Non-parametric methods of option pricing

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ABSTRACT

Non-parametric and computational methods of option pricing have recently attracted attention of researchers. These typically include highly data intensive, model-free approaches that complement traditional parametric methods. Non-parametric and computational methods of option pricing typically include highly data intensive, model-free approaches that complement traditional parametric methods. One characteristic of such methods is their independence of the assumptions of continuous-time finance theory. It is their great strength and a weakness. Strength since it presumes no complex models from which prices are deduced, rather it induces the structure of the problem from data. Weakness as there is no guarantee that the prices obtained from these models will conform to rational pricing. The presentation reviews the current state of non-parametric option pricing, covering; parametric and non-parametric adjustments to Black-Sholes model and purely non-parametric methods.

KEYWORDS

Black-Sholes model, Neural Networks, Classification and Regression Trees, Genetic Programs

1. Problems with traditional parametric option pricing

The Black-Sholes formula presented the first, pioneering tool for rational valuation of options. It was a first option pricing model with all measurable parameters. It is still constantly being adapted for valuation of many other financial instruments. However it and its derivatives show systematic and substantial bias e.g. Galai [1983]. To improve pricing performance, Black-Sholes formula has been generalized to a class of models referred to as the *modern parametric option pricing models* – outlined below.

2. The assumptions of Black-Sholes model

The first approach to improve the pricing results is to relax and make more realistic the assumptions Black-Sholes model is based on. Black [1990] himself considered the assumptions of his model to be unrealistically simple. There are several assumptions, used to derive the original model Black, Sholes [1973], relaxation of which had been reported in the literature:

No dividends	Relaxed by Merton [1973]
No taxes nor transaction costs	
Constant interest rates	Merton [1973]
No penalties for short sales	
Continuous market operation	Merton [1976]
Continuous share price	Cox, Ross [1976]
Lognormal terminal stock price return	Jarrow, Rudd [1982]

In addition, Black-Sholes model assumes; continuous diffusion of the underlying, normal distribution of returns, constant standard deviation/volatility, and no effect on option prices from supply/demand.

2.1. Continuous diffusion of the underlying.

Relaxation of this assumption produced various jump-diffusion models, e.g. Merton [1973], McCulloch [1987]. Ball et. al [1985] concluded that Merton's model improved Black-Sholes only for out-of-money options with less than a month to maturity. A related, recent factor is the long-term memory in asset valuation that complicates model parameter estimation. For example, the market crash of October 1987 significantly altered the markets to make option prices account for another similarly extreme event Evans [1996]. While October 1987 effect still persists in the market option pricing, most parametric model use historical data spanning time windows much too short to account for it.

2.2. Normal distribution of returns.

In practice this assumption seldom holds. Various methods of introducing distributions closer to real ones were investigated, e.g. Corrado [1997], Jarrow, Rudd [1982], Brown [1997]. It appears that the deviations from normality are more complex that these investigated and attempts to introduce a more realistic return distribution are not yet successful Bates [1996].

2.3. Constant standard deviation/volatility.

This assumption too is only a very rough approximation of reality. For example, volatility of SP500 for period 1857-1987 varies between 2% - 20% a month Schwert [1989]. The consequence of this realization is a series of model of volatility. The first type of models assume a deterministic explanation of volatility in terms of: constant elasticity of variance (CEV), displaced-diffusion or treat option as a compound option on the value of the firm.

For the CEV option-pricing model:

$$dS = \phi S dt + \sigma S^{p-1} dW_s \quad \text{where} \quad 0<p<1$$

S is the underlying price, W is the Wiener process, ϕ is the drift and σ the diffusion. CEV model is intended to model observed negative correlation between the underlying's price and it's volatility. Beckers [1980], Gibbons et al. [1988] estimated CEV parameters on stock prices from 1962-85. MacBeth and Merville [1980] found CEV to be more accurate than the Black-Scholes model. However, there are two problems with CEV model; CEV option prices will approach either zero or infinity in the long run, and CEV model allows a positive probability of an underlying's price going down to zero (which makes it unsuitable for index pricing where this seems highly unlikely).

Compound option model derives the negative relationship between stock option price and volatility from capital structure of the firm. The stock is considered a call option on the value of the firm. Accordingly, a call option on the stock is an option on an option.

Displaced-diffusion model derives the relation between volatility of the stock return and the stock price Rubinstein [1981]. This model assumes that volatility and underlying price are positively correlated. MacBeth and Merville [1980] found it more accurate than CEV model.

Deterministic models offer a promise of: reflecting exactly the empirical smiles Bates [1996], capturing the empirical regularities like clustering of volatilities, correlation between volatility and underlying. In addition, under such models markets are dynamically complete so derivative securities can be priced using no-arbitrage arguments without need for general equilibrium and risk-premium models Burashi et al. [1999]. However in literature, specific deterministic models are simplistic and fail to deliver on much of their promise Lajbcygier [1999a].

3. Stochastic volatility models

There are several reasons for the interest in stochastic volatility models. The math for stochastic volatility option-pricing models is an extension of conventional option-pricing mathematics. In principle, such models can explain option-pricing biases. And, finally, research has shown that volatility explains more of option prices than other stochastic variables, such as interest rates.

Bivariate diffusion Both underlying's price and volatility are diffusions. Works in this area include; Hull, White [1987], Scott [1987], Wiggins [1987]. In Stein, Stein [1991] it is assumed a zero correlation between volatility and underlying. Heston [1993] extended the work on non-zero correlation and derived a closed form formula for option prices. Over time, it became apparent that the complexities of stochastic volatility models often result in problems chosen for analysis for their are tractability, and less for their ability to well explain an empirical volatility process. Additionally such processes cannot explain extremely large implied volatility values observed in the market Gallant et al. [1997], e.g. in currency market Bates [1996]. One promising recent approach to enhance bivariate model, introduces a functional dependency of a suitable form between underlying and volatility processes, and calculates from it correction factors to Black-Sholes prices. Basu [1999]

Bivariate Diffusion with Jumps These models are intended to explain the significant negative skew of implied return distributions observed after the 1987 stock market crash. Volatility models alone attribute the skew to the tendency of volatility to fall as stock market rises. Jump models see the skew as result of fears of another crash. Also, volatility models predict a direct relationship between skewness and option maturity – with little skewness for short maturity options. On the other hand, jump models assume finite volatility shocks that are independently distributed and predict inverse relationship between skewness and maturity – with little skewness for long maturity options. Combining the two models Bates [1997] finds that actual, flat-to-declining relationship declines more slowly than that predicted by jump model, and the empirical data lie somewhere in between both models' prices.

Trivariate Diffusion: Underlying, Volatility and Interest Rates A first such model was presented by Bailey et al. [1989]. Amin et al. [1992] added a systematic component in the stock returns. Saez [1997] evaluated the performance of trivariate models and concluded that they generally overpriced near-the-money options and underpriced all other. The only significant difference between trivariate and Black-Sholes prices was in near-the-money options.

Supply/Demand Effect Most of work on modern parametric option pricing assumes that the bias of the conventional parametric option-pricing models can be explained by correct specification for the underlying process. However. Heynen [1994] concludes that observed smile patterns cannot be explained on by alternatively specified asset processes. Including supply/demand into model Follmer et al. [1986], Schweizer [1991] leads to option price biases similar to market ones.

4. Problems with modern parametric option pricing

Modern parametric option-pricing models were expected by many to:

- Be well-specified,
- Consistently outperform other models,
- Be statistically consistent with underlying asset return dynamics,
- Provide a statistical theory of option pricing error, and
- Be elegant and not difficult to estimate

They failed to deliver. Arguably Lajbcygier [1999b], generalizations to Black-Sholes didn't succeed and the resulting models are too complex, have poor out-of-sample performance, and use implausible and/or inconsistent implied parameters. While parametric models provide internal consistency, they do not out-perform simplistic approaches out-of-sample. Even the most complex modern parametric models are imperfect and are outperformed by simple, less general models. They often produce parameters inconsistent with underlying time series and inferior hedging and retain systematic price bias they were intended to eliminate Bakshi [1997], Bakshi [1998].

5. Naïve smile models

Several models, popular with traders do not rely on theoretical arguments. One approach, *relative smile prediction*, states that implied volatilities of a shorter-term option equal volatilities of a corresponding longer-term option with the same moneyness ratio. In the *absolute smile prediction*, a short-term option with given strike price has an implied volatility equal to corresponding longer term option with the same strike price. Jackwerth and Rubinstein [1998] showed that implied volatility smiles can describe the relationship between;

- Option prices at the same point in time and the same time to expiry but different strike prices,
- Option prices at the same point in time and the same strike prices but different time to expiry,
- Option prices with the same strike price and time to expiry but at different points in time, and
- Option prices at the same point in time and the same strike price and time to expiry but with different underlying assets.

According to Jackwerth and Rubinstein [1998] the ultimate objective is to find a single model that can explain all four relationships simultaneously. Based on series of tests applied to variety of models, they conclude that naïve approaches are consistently the best, stochastic deviation models are next best, then there are deterministic volatility models and finally the traditional parametric models.

6. Non-parametric option-pricing models

Prompted by shortcomings of modern parametric option-pricing, new class of methods was created that do not rely on pre-assumed models but instead try to uncover/induce the model, or a process of computing prices, from vast quantities of historic data. Many of them utilize learning methods of Artificial Intelligence. Non-parametric approaches are particularly useful when parametric solution either; lead to bias, or are too complex to use, or do not exist at all.

7. Model-Free Option-Pricing

The purest version of non-parametric option-pricing methods, are model-free methods. They involve no finance theory but estimates option prices inductively using historical or implied variables and transaction data. Although some form of parametric formula usually is involved, at least indirectly, it is not the starting point but a result of an inductive process. There are several methods in this group:

Model-free option pricing with Genetic Programming (GP) In a version most closely following its biological inspiration, a model is represented by a (long) bit string, whose parts are interpreted as coefficients of certain computational process or an option-pricing formula. A set of bit-strings, individuals/option-pricing processes, is then subjected to an iterative process that eliminates the worst performing ones. Surviving bit-strings are mutated (having some bits flipped at random) and crossed-over, i.e. creating new bit-strings by randomly selecting bits from two (or more) others. In some approaches, the length of bit-strings is extended on occasion, by adding at random a new term/component to the process. The mapping of bit-strings into option-pricing processes determines the space within which the best process is selected by Genetic Programming. A version of GP represents its process space as a tree and uses a heuristic walk over this tree, instead of random mutation, to find the best pricing method. An example of work from this category is Keber [1998] which found a GP solution for American put pricing, to be better than any analytical approximation.

Model-free oprion-pricing with kernel regression include ‘smoothers’ – sophisticated processes of averaging data to reduce error, Hardle [1993]. Ait-Sahalia et al. [1995]. Found that kernel methods can provide accurate pricing. Use of kernel methods for American option pricing with stochastic dividends and stochastic volatility was shown to be consistently more accurate than conventional models.

Model-free option-pricing with Artificial Neural Networks (ANN) Malliaris and Salchenberger [1993] trained an ANN that included amongst inputs; a same-day, at-the-money implied volatility, the underlying price and the option price lagged by a day. This ANN was reported to outperform Black-Sholes for out-of-money options but not for the in-the-money options. In another work Hutchinson et al. [1994] used historical volatility over a 60-day window with similar conclusions regarding pricing performance of SP500 options. Qi and Maddala [1995] replaced volatility with previous day's open interest. Hanke [1997] used ANN for assets whose volatility follows a GARCH process. No exact option-pricing model exists for GARCH, however ANN can learn the pricing function very accurately. Anders et al. [1996] compared implied and historical volatilities as ANN input variables and found the use the former to result in much more accurate pricing. They also used a statistical inference White [1989] to prune not-significant connections. Galindo [1998] found ANN to consistently outperform multivariate regression techniques.

ANN option-pricing was shown able to outperform conventional techniques in many different markets. The two problem areas are;

- ANN's inability to model deep-in-the-money options, due to infrequent transactions in that area, and
- Shortest maturity/at-the-money options due to discontinuity of transactions in that region.

8. Non-parametric approaches

The independence of model-free approaches from any finance theory means prices produced by them may not conform to rational pricing and/or may not capture restrictions implied by arbitrage Ghysels [1997]. To improve model-free approaches in this respect, needed constraints have to be introduced Barucci [1997]. There are several ways used to enforce rational pricing into model-free pricing;

The Equivalent Martingale Measure (EMM) adjusts prices to reflect a preference-free, risk-neutral market. Campbell et al. [1997] In risk-neutral economy all assets must earn the same return. Under the risk-adjusted probability distribution the stock price follows a Martingale (a stochastic process where the best forecast of tomorrow's price is today's) and is arbitrage-free.

Non-parametric adjustments to Black-Sholes estimate a portion of the option-pricing model non-parametrically while retaining the conventional option-pricing framework to guarantee rational-pricing.

Generalized Deterministic Volatility estimates unknown volatility either parametrically or non-parametrically and inserts this estimate into a conventional model. The three sample approaches in this category are:

- Implied Binomial Tree Rubinstein [1994], Shimko [1993],
- Generalized Deterministic volatility functions Dumas et al. [1996], and
- Kernel approach Ait-Sahalia te al. [1998] Gourieroux et al. [1994]

Generalized volatility approaches have their costs and benefits. The implied tree approach for example can help with estimation of exotic, path dependent options where no analytical formula exists.

9. Conclusion

Since no perfect option-pricing method was found yet, the interest in finding one continues unabated. There a host of conflicting criteria a perfect option pricing should satisfy; it should reliably out-perform competing methods, it should be simple, elegant. It should provide no arbitrage, rational pricing. It should be well defined and validated…

The traditional models of the Black-Sholes family show biases and their assumptions are unrealistic. The attempt to relax their assumptions often leads to implausible values of implied parameters. The complementary non-parametric approach, where no model is presumed but prices are induced from historic data, doesn't guarantee rational, arbitrage-free prices unless suitable constraints are added. The ideal may be in between, in hybrid or semi-parametric models combining both approaches. Whether such ideal method can ever be found, the quest produces much of very insightful research advancing our understanding of valuation of many financial instruments.

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