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Cointegration, Fractional Cointegration, and Exchange Rate Dynamics

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ABSTRACT

Multivariate tests due to Johansen (1988, 1991) as implemented by Baillie and Bollerslev (1989a) and Diebold, Gardeazabal, and Yilmaz (1994) reveal mixed evidence on whether a group of exchange rates are cointegrated. Further analysis of the deviations from the cointegrating relationship suggests that it possesses long memory and may possibly be well described as a fractionally integrated process. Hence, the influence of shocks to the equilibrium exchange rates may only vanish at very long horizons.

IN THE ARTICLE BY Baillie and Bollerslev (1989a), it is argued that seven different nominal spot and forward exchange rates all contain unit roots in their univariate time series representations. At the same time, however, the spot exchange rates appear to be tied together in the long run through a cointegration-type relationship. The latter finding of Baillie and Bollerslev has attracted particular interest and several studies such as those by Hakkio and Rush (1991) and Sephton and Larsen (1991) have already addressed this issue. Using the same data as the Baillie and Bollerslev article, Sephton and Larsen (1991) describe the evidence for the presence of cointegration as being "fragile" and note that mixed conclusions are reached by truncating the Baillie and Bollerslev sample at different points in time.

Diebold, Gardeazabal, and Yilmaz (1994) henceforth Diebold *et al.*, provide interesting evidence that application of the Johansen (1988, 1991) tests with and without an intercept will result in different inferences on the Baillie and Bollerslev data set. Furthermore, Diebold *et al.* carry out an ex ante forecasting experiment and find that the addition of an error correction term to the martingale model, as implied by the standard cointegration paradigm, fails to reduce the prediction mean square error when compared to a simple martingale model. This therefore leads Diebold *et al.* to conclude that "there exists substantial uncertainty regarding the existence of cointegration relationships among nominal dollar exchange rates." This article provides some additional evidence on the existence of such a long-run relationship among the same seven nominal spot exchange rates. After further analysis it appears that a *form* of cointegration does exist between the exchange rates, so that they do not drift apart in the *long run*. We argue that this form of cointegration is

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associated with long memory and can reasonably be termed "fractional cointegration."

In the next section we briefly discuss the results from applying standard tests for unit roots and cointegration. Section II introduces the concept of fractional cointegration, and reports on some empirical evidence for the existence of such a long-term relationship among the seven nominal exchange rates. Section III concludes.

I. Unit Roots and Cointegration

The central issue in the present controversy, and indeed in many current areas of financial economics and macroeconomics, is that of distinguishing the appropriate order of integration of a time series. Throughout this article, we use the identical data set to Baillie and Bollerslev and Diebold *et al.*, consisting of 1,245 daily nominal exchange rates vis-à-vis the U.S. dollar from March 1, 1980 through January 28, 1985. On the application of standard unit root tests, Baillie and Bollerslev conclude that all the seven nominal exchange rate series are well characterized by $I(1)$ processes. Diebold *et al.* do not disagree with this finding. Indeed, this property of exchange rates and high-frequency asset prices in general has become almost a universally accepted fact. Furthermore, there appears to be almost equally widespread agreement that nominal exchange rates are well described as martingales, so that the past exchange rate movements of a particular rate are of no use in predicting future changes for that same currency.[1] There appears to be no information in own past rates which is of use in predicting the future returns. To further confirm this property, Figure 1 provides a graph of the first 250 autocorrelation coefficients of the logarithm of the deutsche mark–U.S. dollar nominal exchange rate. This correlogram exhibits the typical very slow decline associated with an $I(1)$ process. In contrast, the first ten autocorrelation coefficients of the approximate rate of return, i.e., the first difference of the logarithm for this exchange rate, equal -0.035, 0.040, 0.033, 0.004, 0.019, -0.004, -0.032, 0.024, 0.000, and 0.037, respectively. Although the conventional $T^{-1/2}$ standard error approximation for the estimated autocorrelation coefficients, where T denotes the sample size, is only strictly valid in the case of no temporal dependencies, it can nevertheless be informative even when the process is known to exhibit ARCH effects. This simple checking procedure applied to the first 250 autocorrelations reveals nothing to make us doubt the stylized fact that the deutsche mark–U.S. dollar rate is generated by an $I(1)$ process, so that the nominal returns on an open position in the spot foreign

[1] When nominal rates are observed daily or weekly, they appear to possess substantial time dependent heteroskedasticity, however. For example, Baillie and Bollerslev (1986b) use the identical data to Baillie and Bollerslev (1989a) and estimate GARCH models on the rates of return of the exchange rates.

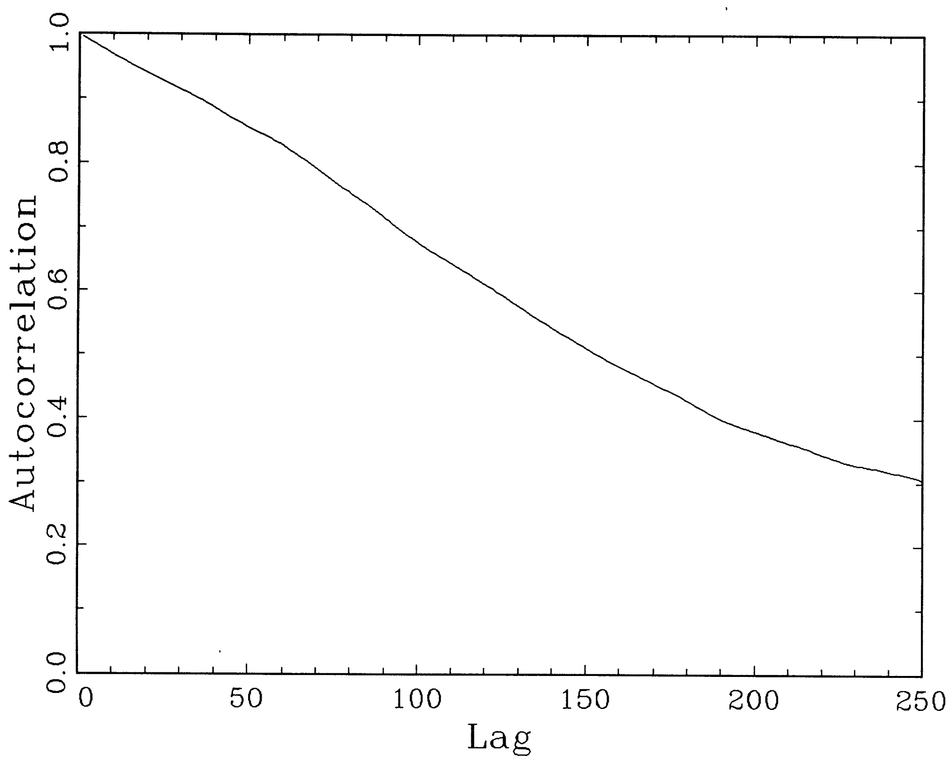

Figure 1. Correlogram for the logarithmic deutsche mark–U.S. dollar exchange rate. The figure graphs the first 250 sample autocorrelations for the logarithm of the nominal deutsche mark–U.S. dollar exchange rate. The data are opening bid prices from March 1, 1980 through January 28, 1985, for a total of 1,245 daily observations.

exchange market may be characterized by an $I(0)$ process. Extremely similar estimated autocorrelation functions for the logarithmic levels and rates of returns are observed for the other six exchange rates, but are omitted for reasons of space. Full details are of course available from the authors on request. It is worth noting that there is no evidence of seasonality or any cyclical patterns in any of the estimated autocorrelation functions for the individual rates.

Now consider the properties of the vector of the logarithms of the seven nominal exchange rates, which we denote as y_t. Throughout this analysis the U.S. dollar rates for the currencies of the seven countries are listed in the following order: West Germany, United Kingdom, Japan, Canada, France, Italy, and Switzerland. Diebold *et al.* apply the test of Johansen (1991) that allows for a drift in the seven-dimensional VAR, to test for the presence of cointegration in the y_t vector. On including the intercept they fail to find evidence for cointegration, while Baillie and Bollerslev, in neglecting the intercept and using Johansen's (1988) procedure, find evidence for one cointegrating vector.

Table I

Analysis of the Cointegrating Vector

The estimate of the cointegrating vector, $\hat{\alpha}$, is obtained from an OLS regression of the logarithm of the U.S. dollar rate for the West German (WG) currency on a constant and the logarithm of the six U.S. dollar rates for the currencies of United Kingdom (UK), Japan (JP), Canada (CN), France (FR), Italy (IT), and Switzerland (SW). The data are opening bid prices from March 1, 1980 through January 28, 1985, for a total of 1,245 daily observations. The Phillips $\hat{Z}_t$ statistic for a unit root in the deviations from the cointegrating linear relationship, $\hat{\alpha}'y_t$, equals -3.95 when computed from twenty-two lagged autocovariances using a Bartlett window. Phillips and Ouliaris (1990) give asymptotic critical values of this test statistic. With six variables in the cointegrating regression the five and ten percent critical values are -4.71 and -4.43, respectively. Critical values of the test statistic are not tabulated for seven variables, as in our case. The critical values are monotonically increasing with the number of variables in the cointegrating regression, however. Since our test statistic lies inside the ten percent critical region for six variables, it is clear that the test would not reject the null hypothesis of no cointegration at any reasonable significance level.

WG	UK	JP	CN	FR	IT	SW	Constant
1.000	−0.070	0.937	0.534	0.003	−0.290	−0.458	5.380

Since previous analysis in Baillie and Bollerslev indicates the existence of at most one linearly independent cointegrating relationship, we estimate the cointegrating vector, α, by ordinary least squares (OLS) as suggested in Engle and Granger (1987). Table I reports the estimates for the full set of the seven exchange rates with the dependent variable, or normalized variable, in the cointegrating relationship taken to be West Germany.[2] If the rates are cointegrated and $\alpha'y_t$ is $I(0)$, this simple procedure will yield super consistent coefficient estimates of the cointegrating vector; see Stock (1987).[3] In the absence of cointegration, however, $\alpha'y_t$ will be $I(1)$ for all nonzero vectors, α. The OLS regression underlying the estimate of $\hat{\alpha}$ will therefore be spurious in the sense of Granger and Newbold (1974). Appropriate critical values that adjust for this bias in the residual based tests for unit roots and lack of cointegration have been derived by Phillips and Ouliaris (1990). The results from this simple univariate approach are detailed in Table I. Counter to our earlier findings reported in Baillie and Bollerslev, the results indicate that on allowing for an intercept in the cointegrating regression, the test statistics do not reject the null hypothesis of a unit root; i.e., no cointegration. This is consistent with the results in Diebold *et al.* based on the more complicated multivariate Johansen procedure.

[2] Very similar results are obtained for the subsystem of the three European Exchange Rate Mechanism countries of West Germany, Italy, and Switzerland. Full details of these results are available from the authors on request but are not reported here for reasons of space.

[3] Asymptotically valid standard errors that allow for normal inference could be constructed as in Stock and Watson (1993). Since our interest centers directly on the temporal dependencies in the $\hat{\alpha}'y_t$ process, we do not pursue this further.

II. Fractional Cointegration

To further understand the reason behind these conflicting findings, the autocorrelation function for the "error correction term," $\hat{\alpha}' y_t$, is plotted in Figure 2. While the autocorrelation coefficients appear to be generated by a stationary process, it is immediately clear that this process exhibits long memory characteristics with long-term cycles in its autocorrelations. The relatively rapid rate of decay of the autocorrelation coefficients is in marked contrast to the apparent $I(1)$ property of the original logarithmic levels of the exchange rate series in Figure 1.

In the classical paradigm for cointegration, and indeed in the Johansen procedure, all the elements of the y_t vector are assumed to be $I(1)$ processes, while the cointegrating linear relationship $\alpha' y_t$ is presumed to be $I(0)$. This is referred to as $CI(1, 1)$. The Granger Representation Theorem underlying the idea of cointegration, however, only requires that the cointegrating vector

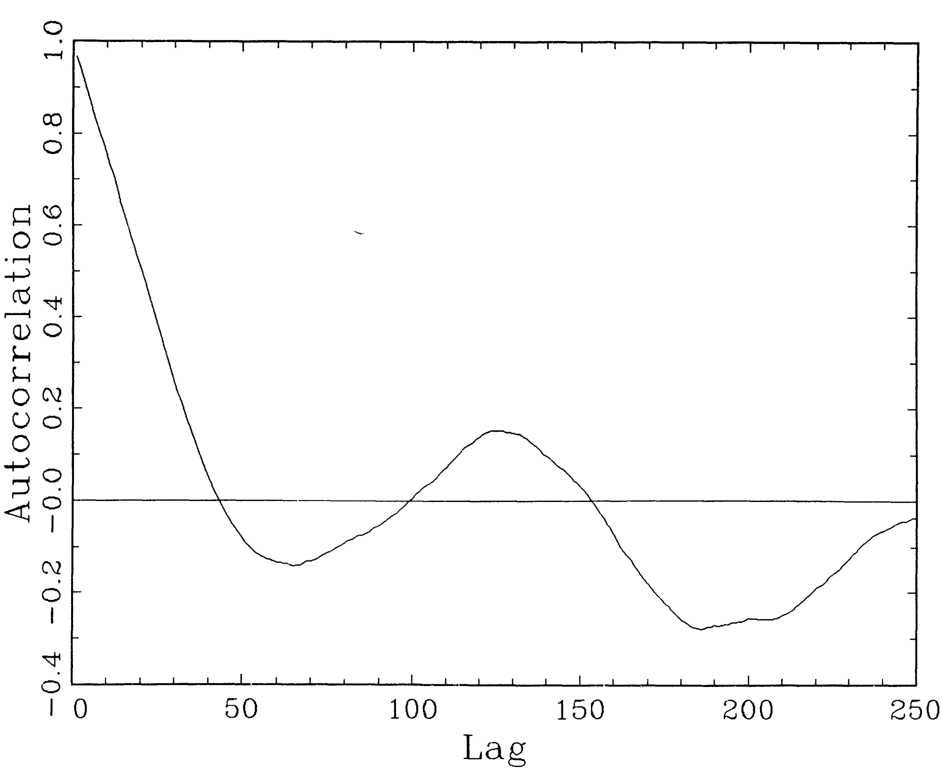

Figure 2. Correlogram for the error correction term. The figure graphs the first 250 sample autocorrelations for the estimated error correction term, $\hat{\alpha}' y_t$. The estimate of the cointegration vector, $\hat{\alpha}$, is obtained from an OLS regression of the logarithm of the U.S. dollar rate for the West German currency on a constant and the logarithm of the six U.S. dollar rates for the currencies of United Kingdom, Japan, Canada, France, Italy, and Switzerland. The data are opening bid prices from March 1, 1980 through January 28, 1985, for a total of 1,245 daily observations.

$\alpha' y_t$ be stationary (see Granger (1981, 1983) and Sowell (1986)). Hence the cointegrating vector must possess a valid Wold decomposition,

$$\alpha' y_t - \mu = \psi(L)\epsilon_t,$$

where μ is a constant, $\psi(L) \equiv \sum_{j=0,\infty} \psi_j L^j$, L denotes the lag operator so that $L^j \epsilon_t \equiv \epsilon_{t-j}$, ψ_j are infinite order moving average coefficients, and ϵ_t is a white noise process. The minimal requirements are square summability so that $\sum_{j=0,\infty} \psi_j^2 < \infty$. There is no necessity for the error correction term $\alpha' y_t$ to be consistent with an $I(0)$ process with moving average coefficients that decline exponentially for large lags. An equally valid paradigm is for the error correction term to respond more slowly to shocks so that deviations from equilibrium are more persistent. One such possibility would be for the deviations from the cointegrating relationship to possess *long memory*, according to which the effect of a shock declines at a slower rate than the usual exponential decay associated with the autocorrelation functions for the class of covariance stationary and invertible ARMA process.

The fractionally integrated process of Granger (1980) is specifically designed to capture such long memory–type behavior. The process z_t is said to be fractionally integrated, or $I(d)$, if

$$(1 - L)^d (z_t - \mu) = u_t,$$

where u_t is an $I(0)$ process. For the process to be covariance stationarity $d < 0.5$, while invertibility requires that $d > -0.5$. Granger and Joyeux (1980) and Hosking (1981) show that for large orders k the moving average coefficients are proportional to k^{d-1}, while the autocorrelation coefficients ρ_k are proportional the k^{2d-1}, and decay at a hyperbolic rate. Also, the cumulative impulse response coefficients corresponding to a shock in the infinite past equals zero for $d < 1$. In contrast, shocks to an $I(1)$ process never die out as reflected by a limiting nonzero cumulative impulse response coefficient (see, e.g., Diebold and Rudebusch (1989)).

The Johansen test, used by Diebold *et al.*, and the residual-based test for cointegration discussed above, are both predicated on the assumption that the error correction term is $I(0)$. It is unclear as to the power of these procedures when the error correction term is $I(d)$ where $0 < d < 1$. Diebold and Rudebusch (1991) examine by simulation methods the power of standard unit root tests when the true data-generating process is fractionally integrated white noise. They conclude that the unit root tests have very low power against fractional alternatives. Similar findings are almost certain to hold true for tests of cointegration when deviations from the cointegrating relationship are fractionally integrated, as recently demonstrated by Cheung and Lai (1993).

Given this difficulty of distinguishing between unit root processes and fractional alternatives, it seems reasonable to consider the error correction term $\alpha' y_t$, to possibly be fractionally integrated. On applying the time domain approximate maximum likelihood estimator described in Chung and Baillie (1993), we estimate the following simple fractional white noise model,[4]

$$(1 - L)^{0.89}(\hat{\alpha}' y_t - \hat{\mu}) = \epsilon_t,$$

The asymptotic standard error for $\hat{d}$ is 0.02, so that $\hat{d} = 0.89$ is over five standard errors away from one.[5] This model appears to provide a reasonable explanation of some of the long-memory characteristics of the error correction term.

For a more complete description of the cyclical components of the autocorrelations evident in Figure 2, it is necessary to use a model that can allow for long-memory harmonic behavior, however. One possible model is the Gegenbauer process introduced by Gray, Zhang, and Woodward (1989). The simplest Gegenbauer processes is of the form,

$$(1 - 2\xi L + L^2)^{\lambda} z_t = \epsilon_t,$$

where ϵ_t is a white noise process. For $|\xi| < 1$ and $0 < \lambda < 0.5$ the process is covariance stationary but exhibits long memory. Clearly the fractionally integrated while noise model discussed above is obtained as a special case with $\xi = 1$ and $\lambda = d/2$. Gray, Zhang, and Woodward (1989) show that for large k the autocorrelations will behave as

$$\rho_k \simeq k^{2\lambda - 1}\sin(\pi\lambda - k\omega_0),$$

where $\omega_0 = \cos^{-1}(\xi)$ determines the harmonic frequency. As yet, virtually no work has considered the estimation of the parameters of the Gegenbauer process. We are currently investigating some methods for obtaining approximate maximum likelihood estimates, that we hope to report in future work.

Consistent with the lack of cointegration, Diebold *et al.* also report no improved Mean Square Error (MSE) predictability from using the deviations from the cointegrating relationship as a lagged error correction term in the martingale models for each of the seven different rates. Their forecasting study only examined up to 126 days ahead forecasts; i.e., roughly one half-year. This is about ten percent of the Baillie and Bollerslev sample. Our results indicate that the impact of long memory is likely to be considerably further ahead, and that adjustment to exchange rate equilibrium will generally take

[4]It follows from Yajima (1988) and Cheung and Lai (1993) that if y_t is $I(1)$ but $d > 0$, so that y_t is fractionally cointegrated, the OLS estimator of α is consistent and converges at the rate of T^{1-d}. Also, from Li and McLeod (1986) and Sowell (1992) if the mean zero stationary and invertible Autoregressive Fractionally Integrated Moving Average (ARFIMA) process z_t is directly observable and $|d| < 0.5$, the Maximum Likelihood Estimate (MLE) of d obtained under the assumption of i.i.d. normal innovations is $T^{1/2}$ consistent.

[5]Estimation of an ARFIMA$(0, d, 0)$ process for the first difference of the deviations from the cointegrating relationship, $(1 - L)(\hat{\alpha}' y_t - \hat{\mu})$, realized an estimated value of $\hat{d} = -0.11$, with an asymptotic standard error of 0.02 also. Hence from a practical point of view it makes no difference whether the series is estimated in levels or first differences. It is interesting to note that the estimate of the standard error calculated from the outer product of the gradients corresponds very closely to the asymptotic value of $\sqrt{(6/\pi^2 T)}$.

several years to complete. Hence, any improvement in forecasting MSE may only be apparent several years into the future.[6] A related phenomenon within the context of deviations from Purchasing Power Parity has been observed by Diebold, Husted, and Rush (1991) and Cheung and Lai (1993).

At this stage is seems premature to hypothesize on the possible reasons behind the apparent long memory in the exchange rate relationship. However, studies such as Baillie and Pecchenino (1991) have also failed to find any evidence for the existence of $CI(1,1)$ cointegration between nominal exchange rates and "fundamentals" associated with standard monetary-type models of exchange rate determination. Yet this analysis keeps open the possibility that the corresponding error correction term may be mean reverting and $I(d)$. It is also possible that the periodic exchange rate realignments may play a role in generating the observed long-memory behavior.

III. Conclusion

The nominal exchange rates used by Baillie and Bollerslev (1989a) appear to be well described as martingales. As Diebold *et al.* correctly note, and counter to our earlier findings, when including an intercept in the vector autoregression test developed by Johansen (1991), the exchange rates show little evidence of $CI(1,1)$ cointegration. However, a more detailed examination of the deviations from the estimated cointegrating relationship reveals that the exchange rates may well be tied together through a long memory $I(d)$-type process, rather than an $I(0)$ process. Hence the inclusion of an error correction term would only reduce the MSE of predictions from a simple martingale model in the very "long run." Part of our current research program involves a more formal investigation of this theory based on a much longer span of data.

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[6] Granger (1983) shows, that if y_t is $CI(1,d)$, i.e., y_t is $I(1)$ and $\alpha' y_t$ is $I(d)$, the error correction representation for y_t takes the form $A(L)(1-L)y_t = -\gamma[1-(1-L)^{1-d}](1-L)^d \alpha' y_t + C(L)\epsilon_t$, where ϵ_t is $I(0)$. Note, that this representation involves only $I(0)$ terms, and that when the square bracket on the right hand side is expanded in terms of powers of L only lagged values of $\alpha' y_t$ will appear.

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