# Partial Adjustment or Stale Prices? Implications from Stock Index and Futures Return Autocorrelations

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# Partial Adjustment or Stale Prices? Implications from Stock Index and Futures Return Autocorrelations

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We investigate the relation between returns on stock indices and their corresponding futures contracts to evaluate potential explanations for the pervasive yet anomalous evidence of positive, short-horizon portfolio autocorrelations. Using a simple theoretical framework, we generate empirical implications for both microstructure and partial adjustment models. The major findings are (i) return autocorrelations of indices are generally positive even though futures contracts have autocorrelations close to zero, and (ii) these autocorrelation differences are maintained under conditions favorable for spot-futures arbitrage and are most prevalent during low-volume periods. These results point toward microstructure-based explanations and away from explanations based on behavioral models.

Arguably one of the most striking asset pricing anomalies is the evidence of large, positive, short-horizon autocorrelations for returns on stock portfolios, first described in Hawawini (1980), Conrad and Kaul (1988, 1989, 1998) and Lo and MacKinlay (1988, 1990b). The evidence is pervasive both across sample periods and across countries, and has been linked to, among other financial variables, firm size [Lo and MacKinlay (1988)], volume [Chordia and Swaminathan (2000)], analyst coverage [Brennan, Jegadeesh and Swaminathan (1993)], institutional ownership [Badrinath, Kale and Noe (1995) and Sias and Starks (1997)], and unexpected cross-sectional return

dispersion [Connolly and Stivers (1999)].[1] These results are puzzling to financial economists precisely because time variation in expected returns is not a high-frequency phenomenon; asset pricing models link expected returns with changing investment opportunities, which, by their nature, are low-frequency events.

As a result, most explanations of the evidence have centered around so-called partial adjustment models in which one group of stocks reacts more slowly to aggregate information than another group of stocks. Because the autocovariance of a well-diversified portfolio is just the average cross-auto-covariance of the stocks that make up the portfolio, positive autocorrelations result. While financial economists have put forth a variety of economic theories to explain this lagged adjustment, many of these models rely on either an underlying behavioral model, that is, irrationality on the part of some agents that matters for pricing, or asymmetric information with market frictions that prevents arbitrageurs from entering the market [see, e.g., Holden and Subrahmanyam (1992), Brennan, Jegadeesh, and Swaminathan (1993), and Jones and Slezak (1999)]. Alternative and seemingly less popular explanations focus on typical microstructure biases [Boudoukh, Richardson and Whitelaw (1994)] or transaction costs which prevent these autocorrelation patterns from disappearing in financial markets [Mech (1993)]. The latter explanation, however, does not explain why these patterns exist in the first place.

This article draws testable implications from the various theories by exploiting the relation between the spot and futures market.[2] Specifically, while much of the existing focus in the literature has been on the statistical properties of artificially constructed portfolios (such as size quartiles), there are numerous stock indices worldwide that exhibit similar properties. Moreover, many of these indices have corresponding futures contracts. Since there is a direct link between the stock index and its futures contract via a no-arbitrage relation, it is possible to show that, under the aforementioned economic theories, the futures contract and the underlying index should exhibit the same time-series properties. In contrast, why might the properties of the returns on the index and its futures contract diverge? If the index is valued using mismeasured prices (e.g., stale prices or bid and ask prices), then the link

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[1] Another powerful and related result is the medium-term continuation of returns, the momentum effect, documented by Jegadeesh and Titman (1993). While this evidence tends to be firm specific, it also produces positive autocorrelation in portfolio stock returns [see Moskowitz and Grinblatt (1999) as a recent example]. Moreover, this evidence holds across countries [e.g., Rouwenhorst (1998)] and across time periods.

[2] Miller, Muthuswamy, and Whaley (1994) and Boudoukh, Richardson, and Whitelaw (1994) also argue that the properties of spot index and futures returns should be different. Miller, Muthuswamy, and Whaley (1994) look at mean reversion in the spot-futures basis in terms of nontrading in S&P 500 stocks, while Boudoukh, Richardson, and Whitelaw (1994) look at combinations of stock indices, like the S&P 500 and NYSE, in order to isolate portfolios with small stock characteristics. While the conclusions in those articles are consistent with this article, those articles provide only heuristic arguments and focus on limited indices over a short time span. This article develops different implications from various theories and tests them across independent, international data series.

between the two is broken. Alternatively, if transaction costs on the individual stocks comprising the index are large enough, then the arbitrage cannot be implemented successfully. This article looks at these possibilities in a simple theoretical framework and tests their implications by looking at spot and futures data on 24 stock indices across 15 countries.

The major results are as follows:

- The return autocorrelations of indices with less liquid stocks (such as the Russell 2000 in the United States, the TOPIX in Japan, and the FTSE 250 in the United Kingdom) tend to be positive even though their corresponding futures contracts have autocorrelations close to zero. For example, the Russell 2000's daily autocorrelation is 22%, while that of its corresponding futures contract is 6%. The differences between these autocorrelation levels are both economically and statistically significant.
- Transaction costs do not appear to explain the magnitude of these auto-correlation differences, as the magnitude changes very little even when adjusting for periods favorable for spot-futures arbitrage. We view this as evidence against the type of partial adjustment models put forth in the literature.
- Several additional empirical facts point to microstructure-type biases, such as stale prices, as the most probable source of the differences between the autocorrelations of the spot index and the futures contracts. For example, in periods of generally high volume, the return autocorrelation of the spot indices drops dramatically. The futures contracts' properties change very little, irrespective of the volume in the market.
- All of these results hold internationally, as well as domestically. This fact is especially interesting given that the cross-correlation across international markets is fairly low, thus providing independent evidence on the issues at hand.

The article is organized as follows. In Section 1, we provide an analysis of the relation between stock indices and their corresponding futures contracts under various assumptions, including the random walk model, a stale price model, and a partial adjustment model. Of special interest, we draw implications for the univariate properties of these series with and without transaction costs. Section 2 describes the data on the various stock indices and futures contracts worldwide, while Section 3 provides the main empirical results of the article. In Section 4 we make some concluding remarks.

## 1. Models of the Spot-Futures Relation

There is a large literature in finance on the relation between the cash market and the stock index futures market, and, in particular, on their lead-lag properties. For example, MacKinlay and Ramaswamy (1988), Stoll and Whaley (1990), and Chan (1992), among others, all look at how quickly

the cash market responds to market-wide information that has already been transmitted into futures prices. While this literature shows that the cash and futures market have different statistical properties, there are several reasons why additional analysis is needed. First, while there is strong evidence that the futures market leads the cash market, this happens fairly quickly. Second, most of the analysis is for indices with very actively traded stocks, such as the S&P 500 or the MMI, which possess very little autocorrelation in their return series. Third, these examinations are at the intraday level and are not concerned with longer horizons that are more relevant for behavioral-based models.

In this section we provide a thorough look at implications for the univariate statistical properties of the cash and futures markets under various theoretical assumptions about market behavior with and without transaction costs. In order to generate these implications, we make the following assumptions:

- The index, $S$, is an equally weighted portfolio of $N$ assets with corresponding futures contract, $F$.[3]
- To the extent possible (i.e., transaction costs aside), there is contemporaneous arbitrage between the spot and futures market. That is, the market is rational with respect to index arbitrage.
- Prices of individual securities, $S_i, i = 1, \ldots, N$, follow a random walk in the absence of any partial adjustment effects. This assumption basically precludes any "equilibrium" time variation in expected returns at high frequencies.
- The dividend processes for each asset, $d_i$, and the interest rate, $i$, are constant.

Under these assumptions, we consider three models. The first is the standard model with no market microstructure effects or partial adjustment behavior. The implications of this model are well known and are provided purely as a benchmark case. The second model imposes a typical market structure bias, namely stale prices, on a subset of the stocks in the index. The third model imposes a lagged adjustment process for some of the stocks in the index. In particular, we assume that some stocks react to market-wide information more slowly due to the reasons espoused in the literature. Transaction costs are then placed on trading in the index to better understand the relation between the cash and futures markets. More detailed derivations of the results for the three models are provided in the appendix.

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[3] The assumption of equal weights is used for simplicity.

## 1.1 Case 1: the random walk model

Applying the cost of carry model and using standard arbitrage arguments, the futures price is simply the current spot price times the compounded rate of interest (adjusted for dividends):[4]

$$F_{t,T} = S_t e^{(i-d)(T-t)},$$

where $F_{t,T}$ is the futures price of the index, maturing in $T-t$ periods, $S_t$ is the current level of the index, $i$ is the continuously compounded rate of interest, $d$ is the continuously compounded rate of dividends paid, and $T$ is the maturity date of the futures contract. Thus, under the cost of carry model, we can write the return on the futures as

$$r_{F_t} = r_{S_t} - i, \tag{1}$$

where $r_F$ is the continuously compounded return (i.e., the log price change) on the futures and $r_S$ is the return on the underlying spot index (see the appendix for details). Note that the change in the continuously compounded interest rate (adjusted for the dividend rate), $\Delta(i-d)$, drops out due to the assumption of constant interest rates and dividend yields (see footnote 4).

Two observations are in order. First, under the random walk model, the autocorrelation of futures returns will mimic that of the spot market, that is, it will be zero. Second, even in the presence of transaction costs, these results should hold, as the futures price should still take on the properties of the present value of the future stock index price, which is the current value of the index in an efficient market.

## 1.2 Case 2: the stale price model

The market microstructure literature presents numerous examples of market structures that can induce non-random walk behavior in security prices. One particular characteristic of the data that has received considerable attention in the literature is nonsynchronous trading. Nonsynchronous trading refers to the fact that stock prices are assumed to be recorded at a particular point in time from period to period, when in fact they are recorded at irregular points in time during these periods. For example, stock indices are recorded at the end of trading using the last transaction price of each stock in the index. If those stocks (i) did not trade at the same time and (ii) did not trade exactly at the close, then the index would be subject to nontrading-induced biases in describing its characteristics. The best known characteristic, of course, is the spurious positive autocorrelation of index returns.

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[4] See, for example, MacKinlay and Ramaswamy (1988). Note that we assume that interest rates and dividend yields are constant. In practice, this assumption is fairly robust due to the fact that these financial variables are significantly less variable than the index itself.

Models of nontrading have appeared throughout the finance literature, including, among others, Fisher (1966), Scholes and Williams (1977), Cohen et al. (1986), Atchison, Butler, and Simonds (1987), Lo and MacKinlay (1990a), and Boudoukh, Richardson, and Whitelaw (1994). There is also substantial empirical literature on the ability of nontrading to explain portfolio autocorrelations at a daily frequency.[5] In perhaps the most definitive study, in which nontrading frequencies are calibrated to those found in intraday data, Kadlec and Patterson (1999) find that nontrading can explain 85%, 52%, and 36% of daily autocorrelations on portfolios of small, random, and large stocks, respectively. In other words, nontrading is important but not the whole story. Similar conclusions are reached by researchers who minimize nontrading by eliminating stocks that do not trade on the relevant days [e.g., Mech (1993), Ogden (1997), and Chordia and Swaminathan (2000)]. Note that ensuring that a stock trades each day is not sufficient to eliminate the nontrading effect, since the last trade may occur at some time other than the close.

Conditioning on trading volume is a natural way to explore the issue of nontrading, but the most relevant study, Chordia and Swaminathan (2000), uses turnover (the number of shares traded divided by the number of shares outstanding) instead of share volume. This methodology leads to the problem that stocks with large share volumes and large numbers of shares outstanding can still be classified as low "volume" stocks. Koutmos (1997) uses volatility, which is correlated with volume, and shows that, across six international markets, index autocorrelations are decreasing in volatility.

If nontrading is unable to explain portfolio autocorrelations completely, what are other potential sources of stale prices? On the NYSE, market makers are rewarded for maintaining price continuity. Specifically this is one dimension on which they are evaluated, and these evaluations play a role when the NYSE decides to which specialist company to assign a new stock (listing). Consequently a specialist may be willing to take a loss on a few small trades in order to avoid updating a price on a thinly traded security too quickly. Alternatively, investors may be slow to remove limit orders after the arrival of new information.[6] Consequently some trades may take place at these stale prices.

On the Nasdaq, there is substantial evidence that SOES bandits (individual investors using the Small Order Execution System) trade profitably by exploiting the stale quotes of Nasdaq market makers [Harris and Schultz (1998)]. These trades lead to transactions at stale prices. Note that, in general, SOES bandits are exploiting information that is already in the market in

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[5] There are also puzzling positive autocorrelations at the weekly frequency for some artificially constructed portfolios, but for our indices, spot autocorrelations are essentially zero at this frequency; therefore extending the horizon is of no interest.

[6] We thank Ron Masulis for pointing out this explanation.

the form of updated quotes from other dealers who make markets in the same stocks. There are a substantial number of SOES bandits—more than 2,000 according to Harris and Schultz (1998). Why don't market makers hire additional employees to maintain updated quotes? The obvious answer is that it is cheaper to allow SOES bandits to provide this service, and to make losses on these trades, than to hire additional employees. Harris and Schultz (1998) also speculate that it is difficult for dealers to structure contracts with their traders to give them the appropriate incentives to maintain updated quotes.

More generally, if there is any cost to either updating quotes or to updating/removing old limit orders, we would expect to see stale prices. Of course, these stale prices are much more likely to be observed for thinly traded stocks. The key point is that these prices are not the result of behavioral biases, but rather of imperfect markets, that is, costs and frictions. That is, the prices do not represent market values at which one could actually trade a reasonable number of shares.

In this article we choose a slightly modified version of the simple model of Lo and MacKinlay (1990a) to illustrate the relation between the spot and futures markets in the presence of stale prices. In any given period there is an exogenous probability $\pi_i$ that the price of stock $S_i$ is not updated, either due to nontrading or some other feature of the market. Furthermore, each security's return, $r_i$, is described by a zero-mean, i.i.d. factor, $m$. Lo and MacKinlay (1990a) show that the measured excess returns on an equally weighted portfolio of $N$ securities, denoted $r_{\widehat{S}}$, can be written as

$$r_{\widehat{S}_t} = \mu + (1 - \pi) \sum_{k=0}^{\infty} \pi^k m_{t-k},$$

where $\mu$ is the average mean return of the N stocks, and $\pi$ is the probability of observing a stale price assuming equal probabilities across the stocks. Of course, the true returns are simply described by

$$r_{S_t} = \mu + m_t,$$

where any idiosyncratic risk has been diversified away.

In a no-arbitrage world, the price of the futures contract will reflect the present value of the stock index at maturity. That is,

$$F_{t,T} = \text{PV}(\widehat{S}_T)e^{i(T-t)} \tag{2}$$

(see the appendix for details). Note that due to nontrading the present value of the index is no longer its true value, but instead a value that partly depends on the current level of staleness in prices. This is because the futures price is based on the measured value of the index at maturity, which includes stale prices. Within the Lo and MacKinlay (1990a) model, prices today have some, albeit a small amount of, information about the staleness of prices in

the distant future. However, as long as the contract is not close to expiration, the effect, which is of order $\pi^{T-t}$, is miniscule. In particular, it is possible to show that the corresponding futures return is

$$r_{F_{t+1}} = (1 - \pi^{T-t})(r_{S_{t+1}} - i) \qquad (3)$$

(see the appendix for details).

Not surprisingly, in contrast to the measured index returns, futures returns will not be autocorrelated, due to the efficiency of the market and the no arbitrage condition between the cash and futures market. However, the futures return will differ slightly from the true excess spot return because it is priced off the measured value of the spot at maturity.

### 1.3 Case 3: the partial adjustment model

As an alternative to market microstructure-based models, the finance literature has developed so-called partial adjustment models. Through either information transmission, noise trading or some other mechanism, these models imply that some subset of securities partially adjust, or adjust more slowly, to market-wide information. While there is some debate about whether these models can be generated in both a reasonable and rational framework, all the models impose some restrictions on trading so that the partial adjustment effects cannot get arbitraged away. There are a number of models that produce these types of partial adjustment effects [e.g., see Holden and Subrahmanyam (1992), Foster and Viswanathan (1993), Badrinath, Kale, and Noe (1995), Klibanoff, Lamont, and Wizman (1998), Chordia and Swaminathan (2000), and Llorente et al. (2001)].

Here we choose one particular model, which coincides well with Section 1.2, namely Brennan, Jegadeesh, and Swaminathan (1993). We assume that the index is made up of two equally weighted portfolios of stocks, $S_1$ and $S_2$, which are full ($S_1$) and partial ($S_2$) response stocks. Brennan, Jegadeesh, and Swaminathan (1993) consider stocks followed by many analysts versus those followed by only a few analysts. Assume that the returns on these two portfolios can be written as

$$r_{1t} = \mu_1 + \beta_1 m_t$$
$$r_{2t} = \mu_2 + \beta_2 m_t + \gamma_2 m_{t-1}.$$

Thus, for whatever reason, the return on the partial response stocks is affected by last period's realization of the market factor. One offered explanation is that market-wide information is only slowly incorporated into certain stock prices, yielding a time-varying expected return that depends on that information. Note that similar to Lo and MacKinlay (1990a) and Section 1.2 above, we have also assumed that these two portfolios are sufficiently well diversified that there is no remaining idiosyncratic risk.

Assume that the index contains a fraction $\omega$ of the fully adjusting stock portfolio and a fraction $1-\omega$ of the partially adjusting portfolio. Under the assumption of no transaction costs and no arbitrage, the returns on the index and its corresponding futures contract can be written as

$$r_{S_t} = \mu + \beta m_t + \gamma m_{t-1}$$
$$r_{F_t} = r_{S_t} - i \tag{4}$$

where

$$\mu = \omega\mu_1 + (1-\omega)\mu_2$$
$$\beta = \omega\beta_1 + (1-\omega)\beta_2$$
$$\gamma = (1-\omega)\gamma_2.$$

The returns on both the stock index and its futures contract coincide and therefore pick up similar autocorrelation properties. In fact, their autocorrelation is

$$\frac{[\omega\beta_1 + (1-\omega)\beta_2](1-\omega)\gamma_2}{[\omega\beta_1 + (1-\omega)\beta_2]^2 + [(1-\omega)\gamma_2]^2}.$$

For indices with relatively few partial-adjustment stocks (i.e., high $\omega$) or low lagged response coefficients (i.e., small $\gamma_2$), the autocorrelation reduces to approximately

$$\frac{(1-\omega)\gamma_2}{\omega\beta_1 + (1-\omega)\beta_2}.$$

With the additional assumption that the beta of the index to the factor is approximately one, an estimate of the autocorrelation is $(1-\omega)\gamma_2$. That is, the autocorrelation depends on the proportion of partially adjusting stocks in the index and on how slowly these stocks respond.

These results should not seem surprising. With the no-arbitrage condition between the cash and futures market, the price of the futures equals the present value of the future spot index, which is just the current value of the index. That is, though the spot price at maturity includes lagged effects, the discount rate does also, leading to the desired result. With nontrading, because the lagged effects are *spurious*, discounting is done at $\mu$, which leads to zero autocorrelation of futures returns.

In response, a behavioralist might argue that the futures return does not pick up the properties of the cash market due to the inability of investors to actually conduct arbitrage between the markets. Of course, the most likely reason for the lack of arbitrage is the presence of transaction costs, that is, commissions and bid-ask spreads paid on the stocks in the index and the futures contract. The level of these transaction costs depend primarily on

costs borne by the institutional index arbitrageurs in these markets. Abstracting from any discussion of basis risk and the price of that risk, we assume here that arbitrageurs buy or sell the index at an additive cost of $\delta$. Thus round-trip transaction costs per arbitrage trade are equal to $2\delta$. In this environment it is possible to show that, in the absence of arbitrage, the futures price must satisfy the following constraints:

$$-\delta(1 + e^{i(T-t)}) \leq F_{t,T} - S_t e^{(i-d)(T-t)} \leq \delta(1 + e^{i(T-t)}). \tag{5}$$

In other words, the futures price is bounded by its no arbitrage value plus/minus round-trip transaction costs.

What statistical properties do futures returns have within the bounds? There is no obvious answer to this question found in the literature. If the futures are priced off the current value of the spot index, then, as described above, futures returns will inherit the autocorrelation properties of the index return. Alternatively, suppose investors in futures markets are more sophisticated, or at least respond to information in $m$ fully. That is, they price futures off the future value of the spot index, discounted at the rate $\mu$. In this case the futures returns will not be autocorrelated, and expected returns on futures will just equal $\mu - i$.

Of course, if the futures-spot parity lies outside the bound, then arbitrage is possible, and futures prices will move until the bound is reached. Futures prices at time $t + 1$ will lie outside the bound (in the absence of arbitrage) under the following condition:

$$|m_t| \geq \frac{\delta \left(1 + e^{i(T-t)}\right)}{(1 - \omega)\gamma_2}. \tag{6}$$

That is, three factors increase the possibility of lying outside the bound: (i) large recent movements in the stock index (i.e., $|m_t|$), (ii) low transaction costs (i.e., $\delta$), and (iii) large autocorrelations in the index (i.e., $(1 - \omega)\gamma_2$). If Equation (6) is met, then, even in the case of sophisticated futures traders, expected returns on futures will not be a constant, but instead capture some of the irrationality of the index. Specifically, if Equation (6) is true, then

$$E_t[r_{F_{t+1}}] = \mu - i + (1 - \omega)\gamma_2 m_t \pm \delta(1 + e^{i(T-t)}). \tag{7}$$

This model generates a particular pattern in expected futures returns. Within the bounds, expected futures returns are flat. Outside the bounds, futures begin to take on the properties of the underlying stock index, and futures returns are positively autocorrelated for more extreme past movements.

While this represents one possible partial adjustment model, there are alternatives. For example, consider a model with two types of traders—rational ones in the futures markets and irrational ones in the spot market.[7] The rational traders in the futures market do not price futures as derivatives per se,

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[7] We thank one of the referees for providing this alternative framework.

but instead price them off the equilibrium fundamentals in the market. Then, inside the arbitrage bounds, the stock index, being driven by "irrational" traders, will reflect the partial adjustment biases and the futures will remain uncorrelated. Outside the bounds, the index arbitrageurs will cause futures and spot prices to move together, and the "irrational" spot index will be forced to pick up the properties of the futures index. In other words, in contrast to the model above, the spot index mimics the futures contract, leading to quite different autocorrelation patterns in the index and corresponding futures. However, both these models have the same implication that, outside the bound, the autocorrelation properties of the spot and futures are the same. It is this common restriction that forms the basis of some of the empirical analysis to come.

### 1.4 Implications

The above models for index and corresponding futures prices are clearly stylized, extreme, and very simple. For example, the Lo and MacKinlay (1988) model of nontrading has been generalized to heterogeneous nontrading and heterogeneous risks of stocks within a portfolio, which provides more realistic autocorrelation predictions [see, e.g., Boudoukh, Richardson, and Whitelaw (1994)]. Which model is best, however, is beside the point for this article. The purpose of the models is to present, in a completely transparent setting, different implications of two opposing schools of thought. The first school believes that the time-varying patterns in index returns are not tradeable (or at least not tradeable in any relevant quantity), and in fact may be completely spurious, that is, an artifact of the way we measure returns. The second school believes that these patterns are real and represent actual prices, resulting from some sort of inefficient information transmission in the market, whether it be irrationality or severe trading frictions with asymmetric information. The implications we draw from these models are quite general and robust to more elaborate specifications of stale pricing or agents' ability to incorporate information quickly.

Figure 1 provides the main implications of the three models—the random walk, stale price, and partial adjustment models—in terms of the difference between expected returns in the spot and futures markets. Given the lagged realization of the factor, $m_{t-1}$, what is the current difference in expected returns, $E[r_{S_t} - r_{F_t}|m_{t-1}]$? As discussed in Section 1.2, in a market microstructure setting, the index returns will be positively autocorrelated, while the futures returns will not be autocorrelated (bid-ask bounce aside). As Figure 1B shows, this leads to a positive slope in the relation between the lagged shock and the difference in expected returns. In contrast, the partial adjustment model of Section 2.3 predicts spot index and futures returns will inherit the same autocorrelation properties, that is, the difference will not depend on the lagged shock. However, in the presence of transaction costs, this flat relation does not necessarily hold everywhere. As Figure 1C

A: Random Walk Model

$E[r_{St}-r_{Ft}|m_{t-1}]$

$m_{t-1}$

B: Stale Price Model

$E[r_{St}-r_{Ft}|m_{t-1}]$

$m_{t-1}$

C: Partial Adjustment Model

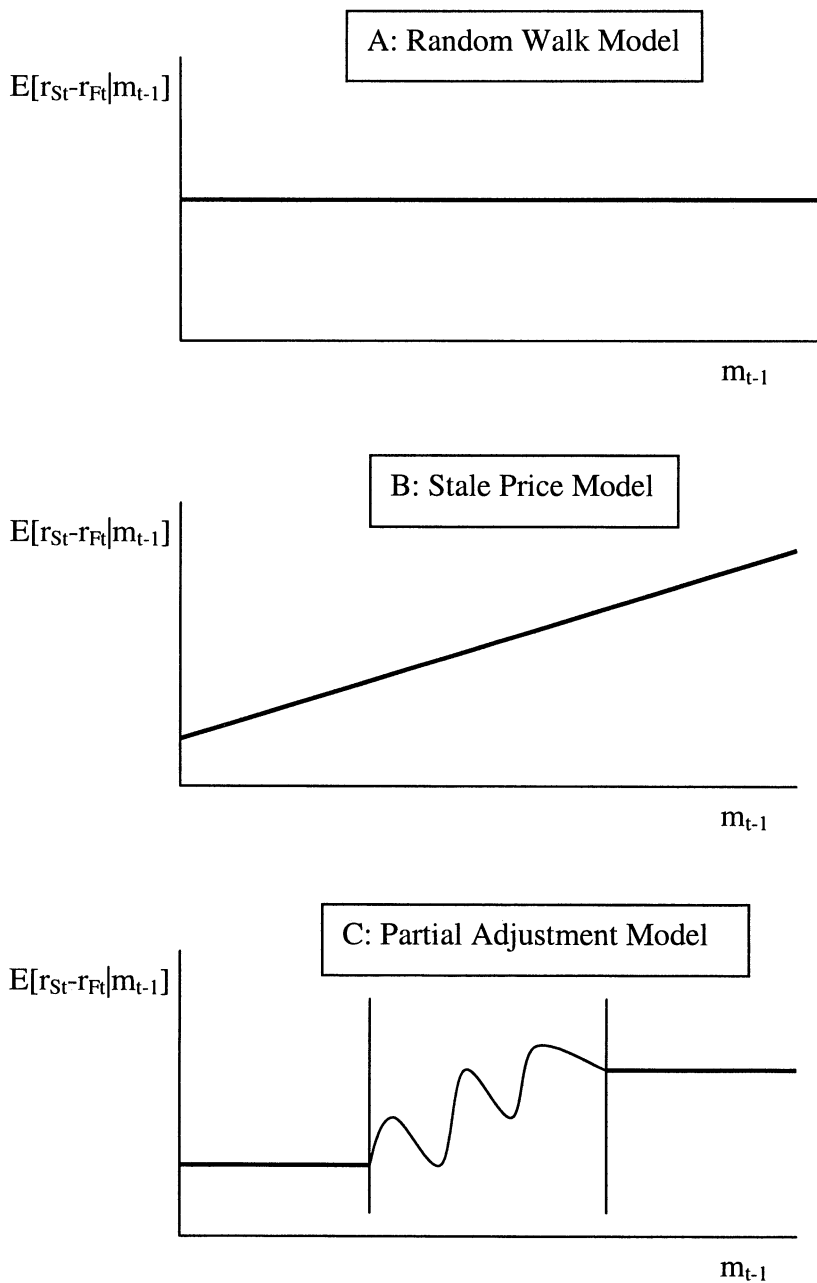$E[r_{St}-r_{Ft}|m_{t-1}]$

$m_{t-1}$

**Figure 1**
A schematic representation of the implications of the three models for the differences in expected returns between the spot and futures markets, conditional on the value of the lagged market factor.

shows, the spot index and futures returns will behave similarly in periods of big stock price movements, when arbitrage is possible, and possibly quite differently in periods of small movements. The squiggly intermediate line indicates that there are no definitive predictions for the relation. Finally, the pure random walk model of Section 1.1 predicts a flat relation for all past values of the factor as shown in Figure 1A.

In addition, we can make another observation about the relative statistical properties of index and futures returns. In the stale price setting, the magnitude of the difference between spot and futures returns will be related to the level of microstructure biases. In particular, as the stale price probability $\pi$ goes down, that is, higher volume, the properties of spot index returns, such as their autocorrelation, should look like the true return process. Moreover, while the properties of the index return change with volume, the properties of the futures return should remain the same for moderate- to long-maturity contracts.

Of course, this implication may also be consistent with partial adjustment models. For example, Klibanoff, Lamont, and Wizman (1998) develop a model to explain the pricing anomalies in closed-end country funds. In their model there are irrational partial adjustment traders in closed-end country funds and rational traders in the markets for the underlying assets; however, they impose a strict prohibition on arbitrage trade across the two markets. While the model does not have any direct implications for volume, they argue that it suggests faster adjustment when volume is high. This prediction is born out in the data. Nevertheless, the relation between autocorrelations and volume is a necessary condition for most reasonable stale price models, so it is worth examining in our dataset.

These observations are the basis for an empirical comparison of spot index and corresponding futures returns. To build up as much independent evidence as possible, this analysis is performed on over 24 indices across 15 countries. Because the daily index returns across these countries are not highly correlated, the results here will have considerably more power to differentiate between the implications of the two schools.

## 2. The Data

All the data are collected from Datastream: specifically, price levels of each stock index and corresponding futures contract at the close of trade every day, daily volume on the overall stock market in a given country, and daily open interest and volume for each futures contract. The data are collected to coincide with the length of the available futures contract. For example, if the futures contract starts on June 1, 1982 (as did the S&P 500), all data associated with this contract start from that date.

The futures data are constructed according to the usual conventions. In particular, a single time series of futures prices is spliced together from individual futures contract prices. For liquidity, the nearest contract's prices are

used until the first day of the expiration month, then the next nearest is used. For a futures contract to be used, we require at least four years of data (or roughly 1,000 observations) to lower the standard errors of the estimators. This leads us to drop a number of countries such as the Eastern European block, emerging markets in Asia such as Thailand, Korea, and Malaysia, as well as some small stock-based indices like the MDAX in Germany. Given this criteria, we are left with 24 futures contracts on stock indices covering 15 countries. Table 1 gives a brief description of each contract, the exchange it is traded on, its country affiliation, its starting date, as well as some summary statistics on the futures returns, open interest, and trading volume. Summary statistics on the underlying index returns are also provided.

Some observations are in order. First, given the wide breadth of countries used in this analysis and the fact that daily returns across countries have relatively small contemporaneous correlations (the mean and median correlation are 0.425 and 0.373, respectively), the data provide considerable independent information about the economic implications described in Section 1. Second, the futures contracts have considerable open interest and daily volume in terms of the number of contracts. Table 1 provides the mean for these contracts, and, for less liquid ones such as the Russell 2000 and Maxi Value Line, these means are still high relative to less liquid stocks, for example, 455 and 197 contracts per day, respectively. The fact that these contracts are liquid allows us to focus primarily on market microstructure biases related to the stocks in the underlying index. Section 3.4 addresses any potential biases related to the futures contracts.

## 3. Empirical Results

In this section we focus on providing evidence for or against the implications derived from the models of Section 1. In particular, we investigate (i) the autocorrelation properties of the spot index and corresponding futures returns, (ii) the relative time-varying properties of spot index and futures returns conditional on recent small and large movements in returns, and (iii) the relation between these time-varying properties and underlying stock market volume.

### 3.1 Autocorrelations
Table 2 presents the evidence for daily autocorrelations of spot index returns and their corresponding futures returns across 24 contracts. The most startling evidence is that, for every contract, the spot index autocorrelation exceeds that of the futures. This cannot be explained by common sampling error, as many of the contracts have low correlations with each other because of the 15-country cross section. Figure 2 presents a scatter plot of the autocorrelations of the futures and spot indices, that is, a graphical representation of these results. On the 45 degree line, the spot and futures autocorrelations

**Table 1**
**Summary statistics**

| Contract | Exchange | Country | No. | Start | Returns | | | | Open interest | | Trading volume | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Index | | Futures | | | | | |
| | | | | | Mn | SD | Mn | SD | Mn | SD | Mn | SD |
| S&P 500 | CME | US | 4220 | 6/1/82 | 0.06 | 1.00 | 0.06 | 1.21 | 274629 | 130926 | 62114 | 32099 |
| Russell 2000 | CME | US | 1510 | 2/11/93 | 0.04 | 0.79 | 0.04 | 0.96 | 5453 | 3993 | 455 | 705 |
| Nikkei 225 | CME | US | 2158 | 10/10/90 | -0.02 | 1.44 | -0.02 | 1.54 | 20348 | 8117 | 1736 | 1907 |
| NYSE | NYSE | US | 4236 | 5/6/82 | 0.05 | 0.92 | 0.05 | 1.20 | 6751 | 3707 | 6207 | 3940 |
| Maxi Value Line | KCBT | US | 2705 | 5/23/88 | 0.05 | 0.67 | 0.05 | 0.92 | 2498 | 2011 | 197 | 304 |
| FTSE 100 | LIFFE | UK | 3731 | 5/3/84 | 0.04 | 0.96 | 0.04 | 1.11 | 94038 | 67982 | 9041 | 10053 |
| FTSE 250 | LIFFE | UK | 1248 | 2/25/94 | 0.02 | 0.57 | 0.02 | 0.64 | 5240 | 1883 | 211 | 648 |
| TOPIX | TSE | Japan | 2570 | 9/5/88 | -0.03 | 1.21 | -0.03 | 1.37 | 61377 | 41801 | 10301 | 6296 |
| Nikkei 225 | OSX | Japan | 2569 | 9/5/88 | -0.03 | 1.46 | -0.03 | 1.46 | 153142 | 76027 | 37204 | 23681 |
| Nikkei 300 | OSX | Japan | 1229 | 2/14/94 | -0.02 | 1.19 | -0.02 | 1.36 | 120847 | 29519 | 7946 | 12511 |
| DAX | EUREX | Germany | 1909 | 7/1/91 | 0.06 | 1.19 | 0.06 | 1.27 | 175678 | 89835 | 18856 | 11907 |
| Swiss Market | EUREX | Switz. | 2072 | 1/19/90 | 0.08 | 1.08 | 0.08 | 1.12 | 23961 | 29148 | 6629 | 8701 |
| AEX | Amsterdam | Neth. | 2601 | 10/24/88 | 0.06 | 1.07 | 0.06 | 1.13 | 21617 | 14500 | 4765 | 5577 |
| BELFOX 20 | BELFOX | Belgium | 1314 | 10/29/93 | 0.07 | 0.89 | 0.07 | 0.94 | 6632 | 4073 | 1430 | 1933 |
| Hang Seng | HKFE | HK | 2741 | 1/18/88 | 0.05 | 1.75 | 0.05 | 2.01 | 31639 | 25014 | 11800 | 12777 |
| IBEX 35 Plus | MEFF | Spain | 1709 | 4/20/92 | 0.08 | 1.29 | 0.08 | 1.50 | 36102 | 21414 | 14519 | 15056 |
| MIB 30 | MIF | Italy | 1052 | 11/28/94 | 0.08 | 1.58 | 0.08 | 1.69 | 19762 | 9261 | 14008 | 9486 |
| OMX | OMX | Sweden | 2286 | 1/2/90 | 0.06 | 1.35 | 0.05 | 1.61 | 59426 | 68909 | 7017 | 13548 |
| ATX | OTOT | Austria | 1610 | 8/7/92 | 0.02 | 1.14 | 0.02 | 1.28 | 30639 | 15681 | 1596 | 1490 |
| Nikkei 225 | SIMEX | Singapore | 3012 | 1/6/87 | -0.01 | 1.45 | -0.01 | 1.69 | 13236 | 11832 | 152469 | 76455 |
| Nikkei 300 | SIMEX | Singapore | 989 | 2/3/95 | -0.02 | 1.27 | -0.02 | 1.36 | 14837 | 8412 | 538 | 1779 |
| Toronto 35 | TFE | Canada | 2005 | 2/25/91 | 0.03 | 0.86 | 0.03 | 1.03 | 14507 | 7193 | 647 | 999 |
| Australia | SFE | Australia | 3825 | 1/3/84 | 0.03 | 1.01 | 0.03 | 1.47 | 23795 | 34821 | 1344138 | 867885 |
| CAC 40 | MATIF | France | 2578 | 10/3/88 | 0.04 | 1.20 | 0.04 | 1.28 | 66795 | 67406 | 6565 | 8983 |

For each index/contract we report the exchange on which the futures contract is traded, the country in which it trades, the number of observations, the start date of the sample (all samples end on 2/3/99 with the exception of the SIMEX-traded Nikkei 225, which ends on 2/1/99), the mean and standard deviation of daily index and futures returns (in percent), and the mean and standard deviation of open interest and trading volume in the futures market (in number of contracts).

**Table 2**
**Daily autocorrelations of index and futures returns**

| Contract | $\rho$(Index) | SE | $\rho$(Futures) | SE | $\chi_1^2$ | $p$-value |
|---|---|---|---|---|---|---|
| S&P 500 | 0.0297 | 0.0272 | −0.0367 | 0.0279 | 27.1078 | 0.0000 |
| Russell 2000 | 0.2247** | 0.0457 | 0.0711 | 0.0400 | 47.4011 | 0.0000 |
| Nikkei 225 (CME) | −0.0345 | 0.0261 | −0.0904** | 0.0264 | 6.5437 | 0.0105 |
| NYSE | 0.0618** | 0.0284 | −0.0582** | 0.0290 | 45.3993 | 0.0000 |
| Maxi Value Line | 0.1965** | 0.0310 | −0.0227 | 0.0359 | 63.2965 | 0.0000 |
| FTSE 100 | 0.0822** | 0.0314 | 0.0197 | 0.0291 | 14.0510 | 0.0002 |
| FTSE 250 | 0.2298** | 0.0657 | 0.1283** | 0.0569 | 6.4278 | 0.0112 |
| TOPIX | 0.1164** | 0.0275 | −0.0037 | 0.0275 | 49.6813 | 0.0000 |
| Nikkei 225 (OSX) | −0.0023 | 0.0250 | −0.0192 | 0.0255 | 1.6923 | 0.1933 |
| Nikkei 300 (OSX) | 0.0327 | 0.0392 | −0.0706* | 0.0349 | 43.7841 | 0.0000 |
| DAX | 0.0266 | 0.0282 | 0.0042 | 0.0299 | 3.5746 | 0.0587 |
| Swiss Market | 0.0577* | 0.0300 | 0.0318 | 0.0288 | 5.5568 | 0.0184 |
| AEX | 0.0346 | 0.0255 | 0.0036 | 0.0279 | 6.0998 | 0.0135 |
| BELFOX 20 | 0.1491** | 0.0394 | 0.0970** | 0.0365 | 4.1320 | 0.0421 |
| Hang Seng | 0.0238 | 0.0438 | −0.0409 | 0.0462 | 20.5247 | 0.0000 |
| IBEX 35 Plus | 0.1341** | 0.0286 | 0.0134 | 0.0310 | 39.0259 | 0.0000 |
| MIB 30 | 0.0265 | 0.0358 | −0.0273 | 0.0366 | 11.7325 | 0.0006 |
| OMX | 0.1175** | 0.0263 | −0.0372 | 0.0370 | 18.3681 | 0.0000 |
| ATX | 0.0976** | 0.0382 | −0.0082 | 0.0399 | 74.1328 | 0.0000 |
| Nikkei 225 (SIMEX) | −0.0065 | 0.0271 | −0.0459 | 0.0308 | 1.2961 | 0.2549 |
| Nikkei 300 (SIMEX) | 0.0302 | 0.0429 | −0.0527 | 0.0425 | 7.8216 | 0.0052 |
| Toronto 35 | 0.0973** | 0.0310 | −0.0751 | 0.0706 | 5.6938 | 0.0170 |
| Australia | 0.1024** | 0.0290 | −0.0794* | 0.0381 | 25.3879 | 0.0000 |
| CAC 40 | 0.0486** | 0.0221 | 0.0125 | 0.0225 | 13.1828 | 0.0003 |

For each contract/index we report the daily autocorrelations of index and futures returns. The $\chi_1^2$ statistic tests $\rho$(Index) = $\rho$(Futures), and we report the corresponding $p$-value. Standard errors are serial correlation and heteroscedasticity-adjusted using Newey and West (1987). *, **Significance at the 10% and 5% levels, respectively.

coincide; however, as the graph shows, all the points lie to the right of this line. Thus all the spot autocorrelations are higher than their corresponding futures.

Note that while the autocorrelations of both the index and futures alone are somewhat difficult to pinpoint due to the size of the standard errors, the autocorrelation differences should be very precisely estimated given the high contemporaneous correlation between the index and futures. This is an important point and one which is crucial to understanding the statistical evidence presented in this article, especially given the difficulty in estimating autocorrelations precisely.

To understand this point more clearly, assume that, for expositional ease, the spot and futures returns are conditionally homoscedastic. Then asymptotically, under the random walk null[8]

$$\sqrt{T} \begin{pmatrix} \hat{\rho}_S \\ \hat{\rho}_F \end{pmatrix} \sim^{asy} N\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \gamma_{S,F}^2 \\ \gamma_{S,F}^2 & 1 \end{pmatrix} \right), \tag{8}$$

where $\gamma_{S,F}$ is the correlation coefficient between the spot and futures return, and $\hat{\rho}$ is the autocorrelation of the returns. Consider a test of whether $\rho_S = \rho_F$.

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[8] See, for example, Richardson (1989). In the actual empirical tests to follow, the statistics are adjusted for conditional heteroscedasticity using the method proposed by Newey and West (1987).
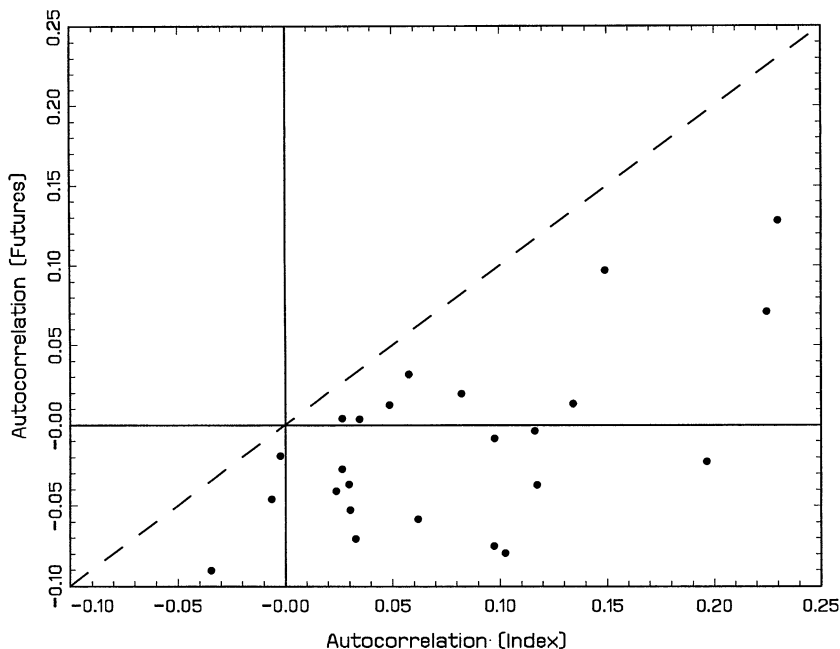
**Figure 2**
Autocorrelations of futures and spot returns for the 24 indices. The dashed line is the 45 degree line on which spot and futures autocorrelations are equal.

By weighting a squared difference of normals by their variance-covariance matrix, the resulting statistic will follow a chi-squared distribution. In this case,

$$T\left(\frac{(\hat{\rho}_S - \hat{\rho}_F)^2}{2(1 - \gamma_{S,F}^2)}\right) \sim \chi_1^2.$$

For $\gamma_{S,F}$ close to 1, which is clearly the case here, small differences between the autocorrelation estimators provide strong evidence against the null. For example, $\gamma_{S,F} = 0.99$ versus $\gamma_{S,F} = 0.95$ and $\gamma_{S,Ft} = 0.50$ produces a statistic 5 and 38 times larger, respectively.

Several observations are in order. First, other than the Nikkei 225 (which has a marginally negative value), all of the spot index returns are positively autocorrelated. Some of these indices, such as the Russell 2000 (small firm U.S.), Value Line (equal-weighted U.S.), FTSE 250 (medium-firm U.K.), TOPIX (all firms Japan), OMX (all firms Sweden), and Australian All-Share, have fairly large autocorrelations—0.22, 0.19, 0.21, 0.10, 0.12, and 0.10, respectively. Of interest, these indices also tend to be ones that include large weights on firms which trade relatively infrequently. In contrast, the value-weighted indices with large, liquid, actively traded stocks, such as the S&P

500 (largest 500 U.S. firms), FTSE 100 (100 most active U.K. firms), Nikkei 225 (225 active Japanese stocks), and DAX (active German firms), are barely autocorrelated—0.03, 0.08, −0.01, and 0.02, respectively.

Second, a quick perusal of Figure 2 shows that autocorrelations in the futures market tend to be positively related to autocorrelations in the spot market. That is, the points do not appear to be scattered randomly around the horizontal line that represents zero autocorrelation in the futures market. This result is to be expected if there is any sampling error in the estimation, even if the true futures autocorrelations are all zero. Equation (8) illustrates the effect of the high contemporaneous correlation between the spot and futures markets on the correlation of the autocorrelation estimates. Basically the autocorrelation estimates in the spot and futures markets will pick up the same estimation error. This feature of the data is why it is so important to focus on the difference between the spot and futures autocorrelations and to take account of their relationship to the common underlying fundamentals.

Third, in terms of formal statistical tests, for 21 of 24 contracts we can reject the hypothesis that the spot index autocorrelation equals that of its futures contract at the 5% level. To the extent that this is one of the main comparative implications of market microstructure versus partial adjustment models, this evidence supports the microstructure-based explanation.[9] The evidence is particularly strong as 17 of the differences are significant the 1% level. These levels of significance should not be surprising given that the index and its futures capture the same aggregate information, yet produce autocorrelation differences of at least 10% on a daily basis in 12 cases.

Fourth, and finally, Equation (8) can be rewritten to reflect the asymptotic relation between autocorrelation estimators across countries. In particular,

$$\sqrt{T}\begin{pmatrix}\hat{\rho}_i\\\hat{\rho}_j\end{pmatrix} \sim^{asy} N\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & \gamma_{i,j}^2\\\gamma_{i,j}^2 & 1\end{pmatrix}\right),$$

where $\gamma_{i,j}$ is the correlation coefficient between the returns on country $i$ and country $j$. Consider two of the a priori expected stale price portfolios, the Russell 2000 and the TOPIX. The correlation between their returns is 0.23, which leads to a correlation between their autocorrelation estimators of just 0.05. Thus the fact that both indices show significant differences between autocorrelations in the futures and spot markets provides powerful evidence precisely because the series are fairly independent.

### 3.2 Time-varying patterns of returns

The results in Section 3.1 are suggestive of differences between the properties of spot index and futures returns. While this tends to be inconsistent with

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[9] Of course, due to bid-ask bounce, futures returns should have negative autocorrelations, which could partially explain the differences even without index microstructure biases. Section 3.4 looks at the extent to which futures biases can explain the result.

partial adjustment-based explanations of the data, we showed in Section 1.3 that it is possible to construct a reasonable scenario in which large differences can appear. Specifically, the reason why partial adjustment models imply a one-to-one relation between spot and futures returns is that they are linked via spot-futures arbitrage. If spot-futures arbitrage is not possible due to transaction costs, then theoretically spot and futures prices might diverge if their markets are driven by different investors. Figure 1C illustrates the implication of this transaction-based model. Conditional on extreme recent movements, the statistical properties of spot and futures returns should be similar; for small movements, they can follow any pattern, including the spot return and futures return having quite different autocorrelation patterns.

In order to test this implication directly, consider a piecewise linear regression of the difference between the spot and futures return on the most recent lag of the futures return.[10] In particular,

$$r_{S,t+1} - r_{F,t+1} = a + b_1 r_{F,t} + (b_2 - b_1) \max[0, r_{F,t} - a_1]$$
$$+ (b_3 - b_2) \max[0, r_{F,t} - a_2] + \epsilon_{t+1}, \qquad (9)$$

where $a_1$ and $a_2$ are the breakpoints of the piecewise regression. These breakpoints are equivalent to the transaction costs bounds described in Section 1.3. Here we initially choose these points as $-1.0\%$ and $1.0\%$, respectively. Thus any daily return of plus/minus 1% or greater in magnitude allows index arbitrage to take place. The coefficients $b_1, b_2$, and $b_3$ reflect the slopes of the piecewise relation. In the context of Figure 1C, $b_1$ and $b_3$ are zero, while $b_2$ is not specified under the partial adjustment model. In the stale price model, bid-ask bounce aside, these coefficients are all greater than zero (see Figure 1B).

Table 3 presents the regression results from Equation (9) for each contract across the 15 countries. From the partial adjustment viewpoint, Equation (9) implies, as its null hypothesis, a series of multivariate inequality constraints, $H_0 : b_1 = 0$ and $b_3 = 0$ versus the alternative $H_A : b_1 \geq 0$ and $b_3 \geq 0$. Eighteen of the 21 contracts reject the partial adjustment theory at conventional 5% levels using an inequality restrictions-based test statistic [see Wolak (1987) and Boudoukh, Richardson, and Smith (1993) for a description of the test methodology]. Moreover, some of these rejections are so strong that the $p$-values are zero to four decimal places. Just as important, however, is the fact that almost all the $b_1$ and $b_3$ coefficients are positive (i.e., only 3 negative estimates among 42), which again implies that the time variation of the expected spot index returns is greater than that of its corresponding futures

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[10] These tests can only be performed for contracts in which both the futures and spot trade contemporaneously, thus ruling out, for example, the Nikkei futures contract that trades on the Chicago Mercantile Exchange. Also note that there is some additional noise because the spot and futures contracts do not close simultaneously. For example, in the U.S. markets, many of the relevant futures contracts close 15 minutes after the spot market.

**Table 3**
**Time variation in the spot-futures return spread**

| Contract | $b_1$ | SE | $b_2$ | SE | $b_3$ | SE | $\chi_2^{-2}$ | $p$-value |
|---|---|---|---|---|---|---|---|---|
| S&P 500 | 0.0605* | 0.0341 | 0.1321** | 0.0121 | −0.0115 | 0.0471 | 5.6711 | 0.0257 |
| Russell 2000 | 0.1604** | 0.0336 | 0.1463** | 0.0194 | 0.1247** | 0.0576 | 24.7584 | 0.0000 |
| NYSE | 0.1012** | 0.0432 | 0.1782** | 0.0151 | 0.0707 | 0.0655 | 5.7518 | 0.0249 |
| Maxi Value Line | 0.3487** | 0.0870 | 0.1371** | 0.0225 | 0.0923 | 0.0562 | 19.0777 | 0.0000 |
| FTSE 100 | 0.1749** | 0.0290 | 0.0570** | 0.0123 | 0.0777** | 0.0358 | 49.6349 | 0.0000 |
| FTSE 250 | −0.0116 | 0.0519 | 0.0206 | 0.0276 | 0.4934** | 0.2397 | 5.0722 | 0.0336 |
| TOPIX | 0.1302** | 0.0388 | 0.1888** | 0.0172 | 0.0379 | 0.0297 | 11.3162 | 0.0014 |
| Nikkei 225 (OSX) | 0.0246 | 0.0272 | 0.0792** | 0.0206 | 0.0089 | 0.0373 | 0.8164 | 0.3467 |
| Nikkei 300 (OSX) | 0.0654 | 0.0411 | 0.1408** | 0.0212 | 0.0734 | 0.0385 | 7.3977 | 0.0091 |
| DAX | 0.1037** | 0.0364 | 0.0023 | 0.0153 | 0.0381** | 0.0171 | 13.0319 | 0.0005 |
| Swiss Market | 0.0176 | 0.0199 | 0.0732** | 0.0114 | 0.0344** | 0.0171 | 4.6283 | 0.0408 |
| AEX | 0.0107 | 0.0283 | 0.0501** | 0.0115 | 0.0342** | 0.0173 | 4.0076 | 0.0566 |
| BELFOX 20 | −0.0635 | 0.0619 | 0.0920** | 0.0208 | 0.0150 | 0.0421 | 0.2673 | 0.5132 |
| Hang Seng | 0.0836** | 0.0334 | 0.1622** | 0.0298 | 0.0838** | 0.0340 | 8.3740 | 0.0063 |
| IBEX 35 Plus | 0.1084** | 0.0270 | 0.1451** | 0.0268 | 0.2147** | 0.0681 | 24.7747 | 0.0000 |
| MIB 30 | 0.0351 | 0.0249 | 0.1020** | 0.0258 | 0.0767** | 0.0290 | 8.1925 | 0.0064 |
| OMX | 0.1962** | 0.0612 | 0.0025 | 0.0300 | 0.2675** | 0.0984 | 19.9442 | 0.0000 |
| ATX | 0.1361** | 0.0231 | 0.0908** | 0.0168 | 0.1284** | 0.0259 | 58.3805 | 0.0000 |
| Toronto 35 | 0.1111** | 0.0663 | 0.0533 | 0.0398 | 0.4360** | 0.1809 | 12.8412 | 0.0005 |
| Australia | 0.7034** | 0.0993 | 0.1889** | 0.0550 | 0.4208** | 0.1250 | 55.7928 | 0.0000 |
| CAC 40 | 0.0068 | 0.0240 | 0.0730** | 0.0129 | 0.0069 | 0.0135 | 0.3000 | 0.4990 |

For each index/contract we report the correlation of the spot-futures return spread with the lagged futures return. Large movements in lagged futures returns are based on the cutoff points −1.0% and 1.0%. The regression is

$$r_{t+1}^S - r_{t+1}^F = b_0 + b_1 r_t^F + (b_2 - b_1)\,\mathrm{Max}[0, r_t^F - a_1] + (b_3 - b_2)\,\mathrm{Max}[0, r_t^F - a_2] + \epsilon_{t+1}^{S-F},$$

where $a_1 = -0.01$ and $a_2 = 0.01$. The $\chi_2^{-2}$ statistic tests $H_0 : b_1 = b_3 = 0$ versus $H_a : b_1 \geq 0, b_3 \geq 0$, and we report the corresponding $p$-value. Standard errors are serial correlation and heteroscedasticity-adjusted using Newey and West (1987).
*, **Significance at the 10% and 5% levels, respectively.

contract and that the spot returns are more positively autocorrelated. To the extent that the microstructure-based theory would imply that all three coefficients $(b_1, b_2, b_3)$ should be positive, 60 of 63 are. Since these coefficients represent relations over different (and apparently independent) data ranges and across 15 somewhat unrelated countries, this evidence is, in our opinion, quite strong. Thus, even for the circumstances most favorable to spot-futures arbitrage, there is little evidence that futures and spot index returns behave similarly, the opposite of the implication of the partial adjustment model.

One problem with the tests in Table 3 is that, while they can reject the strict partial adjustment model of Section 1.3, they cannot reject a model with both partial adjustment and stale price effects. In other words, the tests show that stale prices are important, but they do not show conclusively that there are no partial adjustment effects as well. The autocorrelation evidence in Table 2 and Figure 2 partially addresses this issue. Specifically, average futures autocorrelations are approximately zero, which is consistent with the absence of any partial adjustment effects. Of course, the individual standard errors are quite large, so the results are not definitive. A different approach is to examine the magnitudes of the coefficients $b_1$, $b_2$, and $b_3$. Under the

stale price model, there is no reason to expect the coefficients to be different. In contrast, if there are partial adjustment effects as well, then the coefficients for large moves ($b_1$ and $b_3$) should be less than the coefficients for small moves ($b_2$) because these effects should be eliminated when index arbitrage is feasible. In Table 3, the average coefficients across the 21 contracts are 0.12, 0.10, and 0.13 for $b_1$, $b_2$, and $b_3$, respectively. Basically there is no evidence of partial adjustment effects. The robustness of these results to different hypothesized levels of transaction costs can be checked by increasing the magnitudes of the breakpoints. For breakpoints up to 2.5%, the same basic relation between the coefficients holds. In fact, the average $b_2$ coefficient is stable, while the average coefficients for extreme returns increase in magnitude slightly, exactly the opposite of the implication of a model with partial adjustment. Of course, as the breakpoints increase in magnitude, there are fewer observations in the extremes, and the standard errors increase commensurately. Consequently it makes little sense to go beyond 2.5% when we are already estimating coefficients with an average of only 60 observations.

A second potential point of discussion is that Equation (9) describes a linear (albeit piecewise) relation between differences in the spot and futures return and the lagged return. For further evidence, Figure 3 provides a
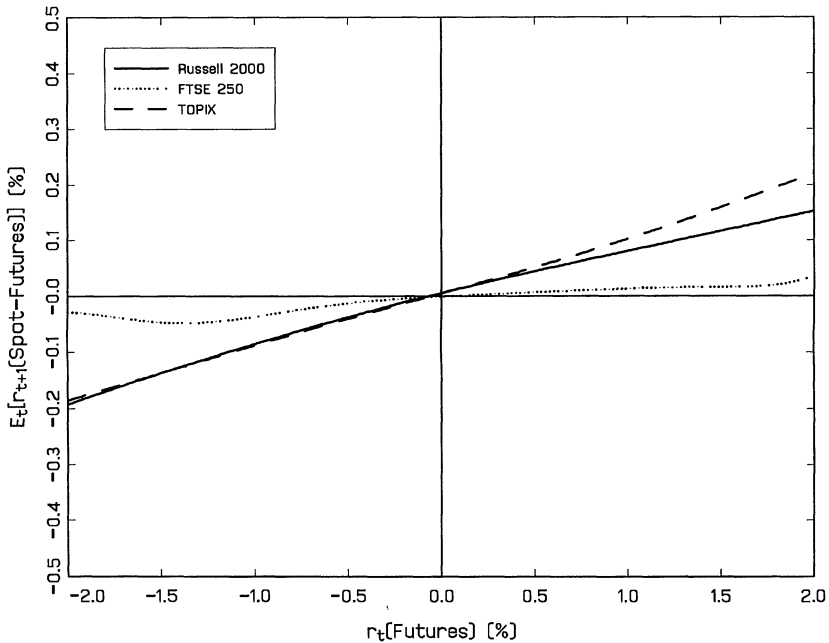


**Figure 3**
Nonparametric kernel estimates of the relation between the spot-futures return spread ($r_{t+1}^S - r_{t+1}^F$) and the lagged return on the futures ($r_t^F$) for three indices: the Russell 2000, the FTSE 250, and the TOPIX.
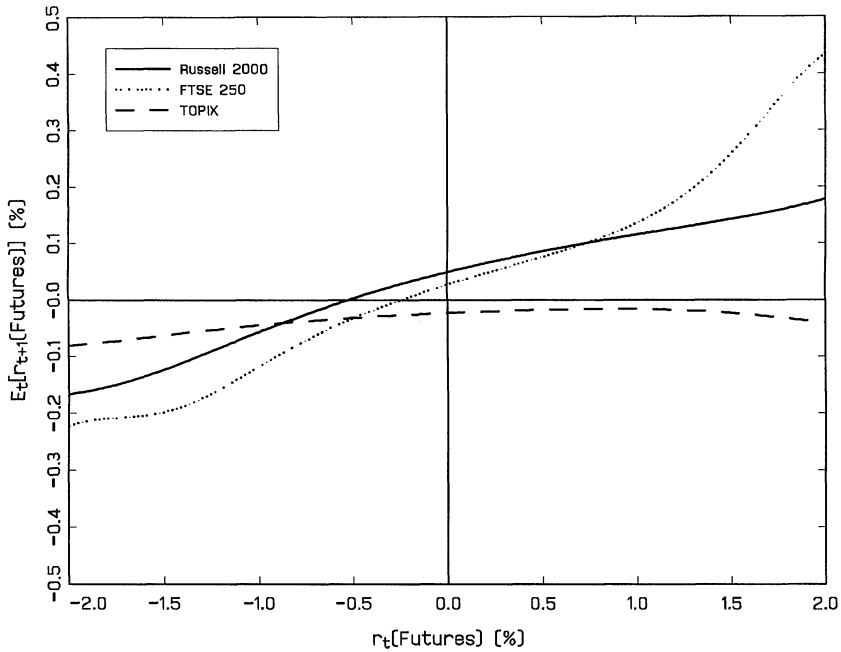
**Figure 4**
Nonparametric kernel estimates of the relation between the futures return ($r^F_{t+1}$) the lagged futures return ($r^F_t$) on three futures contracts: the Russell 2000, the FTSE 250, and the TOPIX.

nonparametric graphical presentation of these results for three contracts which contain illiquid stocks, namely the Russell 2000, TOPIX, and FTSE 250. The graph represents a kernel estimation of the mean of $r_{S_{t+1}} - r_{F_{t+1}}$, conditional on the value of $r_{F_t}$. As seen from these three somewhat independent lines, the implications of the partial adjustment model (i.e., Figure 1C) are not borne out. Time variation of differences in expected spot and futures returns occurs for all values of returns, especially for high values, which contradicts the partial adjustment hypothesis.[11]

As related evidence, Figure 4 graphs the relation between expected futures returns and their lags for the same indices and using the same nonparametric estimation technique. Consistent with a random walk, there is not much time variation in the estimated expected return. While this is consistent with some partial adjustment models, it is clearly a necessary condition of any stale price model.

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[11] The exception here is the FTSE 250, which is fairly flat except for extreme negative values, which again contrasts with partial adjustment theories. We view these results more in the context of there being only data available post-1994, so that there is a shortage of observations. Thus the results fall into the so-called Star Trek region of the data, and are unreliable.

### 3.3 Autocorrelations and volume

One obvious implication of the stale price model of Section 1.2 is that there should be some relation between the spot index properties and volume on that index, whereas the futures should for the most part be unrelated to volume. Of course, partial adjustment models may also imply some correlation between volume and autocorrelations [e.g., as in Klibanoff, Lamont, and Wizman (1998) and Chordia and Swaminathan (2000)], but it is clearly a necessary result of the stale price explanation.

In order to investigate this implication, we collected data from Datastream on overall stock market volume for each of the 15 countries. While this does not represent volume for the stocks underlying the index, it should be highly correlated with trading in these stocks because all the indices we look at are broad-based, market indices. That is, on days in which stock market volume is low, it seems reasonable to assume that large, aggregate subsets of this volume will also be relatively low. During the sample periods for each country, there has been a tendency for volume to increase (partly due to increased equity values and greater participation in equity markets). The standard approach is to avoid the nonstationarity issue and look at levels of detrended volume.

In order to investigate the effect of trading volumes on autocorrelations of the spot index and its futures return, we consider the following nonlinear regressions:

$$
\begin{aligned}
r_{S_{t+1}} &= \alpha_0^s + \left[\alpha_1^s + \alpha_2^s(\max(\mathrm{vol}^s) - \mathrm{vol}_t^s)\right]r_{S_t} + \epsilon_{t+1}^s \\
r_{F_{t+1}} &= \alpha_0^f + \left[\alpha_1^f + \alpha_2^f(\max(\mathrm{vol}^s) - \mathrm{vol}_t^s)\right]r_{F_t} + \epsilon_{t+1}^f,
\end{aligned}
\tag{10}
$$

where $\max(\mathrm{vol}^s)$ is the maximum volume of the stock market during the sample period. Note that these regressions represent fairly logical representations of the relation between next period's return and current returns and volume. Specifically there are two components to the time variation of expected returns: (i) the magnitude of last period's return, and (ii) the level of volume in the market.

The hypothesis that the trading volume is a factor that influences autocorrelation differentials can be represented as follows:

***Hypothesis 1.*** *The trading volume reduces the autocorrelation of the spot, but not the futures contract*:

$$\alpha_2^s > 0$$
$$\alpha_2^f = 0.$$

***Hypothesis 2.*** *We can interpret $\alpha_1^s$ and $\alpha_1^f$ as the autocorrelations of the spot index and the futures contract returns when the trading volume of the*

*spot is at its highest level. In that case, the autocorrelation of the spot as well as the futures should be close to zero*:

$$\alpha_1^s = 0$$
$$\alpha_1^f = 0.$$

Some observations are in order. Hypothesis 1 is an obvious implication of index returns being driven by stale prices, and the most important component of our hypotheses. Note that it is possible that $\alpha_2^s = 0$, in which case $\alpha_1^s$ represents the autocorrelation of the index return in a world in which volume plays no role. With respect to Hypothesis 2, it appears to be redundant given Hypothesis 1. However, we want to be able to test whether the negative relation is strong enough to bring forth the desired result that the spot index return autocorrelation becomes zero at the highest level of trading volume. Finally, an important hypothesis to test is whether the futures contract's autocorrelation is independent of trading volume.

Table 4 provides results for each of the 24 stock indices across the 15 countries. First, there is a negative relation between the trading volume and the autocorrelation of the spot index return for most of the countries (i.e., $\alpha_2^s > 0$). While the estimators are individually significant at the 5% level for only seven of the indices (e.g., the Russell 2000's estimate is 0.54 with standard error 0.19), 23 of 24 are positive. Moreover, relative to the futures return coefficient on volume (i.e., $\alpha_2^f$), about two-thirds have values of $\alpha_2^s > \alpha_2^f$. While only eight of these are individually significant at the 5% level (i.e., S&P 500, Russell 2000, NYSE, FTSE 250, Switzerland, Hong Kong, and Belgium), only one contract goes in the direction opposite to that implied by the stale price theory.

Second, independent of volume, the relation between futures return autocorrelations and trading volume is very weak. Even though many of the autocorrelation coefficients, $\alpha_2^f$, are positive, they tend to be very small in magnitude and are thus both economically and statistically insignificant. Furthermore, the estimates at high volume levels imply negative autocorrelation in futures returns. Combining the estimates of $\alpha_1^f$ and $\alpha_2^f$ together in equation (10) implies that the autocorrelations of futures returns are rarely positive irrespective of volume levels. This result is consistent with the bid-ask bounce effect which will be looked at in Section 3.4.

Third, at the highest level of trading, the autocorrelations of the spot and futures return are for the most part insignificantly different from zero. Only five contracts violate this hypothesis. However, for each of these cases, the autocorrelations are negative at high volume, and thus do not contradict the stale price theory per se. In fact, 19 of the 24 indices imply negative autocorrelation of the spot index return during periods of highest volume. While these autocorrelations are not estimated precisely, it does point out

**Table 4**
**Daily autocorrelations and trading volume**

| Contract | $\alpha_1^S$ | SE | $\alpha_1^F$ | SE | $\alpha_2^S$ | SE | $\alpha_2^F$ | SE | $\chi_2^2$ | p-value | z-stat | p-value |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S&P 500 | -0.0912 | 0.1038 | -0.0541 | 0.0839 | 0.0791 | 0.0553 | 0.0122 | 0.0464 | 0.9858 | 0.6108 | 2.7059 | 0.0034 |
| Russell 2000 | -0.0909 | 0.1374 | -0.0649 | 0.1255 | 0.5395*** | 0.1894 | 0.2231 | 0.1751 | 0.4411 | 0.8021 | 2.6527 | 0.0040 |
| Nikkei 225 (CME) | -0.0776 | 0.1461 | -0.0423 | 0.1875 | 0.0095 | 0.0262 | -0.0070 | 0.0340 | 0.2826 | 0.8682 | 0.4913 | 0.3116 |
| NYSE | -0.0645 | 0.1078 | -0.0447 | 0.0825 | 0.0831 | 0.0570 | -0.0100 | 0.0449 | 0.3587 | 0.8358 | 3.0441 | 0.0012 |
| Maxi Value Line | -0.0368 | 0.1351 | -0.1708 | 0.1436 | 0.2412** | 0.1221 | 0.1543 | 0.1269 | 1.6147 | 0.4460 | 0.7341 | 0.2315 |
| FTSE 100 | -0.0446 | 0.1976 | -0.0328 | 0.1572 | 0.0877 | 0.1259 | 0.0351 | 0.1026 | 0.0511 | 0.9748 | 1.0791 | 0.1403 |
| FTSE 250 | 0.0812 | 0.2122 | 0.1036 | 0.1713 | 0.2236 | 0.2462 | 0.0363 | 0.2028 | 0.7400 | 0.6907 | 1.9662 | 0.0246 |
| TOPIX | 0.0679 | 0.0800 | -0.1434** | 0.0743 | 0.0140 | 0.0232 | 0.0403** | 0.0197 | 11.8470 | 0.0027 | -1.2971 | 0.9027 |
| Nikkei 225 (OSX) | -0.0758 | 0.0758 | -0.1159 | 0.0822 | 0.0208 | 0.0215 | 0.0270 | 0.0229 | 2.1787 | 0.3364 | -0.5658 | 0.7143 |
| Nikkei 300 (OSX) | -0.1208 | 0.1290 | -0.2047 | 0.1338 | 0.1156 | 0.0885 | 0.0987 | 0.0919 | 3.8412 | 0.1465 | 0.4310 | 0.3332 |
| DAX | -0.1711 | 0.2144 | -0.3306** | 0.1905 | 0.0733 | 0.0787 | 0.1246* | 0.0688 | 5.9837 | 0.0502 | -1.4791 | 0.9304 |
| Swiss Market | -0.0408 | 0.0878 | 0.0253 | 0.0663 | 0.0568 | 0.0437 | 0.0037 | 0.0314 | 2.5059 | 0.2857 | 2.1962 | 0.0140 |
| AEX | 0.1636** | 0.0855 | -0.0017 | 0.1169 | -0.0269 | 0.0141 | 0.0012 | 0.0206 | 14.4877 | 0.0007 | -2.3704 | 0.9911 |
| BELFOX 20 | -0.1387 | 0.2194 | 0.1018 | 0.1579 | 0.1416 | 0.1023 | -0.0023 | 0.0732 | 2.8050 | 0.2460 | 1.9055 | 0.0284 |
| Hang Seng | -0.2910** | 0.1232 | -0.2382 | 0.1471 | 0.1920** | 0.0607 | 0.1188 | 0.0706 | 12.2195 | 0.0022 | 3.5141 | 0.0002 |
| IBEX 35 Plus | 0.0872 | 0.1570 | 0.0632 | 0.1310 | 0.0205 | 0.0666 | -0.0215 | 0.0557 | 0.3119 | 0.8556 | 1.0146 | 0.1551 |
| MIB 30 | -0.1473 | 0.1332 | -0.2030 | 0.1246 | 0.1538 | 0.1052 | 0.1551 | 0.0994 | 4.3753 | 0.1122 | -0.0391 | 0.5156 |
| OMX | 0.0610 | 0.2270 | -0.2349 | 0.2342 | 0.0154 | 0.0584 | 0.0538 | 0.0619 | 2.2468 | 0.3252 | -0.6963 | 0.7569 |
| ATX | -0.3985 | 0.2680 | -0.3835 | 0.2846 | 0.1840** | 0.0907 | 0.1376 | 0.0939 | 2.2953 | 0.3174 | 1.6024 | 0.0545 |
| Nikkei 225 (SIMEX) | -0.0750 | 0.0759 | -0.2594*** | 0.1014 | 0.0206 | 0.0215 | 0.0575 | 0.0250 | 6.8801 | 0.0321 | -1.6904 | 0.9545 |
| Nikkei 300 (SIMEX) | -0.1209 | 0.1403 | -0.1168 | 0.1449 | 0.1171 | 0.0977 | 0.0498 | 0.1072 | 0.7526 | 0.6864 | 1.0010 | 0.1584 |
| Toronto 35 | -0.0930 | 0.0763 | -0.3778* | 0.2103 | 0.2435*** | 0.0815 | 0.4135* | 0.2260 | 3.9944 | 0.1357 | -0.7891 | 0.7850 |
| Australia | -0.2411** | 0.0911 | -0.6318** | 0.2732 | 0.1388*** | 0.0348 | 0.1977 | 0.0924 | 7.6683 | 0.0216 | -0.7834 | 0.7833 |
| CAC 40 | -0.1314 | 0.0955 | -0.1198 | 0.1033 | 0.0686** | 0.0342 | 0.0500 | 0.0378 | 1.9071 | 0.3854 | 0.9692 | 0.1662 |

For each index/contract we report the regression coefficients describing the relationship between the daily autocorrelations of spot and futures returns and lagged trading volume in the spot market. The regressions are

$$r_{t+1}^S = \alpha_0^S + [\alpha_1^S + \alpha_2^S[\max(\text{vol}^S) - \text{vol}_t^S]]r_t^S + \epsilon_{t+1}^S$$
$$r_{t+1}^F = \alpha_0^F + [\alpha_1^F + \alpha_2^F[\max(\text{vol}^S) - \text{vol}_t^S]]r_t^F + \epsilon_{t+1}^F.$$

The $\chi_2^2$ statistic tests $\alpha_1^S = \alpha_1^F = 0$, the z-statistic tests $\alpha_2^S - \alpha_2^F > 0$, and we report the corresponding p-values. Standard errors are serial correlation and heteroscedasticity-adjusted using Newey and West (1987). *, **Significance at the 10% and 5% levels, respectively.

that adjustments for trading volume lead to changes in the level of auto-correlations. For example, the Russell 2000's autocorrelation changes from Table 2's estimate of 0.22 to −0.09 at highest volume levels in Table 4. The most obvious explanation for the negative values is misspecification of the regression model in Equation (10).

In order to address this issue, we perform a nonparametric analysis of the effect of trading volumes on autocorrelations of the spot and futures return for the Russell 2000 contract. Specifically, using multivariate density estimation methods, we look at the expected return differential, $r_{S_{t+1}} - r_{F_{t+1}}$, on detrended market volume and the most recent stock market innovation, estimated by the futures return $r_{F_t}$. For multidimensional estimation problems like this, it is important to document the area of relevant data. Figure 5 provides a scatter plot of detrended volume and futures returns, which represents the applicable space. Any results outside this area should be interpreted cautiously.

Figure 6 graphs the relation between spot-futures return spreads and past returns and volume, that is, the nonparametric alternative to the regressions described in Equation (10). For low volume periods, the differential is positive and particularly steep when past returns are high, and negative when past returns are low. In other words, low volume periods seem to be an important
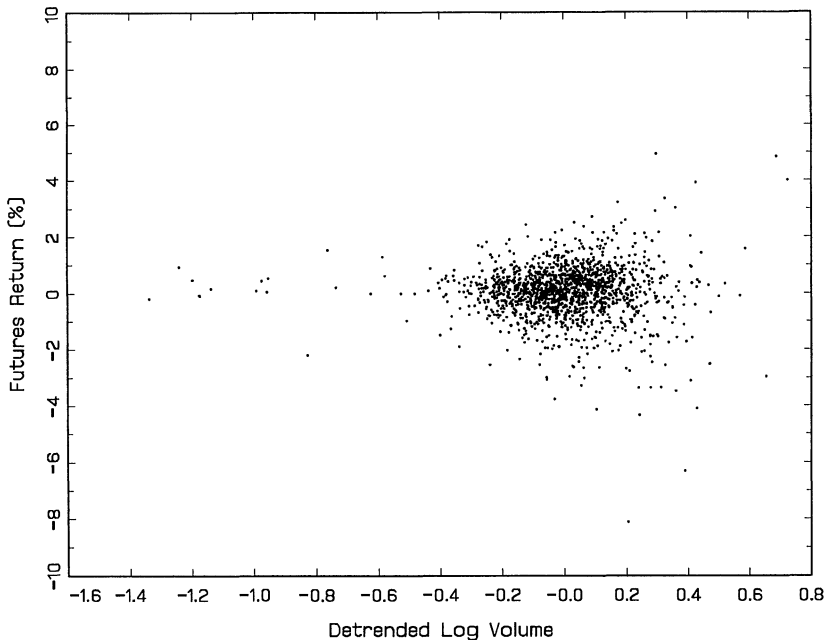


Figure 5
A scatter plot of detrended log trading volume ($\ln(1 + vol_t^S)$) in the U.S. spot market and the futures return ($r_t^F$) on the Russell 2000.
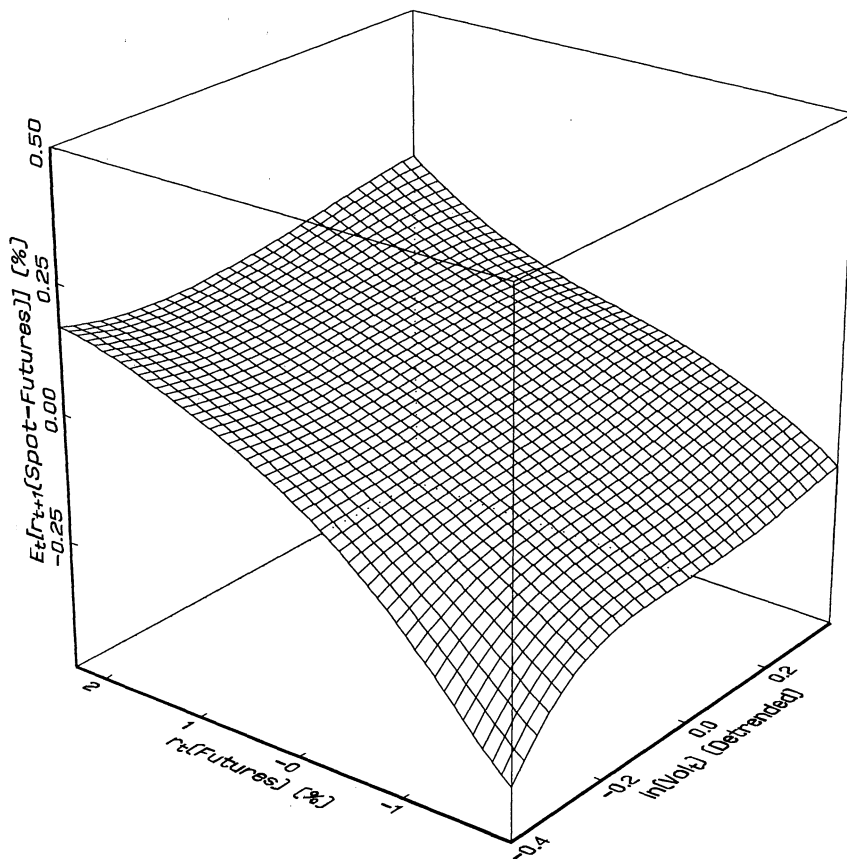
**Figure 6**
A three-dimensional plot of the nonparametric kernel estimate of the relation between the spot-futures return spread $(r_{t+1}^S - r_{t+1}^F)$, the lagged return on the futures $(r_t^F)$ and detrended log trading volume $(\ln(1 + vol_t^S))$ for the Russell 2000.

factor describing differences in the statistical properties of spot and futures returns. Of interest, for average and heavy-volume days, there appears to be little difference in their time-varying properties.

As a finer partition of this graph, Figure 7 presents cut-throughs of the relation between spot-futures return differentials and past market innovations for four different levels of volume within the range of the data. As seen in Figure 7, while there are positive differentials for all levels of volume (as consistent with the one-dimensional analysis of Sections 3.1 and 3.2), the most striking evidence takes place during low-volume periods. To the extent that low-volume periods are associated with stale prices, these results provide evidence supportive of the type of model described in Section 1.2.
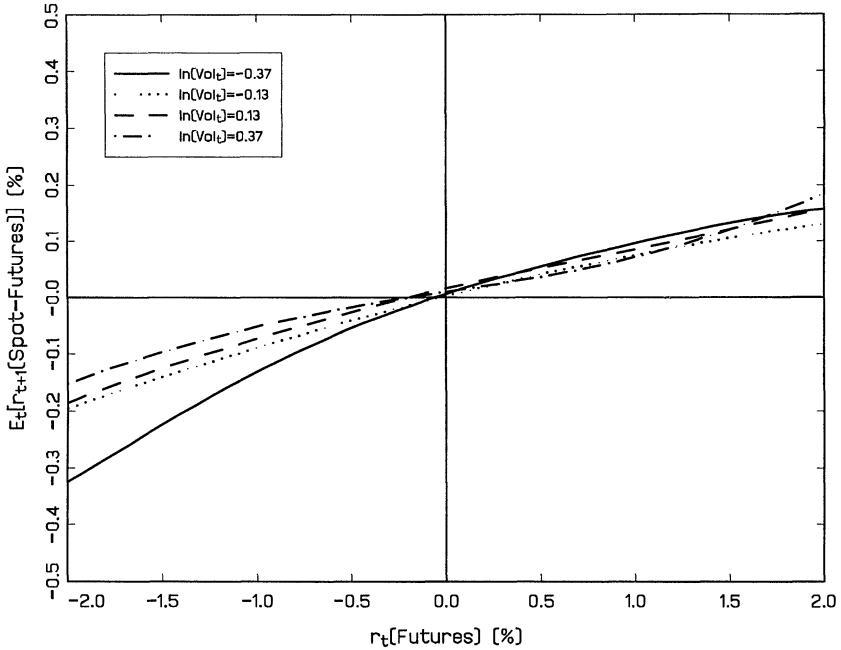
**Figure 7**
A two-dimensional cut-through of the relation between the spot-futures return spread $(r_{t+1}^S - r_{t+1}^F)$ and the lagged futures return $(r_t^F)$ for the Russell 2000 at different values of the detrended lagged log volume $(\ln(1 + vol_t^S))$.

It is, of course, possible for researchers to devise a partial adjustment model that fits these characteristics as well, but they must do so in the presence of spot-futures arbitrage.

### 3.4 Can bid-ask bounce explain the autocorrelation differences?

One possible explanation for the differences between spot index and futures return autocorrelations is that the futures contracts themselves suffer from microstructure biases. That is, a skeptic might argue that the true autocorrelation is large and positive, yet the futures autocorrelation gets reduced by bid-ask bounce and similar effects. In fact, it is well known that bid-ask bias leads to negative serial correlation in returns [see, e.g., Blume and Stambaugh (1983) and Roll (1984)]. How large does the bid-ask spread need to be to give credibility to this explanation?

Consider a variation of the Blume and Stambaugh (1983) model in which the measured futures price, $F^m$, is equal to the true price, $F$, adjusted for the fact that some trades occur at the bid or ask price, that is,

$$F_t^m = F_t(1 + \theta_t),$$

where $\theta_t$ equals the adjustment factor. In particular, assume that $\theta_t$ equals $\frac{s}{2}$ with probability $\frac{p}{2}$ (i.e., the ask price), $-\frac{s}{2}$ with probability $\frac{p}{2}$ (i.e., the bid price), or 0 with probability $1 - p$ (i.e., a trade within the spread). Here $s$ represents the size of the bid-ask spread, and can be shown to be directly linked to the volatility of $\theta_t$. Specifically we can show that $\sigma_\theta^2 = p\frac{s^2}{4}$. In words, the additional variance of the futures price is proportional to the size of the spread and the probability that trades take place at the quotes. Using the approximation $\ln(1 + x) \approx x$, it is possible to show that the implied autocorrelation of futures returns is given by

$$\frac{-ps^2}{4\sigma_{R_F}^2 + 2ps^2}. \tag{11}$$

Table 5 reports the autocorrelation differences between the stock index and futures returns. If these differences were completely due to bid-ask bias in the futures market, then Equation (11) can be used to back out the relevant bid-ask spread. The last two columns of Table 5 provide estimates of the size

**Table 5**
**Futures return autocorrelations and the bid-ask spread**

| | | | | Implied AC difference | | Implied spread | |
|---|---|---|---|---|---|---|---|
| Contract | AC difference | Volume | Spread | Low probability | High probability | Low probability | High probability |
| S&P 500 | 0.0653 | 1.1911 | 0.037 | 0.0001 | 0.0002 | 0.86 | 0.61 |
| Russell 200 | 0.1487 | 0.9418 | 0.150 | 0.0032 | 0.0063 | 1.03 | 0.73 |
| Nikkei 225 (CME) | 0.0564 | 1.5385 | 0.060 | 0.0002 | 0.0004 | 1.03 | 0.73 |
| NYSE | 0.1171 | 1.1799 | 0.040 | 0.0001 | 0.0003 | 1.14 | 0.81 |
| Maxi Value Line | 0.2147 | 0.9025 | 0.100 | 0.0015 | 0.0031 | 1.18 | 0.84 |
| FTSE 100 | 0.0574 | 1.0964 | 0.150 | 0.0023 | 0.0047 | 0.74 | 0.53 |
| FTSE 250 | 0.0881 | 0.6330 | NA | NA | NA | 0.53 | 0.38 |
| TOPIX | 0.1099 | 1.3280 | 0.038 | 0.0001 | 0.0002 | 1.25 | 0.88 |
| Nikkei 225 (OSX) | 0.0134 | 1.4166 | 0.059 | 0.0002 | 0.0004 | 0.46 | 0.33 |
| Nikkei 300 (OSX) | 0.0892 | 1.3256 | 0.500 | 0.0178 | 0.0356 | 1.12 | 0.79 |
| DAX | 0.0239 | 1.2450 | 0.060 | 0.0003 | 0.0006 | 0.54 | 0.38 |
| Swiss Market | 0.0241 | 1.0970 | 0.014 | 0.0000 | 0.0000 | 0.48 | 0.34 |
| AEX | 0.0322 | 1.1164 | 0.089 | 0.0008 | 0.0016 | 0.57 | 0.40 |
| BELFOX 20 | 0.0502 | 0.9214 | 0.217 | 0.0069 | 0.0139 | 0.58 | 0.41 |
| Hang Seng | 0.0653 | 1.9631 | 0.037 | 0.0000 | 0.0001 | 1.42 | 1.00 |
| IBEX 35 Plus | 0.1194 | 1.4750 | 0.070 | 0.0003 | 0.0006 | 1.44 | 1.02 |
| MIB 30 | 0.0487 | 1.6605 | 0.068 | 0.0002 | 0.0004 | 1.04 | 0.73 |
| OMX | 0.1536 | 1.5853 | 0.032 | 0.0001 | 0.0001 | 1.76 | 1.24 |
| ATX | 0.0999 | 1.2468 | 0.308 | 0.0076 | 0.0153 | 1.11 | 0.79 |
| Nikkei 225 (SIMEX) | 0.0270 | 1.6491 | 0.060 | 0.0002 | 0.0003 | 0.77 | 0.54 |
| Nikkei 300 (SIMEX) | 0.0526 | 1.3230 | 0.375 | 0.0100 | 0.0201 | 0.86 | 0.61 |
| Toronto 35 | 0.1640 | 1.0129 | 0.127 | 0.0020 | 0.0039 | 1.16 | 0.82 |
| Australia | 0.1769 | 1.4475 | 0.032 | 0.0001 | 0.0001 | 1.72 | 1.22 |
| CAC 40 | 0.0345 | 1.2503 | NA | NA | NA | 0.66 | 0.46 |

For each contract/index we report the difference between the autocorrelation of returns in the spot and futures markets and the actual estimated bid-ask spread of each futures contract. Using a Blume and Stambaugh (1993) bid-ask model, we provide (i) implied autocorrelation differences based on actual spreads, and (ii) the implied spread necessary to explain the actual autocorrelation differences. Low probability (i.e., 50%) and high probability (i.e., 100%) states refer to the probability that the trades take place at the quotes rather than within the quoted spreads.

of this spread in percentage terms of the futures price. The two columns represent two different values of $p$, the probability of trading at the ask or bid, equal to either 0.5 or 1.0. Of course, a value of 1.0 is an upper bound on the effect of the bid-ask spread. The implied spreads in general are much larger than those that occur in practice. To see this, we document actual spreads at the end of the sample over a week period, and find that they are approximately one-tenth the magnitude (see column 4 of Table 5). Alternatively, using the actual spreads, and the above model, we report implied autocorrelation differences that are all close to zero. Therefore the differences in the autocorrelations across the series are clearly not driven by bid-ask bounce in the futures market.

## 4. Conclusion

The simple theoretical results in this article, coupled with the supporting empirical evidence, lead to several conclusions. First, there are significant differences between the statistical properties of spot index and corresponding futures returns even though they cover the same underlying stocks. This is true even accounting for the presence of transaction costs by examining large market movements. Second, an important factor describing these different properties is the level of volume in the market. These two empirical facts can most easily be associated with market microstructure-based explanations, as partial adjustment models do not seem to capture these characteristics of the data.

The unique aspect of this article has been to differentiate, rather generally, implications from two very different schools of thought and provide evidence thereon. Our conclusion is generally not supportive of the partial adjustment models that have become popular as of late.

What then is going on in the market that can describe these large daily autocorrelations of portfolio returns? Previous authors [e.g., Conrad and Kaul (1989) and Mech (1993), among others] have performed careful empirical analyses of nontrading by taking portfolios that include only stocks that have traded. Their results, though somewhat diminished, suggest autocorrelations are still positive and large for these portfolios. However, as pointed out in the article, stale price models are more general. Rational models predict that the price of a security is the discounted value of its future cash flow. Within this context, how should we view a trade for 100 shares when there is little or no other trading? Does it make sense to discard a theory based on a single investor buying a small number of shares at a stale overvalued or undervalued (relative to market information) price, or a dealer failing to adjust quotes for a small purchase or sale? Our view is that the important issue is how

many shares can trade at that price (either through a large order or numerous small transactions). What would researchers find if we took portfolios of stocks that trade meaningfully, and then what would happen if these portfolios got segmented via size, number of analysts, turnover, et cetera? These are questions which seem very relevant given the results of this article.

# Appendix

### Case 1: the random walk model
Assume that the ex dividend, log price process of an equally weighted portfolio follows a random walk with drift

$$s_{t+1} = s_t + \mu - d + m_{t+1}, \tag{A1}$$

where $s_t = \ln S_t$ is the log of the ex dividend portfolio price, $\mu$ and $d$ are the constant expected return and dividend yield on the portfolio, respectively, and $m_{t+1}$ is the mean-zero, i.i.d. market factor. From the cost of carry model, the price of a futures contract on this portfolio is

$$F_{t,T} = S_t e^{(i-d)(T-t)}, \tag{A2}$$

where $F_{t,T}$ is the futures price on a contract at time $t$ that expires at time $T$, and $i$ and $d$ are the periodic, risk-free, interest rate and dividend yield, respectively, both of which are assumed to be constant.

The futures price follows from the absence of arbitrage. A futures contract can be replicated by buying the underlying portfolio and borrowing to finance this purchase. The payoff on the futures contract is $S_T - F_{t,T}$, and the payoff on the replicating portfolio is $S_T - S_t e^{(i-d)(T-t)}$. Equating these two payoffs yields Equation (A2).

The one-period returns on the portfolio and the futures contract are closely related. Specifically,

$$r_{F_{t+1}} = r_{S_{t+1}} - i, \tag{A3}$$

where

$$r_{F_{t+1}} = \ln \frac{F_{t+1,T}}{F_{t,T}} \tag{A4}$$

$$r_{S_{t+1}} = \ln \frac{S_{t+1}}{S_t} + d = \mu + m_{t+1}. \tag{A5}$$

The futures return equals the excess return on the underlying portfolio.

### Case 2: the stale price model
Assume that in every period a fraction $\pi$ of the prices of the securities within the portfolio are not updated, that is, the observed portfolio price reflects stale prices. The true price and return processes are identical to those in case 1, that is,

$$s_{t+1} = s_t + \mu - d + m_{t+1} \tag{A6}$$

$$r_{S_{t+1}} = \mu + m_{t+1}, \tag{A7}$$

and the measured (i.e., observed) price, $\hat{s}$, and return processes can be written as [see Lo and MacKinlay (1990a)]

$$\hat{s}_{t+1} = \hat{s}_t + \mu - d + (1 - \pi)\sum_{k=0}^{\infty}\pi^k m_{t+1-k} \tag{A8}$$

$$r_{\widehat{S}_{t+1}} = \mu + (1 - \pi)\sum_{k=0}^{\infty}\pi^k m_{t+1-k}. \tag{A9}$$

Note from Equations (A6) and (A8) that

$$s_T = s_t + (T - t)(\mu - d) + \sum_{k=0}^{T-t-1} m_{T-k} \tag{A10}$$

$$\hat{s}_T = \hat{s}_t + (T - t)(\mu - d) + \sum_{k=0}^{T-t-1}(1 - \pi^{k+1})m_{T-k} + (1 - \pi^{T-t})\sum_{k=T-t}^{\infty}\pi^{k-T+t+1}m_{T-k}. \tag{A11}$$

The observed price is subject to the same shocks as the true price, but the weights on these shocks start at $(1 - \pi)$ and gradually accumulate to 1 as the prices within the portfolio are updated. As a result, changes in the observed price process always reflect the full history of shocks.

The corresponding futures contract is cash settled on the basis of the observed price $\widehat{S}_T$, not the true price $S_T$; consequently it is priced off the observed prices. Specifically, analogously to the cost of carry model in case 1 above,

$$F_{t,T} = \mathrm{PV}_t(\widehat{S}_T)e^{i(T-t)}, \tag{A12}$$

where $\mathrm{PV}_t(\widehat{S}_T)$ is the present value at time $t$ of the observed price at time $T$. Shocks after time $t$ are risky and hence need to be discounted at $\mu$, whereas additional accumulations on historical shocks are riskless and hence are discounted at the riskless rate $i$. Therefore, from Equation (A11),

$$\mathrm{PV}_t(\hat{s}_T) = \hat{s}_t - (T - t)d + \pi\left(\frac{1 - \pi^{T-t}}{1 - \pi}\right)(\mu - i) + \sum_{k=T-t}^{\infty}(1 - \pi^{T-t})\pi^{k-T+t+1}m_{T-k}. \tag{A13}$$

Consequently,

$$r_{F_{t+1}} = \mathrm{PV}_{t+1}(\hat{s}_T) - \mathrm{PV}_t(\hat{s}_T) - i = (1 - \pi^{T-t})(r_{S_{t+1}} - i). \tag{A14}$$

The argument above is an equilibrium argument rather than an arbitrage argument for futures valuation. In the case of stale prices, the arbitrage strategy is more complex than in the random walk case because the contract is settled based on a nontraded asset, that is, it is not possible to buy $\widehat{S}_t$. The key to replicating the futures contract is to replicate the exposure to the same shocks as the futures contract. From Equation (A11), it is clear that the exposure to future shocks declines over the life of the contract. Consequently the strategy is to purchase $(1 - \pi^{T-t})$ of the underlying asset initially, financed by borrowing. In each subsequent period $t + j$, the exposure is reduced to $(1 - \pi^{T-t-j})$ with the proceeds from the sale used to pay off some of the initial borrowing. At the maturity of the futures contract, this strategy has a payoff of

$$\sum_{k=1}^{T-t}\left[(1 - \pi^k)(S_{T+1-k} - S_{T-k}) + (1 - \pi^k)d\right] - \sum_{k=1}^{T-t}(1 - \pi^k)i, \tag{A15}$$

where the first term is the payoff from the net position in the underlying asset and the second term is the repayment of the net borrowing. This payoff reduces to

$$\sum_{k=1}^{T-t}\left[(1 - \pi^k)(\mu - i) + (1 - \pi^k)m_{T+1-k}\right]. \tag{A16}$$

Equating this quantity to the payoff to the futures contract, $\widehat{S}_T - F_{t,T}$, where $\hat{s}_T$ is given in Equation (A11), yields the desired result, which is identical to the equilibrium result in Equation (A12).

### Case 3: the partial adjustment model

Assume that the log price process on the underlying index follows the process

$$s_{t+1} = s_t + \mu - d + \beta m_{t+1} + \gamma m_t. \tag{A17}$$

Note that this process represents both the true and observed prices, that is, trading is possible at these prices. With no transaction costs, the standard cost of carry arbitrage argument holds, and the futures contract is priced at the current spot price as in case 1:

$$F_{t,T} = S_t e^{(i-d)(T-t)}. \tag{A18}$$

Consequently there is also the same relation between futures and spot returns,

$$r_{F_{t+1}} = r_{S_{t+1}} - i \tag{A19}$$

$$r_{S_{t+1}} = \mu - d + \beta m_{t+1} + \gamma m_t, \tag{A20}$$

but in contrast to the random walk model, both returns are autocorrelated due to partial adjustment in the spot market.

Now assume that there are fixed additive transaction costs $\delta$ in the spot market (but not in the futures market or loan market). Replicating a long futures position requires buying the underlying index initially at a cost of $S_t + \delta$ and then selling it at the maturity of the futures contract at $S_T - \delta$. Similarly, replicating a short futures position requires shorting the index initially and then closing out the position at time $T$. The payoffs to these long and short strategies provide upper and lower bounds on the futures price, respectively. Specifically

$$-\delta(1 + e^{i(T-t)}) \leq F_{t,T} - S_t e^{(i-d)(T-t)} \leq \delta(1 + e^{i(T-t)}). \tag{A21}$$

The futures price is bounded by its value with no transaction costs plus or minus round-trip transaction costs.

When the futures price is within the transaction cost bounds, the model does not have definitive implications for the time-series properties of futures returns. Outside these bounds, index arbitrage guarantees that the futures will track the underlying index. In particular, if the spot price follows the process in Equation (A17), futures returns will be positively autocorrelated.

An alternative model is that Equation (A17) holds within the bounds, but outside the bounds index arbitrage forces the index prices to track futures prices, which are not subject to partial adjustment. In this case, futures returns should exhibit no autocorrelation in any range, spot returns should be positively autocorrelated within the transaction cost bounds and exhibit zero autocorrelation outside the bounds.

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