and quantify the contribution of each in generating negatively serially correlated returns. While the table only includes evidence for the variance ratio statistic when the model is calibrated to consumption, these results are representative of the other cases that we do not report to economize on space.

We compare our results to those obtained when the endowment is assumed to follow a geometric random walk with drift and i.i.d. innovations with variance set equal to the sample values obtained from the consumption data. The table reports the median value of a Monte Carlo experiment with 10,000 replications, along with the percentage of the empirical distribution that lies below the value computed from the data.

To isolate the effects of the nonlinear endowment process, we first examine the implications for linear utility. A comparison of columns (3) and (4) reveals that the Markov switching model makes the small sample bias larger than it is in the case of the geometric random walk. While the variance ratio statistics are very near their population values of

TABLE 8—SERIAL CORRELATION IN EXCESS RETURNS, EQUILIBRIUM RETURNS IMPLIED BY THE MODEL CALIBRATED TO CONSUMPTION

		Linear Utility		Concave Utility		
		$T=116$		$T=116$		
k	Actual	Median	p-Value	Median	p-Value	Population
(1)	(2)	(3)	(4)	(5)	(6)	(7)
			Variance Ratios:			
2	1.0492	0.9866	0.7048	0.9878	0.7423	0.9993
3	0.9300	0.9602	0.4712	0.9729	0.4366	0.9988
4	0.9121	0.9325	0.5080	0.9514	0.4678	0.9985
5	0.8639	0.9124	0.4721	0.9424	0.4184	0.9982
6	0.7848	0.8998	0.3937	0.9231	0.3308	0.9981
7	0.7246	0.8793	0.3543	0.9044	0.2846	0.9980
8	0.7123	0.8577	0.3736	0.8968	0.3021	0.9979
9	0.7038	0.8421	0.3924	0.8854	0.3245	0.9978
10	0.7148	0.8345	0.4363	0.8665	0.3842	0.9977
			Regression Coefficients:			
1	0.0511	−0.0133	0.6809	−0.0114	0.7209	−0.692e−03
2	−0.1193	−0.0407	0.2871	−0.0281	0.2229	−0.783e−03
3	−0.1021	−0.0558	0.3834	−0.0387	0.3330	−0.714e−03
4	−0.1230	−0.0663	0.3609	−0.0499	0.3224	−0.616e−03
5	−0.0973	−0.0778	0.4504	−0.0654	0.4268	−0.527e−03
6	0.0557	−0.0894	0.7838	−0.0816	0.7569	−0.454e−03
7	0.1094	−0.0952	0.8470	−0.0956	0.8401	−0.396e−03
8	0.0904	−0.1090	0.8298	−0.1033	0.8226	−0.349e−03
9	0.1202	−0.1248	0.8618	−0.1225	0.8527	−0.312e−03
10	0.0863	−0.1436	0.8331	−0.1342	0.8133	−0.281e−03

Notes: Column 1: Horizon, in years. Column 2: Statistics computed using historical excess returns. Column 3: Median of Monte Carlo distribution of statistics for 116 equilibrium excess returns generated with linear utility. Column 4: Percentage of Monte Carlo distribution having values less than the value in column 3. Column 5: Median of Monte Carlo distribution of statistics of 116 equilibrium excess returns generated with concave utility. Column 6: Percentage of Monte Carlo distribution with values less than the value in column 5. Column 7: Population statistics for equilibrium excess returns.

1.0 for $T=1160$, the distribution shifts farther down in a sample of $T=116$ when the endowment follows the Markov switching process. The results in column (5) show that for the Markov switching model, roughly 10 percent more of the empirical distribution lies below the data than for the geometric random walk model reported in column (1).

The effect of consumption smoothing alone can also be deduced from the table. Columns (7) and (9) of Table 7 replicate the well known result that if the endowment process follows a geometric random walk, concave utility cannot produce mean reversion. This can also be seen from our solution for returns in equation (13). When there is no switching of states, the function ρ is constant. Because the ε_t's are independent, returns are serially uncorrelated regardless of the elasticity of intertemporal substitution.

Columns (7) and (8) report on the importance of the interaction between consumption smoothing and the Markov switching model. Comparing column (8) to column (3) shows the importance of the combined impact of these two effects in small samples. By comparison with the model calibrated to the geometric random walk with linear utility, the nonlinear model with concave utility yields roughly an additional 25 percent of the empirical distribution below the data when $T=116$. Furthermore, the results in column (10) show that even in large samples, consumption smoothing implies negative serial correlation in returns when the endowment follows the Markov switching process.

D. *Excess Returns*

Table 8 reports results on mean reversion in excess returns when the model is calibrated to consumption.[20] Poterba and Summers also find evidence of mean reversion in excess returns. The top panel of the table shows the results for the variance ratio statistics. Column (2) of table reports the values computed from historical excess returns. These are only slightly larger than the values for real equity returns reported in Table 5. Although the model generates time-varying excess returns for the parameter values that we consider (i.e., the implied variance of excess returns are not zero), they do not exhibit very much serial correlation. However, the estimates in column (2) still lie well within the standard 95 percent confidence intervals of the median of the Monte Carlo distributions.

The bottom panel of the table reports the results for the regression coefficients although Fama and French do not report results for excess returns. From column (2), it can be seen that the coefficients computed from historical excess returns are substantially different from those computed with real equity returns, and are strikingly different from any of the results reported by Fama and French. Rather than becoming more and more negative as the horizon lengthens, they become positive at six years, and grow larger and larger. Nevertheless, because of small sample bias in the calculation of the regression coefficients at long horizons, the data still lie within a 90 percent confidence interval of the median of our Monte Carlo distributions.

III. Conclusion

This paper demonstrates that the findings of Poterba and Summers (1988) and Fama and French (1988), that stock prices are mean reverting, are consistent with an equilibrium model of asset price determination. The question we addressed was whether the empirically observed serial correlation properties of stock returns can be generated by an equilibrium model of asset pricing. Monte Carlo distributions of Poterba and Summers' variance ratio statistics and Fama and French's long-horizon return regression coefficients are generated using equilibrium returns derived from the Lucas (1978) model and Hamilton's (1989) Markov switching process governing consumption, dividends, and GNP. When economic agents care about smoothing their consumption, the equilibrium model implies that stock prices are mean reverting. It is possible that this is what was detected by Poterba and Summers and Fama and French. However, even with a linear utility function, the variance ratios and regression coefficients calculated with the historical Standard and Poors' returns data are not significantly different from the median of our Monte Carlo distribution. This latter result underscores the problem that 116 annual observations do not contain very much information when computing statistics based on returns at five- or ten-year horizons. Both the bias and the size of confidence intervals generated by sampling variation grow as the *effective* sample size gets smaller. The implication for testing the null against local alternatives is complementary to Summers' (1986) point that most tests of market efficiency have virtually no power against what he calls fad alternatives. Since we have shown that a properly constructed equilibrium model can generate rational assets prices that exhibit a good deal of negative serial correlation, it follows that, given the available data, the test of any fad model will have very little power against the rather wide class of equilibrium alternatives. More precise estimates and more powerful tests can only come through the passage of time and not by sampling the data more frequently.[21] If there had been a well-functioning asset market since the time of the Norman invasion (A.D. 1066) and we had all the necessary price and dividend data, then we might begin to distinguish among some of the competing theories. We conclude that

[20] The results with the model calibrated to dividends and GNP are similar and are not reported to save space.

[21] That is, in computing the autocorrelation of ten-year returns, what is needed is more ten-year time periods and not weekly or daily observations. All we can do is wait.

the evidence drawn from variance ratios and return regression coefficients are not sufficient to rule out equilibrium models. It is important to emphasize, however, that these results do not prove that the equilibrium model is true, since it is impossible to prove rationality or irrationality.

DATA APPENDIX

The dividend data are annual observations from the Standard and Poors' index from 1871 to 1985 deflated by the CPI. This is the Standard and Poors' historical data used by Poterba and Summers. Observations on returns and the CPI from 1871 to 1926 are from Jack Wilson and Charles Jones (1987) and Wilson and Jones (1988), respectively. From 1926 to 1985, the data are from Roger Ibbotson and Rex Sinquefeld (1988). Observations on nominal dividends are those used by Campbell and Shiller (1987). We use these data as a benchmark because they represent the longest available time-series, and we believe that the characteristics of these data are representative of equity returns and dividend disbursements in general. Also, the Standard and Poors' index is one of the data sets used by Poterba and Summers, so a direct comparison can be made with some of their results. We follow both Poterba and Summers and Fama and French in deflating by the CPI.

The risk-free rate series is the *ex post* real return to holding a one-year U.S. Treasury security, or the equivalent, computed by subtracting realized inflation from the nominal interest rate. The nominal interest rate series was computed using data from four separate sources. The constructed series is intended to come as close as possible to a measure of the yield to maturity on a one-year U.S. security. For the period from 1920 to 1929, the basic data are drawn from the column giving the compound annual return from rolling "3- to 6-month Treasury notes and certificates," Table No. 122, page 460, of the *Banking and Monetary Statistics of the United States*. For a given year, the one-year yield was computed by assuming that the security was rolled over in July. For the period from 1930 to 1950, the data are the one-year yield for December of the previous year reported in the appendix to Stephen Cecchetti (1988). From 1951 to 1985, the data are the one-year zero coupon yield reported in J. Huston McCulloch (1988). For the period from 1871 to 1919, there is no direct information on government yields. To obtain a consistent series, we began by regressing the one-year yields from 1920 to 1987 (as described above) on the commercial paper rate constructed by Campbell and Shiller (1987). Since the commercial paper rate series extends back to 1871, we were able to estimate the implied government yield as the fitted values from this regression.

The real GNP data are constructed by combining data from 1869 to 1928 from Christina Romer (1989) with data from 1909 to 1928 from Romer (1985), and observations from 1929 to 1985 from the National Income and Product Accounts.

The consumption data are constructed by splicing the Kendrick consumption series reported in Nathan Balke and Robert Gordon (1986), from 1889 to 1928, with the National Income and Product Accounts data from 1928 to 1985. This series is the longest available series on aggregate personal consumption expenditure we are aware of.

In order to express quantities in per capita terms, we divided each time-series by annual population estimates. The estimates used are as follows. From 1869 to 1938 the data are from the *Historical Statistics of the United States*, Series A7 from 1869 to 1928 (with the data in footnote 1 for 1917 to 1919), and Series A6 from 1929 to 1938. From 1938 to 1985 the data are from the *Economic Report of the President*, 1989, Table B-31.

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