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# *Maximizing futures returns using fixed fraction asset allocation*

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While considerable evidence has been produced concerning the efficacy of trading rules in futures markets, the results have generally not allowed for the reinvestment of profits as might be observed for real traders. Similarly, the determination of the appropriate capital allocation required per futures contract traded has been largely unstructured so making reported percentage returns questionable. This paper provides evidence of the profitability of a simple and publicly available trading rule in five futures markets but more importantly incorporates the ability to reinvest any profits via the 'Optimal $f$' technique described by Vince (1990). The results indicate that money management in speculative futures trading plays a more important role in trading rule profitability than previously considered by providing dramatic differences in profitability depending on how aggressively the trader capitalizes each futures contract.

## I. INTRODUCTION

The research generally presented in the area of trading rule performance in futures markets has treated profits and losses in a static manner with respect to position sizes. That is, a trading rule is devised, tested and reported using a fixed number of futures contracts for every trade throughout the test. Examples using this approach include Leuthold (1972), Stevenson and Bear (1970) through to Lukac *et al.* (1988) and Taylor (1993); while more recent examples can be found in Buckle *et al.* (1999), Raj (2000), Kwan *et al.* (2000) and Wang (2000).

While the use of a single contract is a useful simplifying assumption for determining issues such as the efficacy of a trading rule or other forecasting techniques, this simplistic one-contract only approach creates two problems. First, little information is revealed about the amount of capital required to generate those profits, so providing limited information as to the 'percentage returns' by reporting only dollar returns. Second, it ignores the basic behavioural elements of a trader's attitude to risk and how position sizes may alter depending on recent successful/unsuccessful trades.

The fixed contract approach provides a useful approximation for assessment of trading rule(s), however its ability to reflect reality is intuitively unappealing as some reasonable approximation of human behaviour, as the trader is faced with the emotional issues relating to profits and losses on a real trading account. Recent research in behavioural finance (for example, Locke and Mann, 2001) demonstrates that changes to a trader's position sizes are often related to recent wins and losses. As would be expected under utility theory, traders will increase risk when making profits and decrease risk when facing losses.

Trading practitioners are faced with the issue of determining appropriate position sizing for a given level of trading capital. For example, Kwan *et al.* (2000) argue a common theme in the futures trading literature that the

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returns are those generated from profits produced where a riskless US Treasury Bill is used as the margin security for the futures contract. It could well be argued that the use of a \$100 000 US Treasury Bill would be inefficiently used if only one contract were traded on each transaction in markets such as Corn where low price volatility sees the initial margin of only \$405 per contract.[1] This element has been so far ignored by the literature where such a proxy return figure has been applied. Conversely, it would be naïve in the extreme to assume that the position size should be \$100 000/\$405 = 247 futures contracts per trade.

While psychology would arguably play a strong role in determining a trader's risk levels, traders may adopt some form of betting strategy where they feel they have, in gambling parlance, some 'edge over the house'. With the belief of some 'edge' in a particular game, it is reasonable to treat speculative futures trading as analogous to games of chance where the futures trader's position size can be managed in the same way as the betting stakes available to a blackjack player who believes they have identified some edge in the game.

If the trader believes he has identified some form of exploitable market inefficiency he must then determine how best to manage his trading capital to maximise his utility from trading. By applying concepts drawn from gaming mathematics, the technique of Optimal $f$ for determining position sizes has emerged in trading practitioner literature. This technique essentially aims to identify the number of futures contracts that should be traded for a given trading rule, to maximize the geometric rate of return. For the purposes of this paper it is assumed that a trader's utility is maximized when the profits from trading activities are maximized irrespective of the risks generated by that part of the portfolio allocated to futures trading.

This study provides the first empirical evidence as to the value of the Optimal $f$ technique for portfolio management in a futures trading context and demonstrates the impact of position sizing and reinvestment rates on the portfolio's final balance in the S&P500, US T-Bonds, British Pound, COMEX Gold and Corn futures markets. Therefore a simple and robust technique for determining percentage returns is demonstrated and the ability to reinvest profits provides results more consistent with behavioural finance.

The remainder of the paper is organized as follows. The next section outlines basic principles of gaming mathematics, which then leads into the concept of the optimal fixed fraction ('optimal $f$') rule of reinvestment. Section III then outlines a simple futures market application of the optimal $f$ technique and the associated results of this hypothetical trading rule experiment. The final section presents a summary and conclusion.

## II. GAMING MATHEMATICS AND OPTIMAL $f$

The history of gaming mathematics is indeed extensive wherein authors attempt to grapple with maximizing returns from payoffs governed by probabilistic properties. Examples include Bernoulli's work in the 18th century (in Sommer, 1954), Latane (1959), Thorp and Walden (1966), Thorp (1966, 1969, 1974 and 1980) and Connolly (1999).

If the returns from futures trading are viewed as being a probabilistic process one can identify various characteristics about trading rule performance. These characteristics include percentage of winning trades, maximum loss, maximum profit, average size of wins/losses and so on. While these characteristics are readily observable in most games of chance, historical results can be modelled in futures trading from testing a trading rule over a given data set. Armed with this information one can apply concepts developed in gaming mathematics to the position size problem in futures trading.

Kelly (1956) approached the problem from the perspective of a gambler with access to sporting results before they become available to the wider public. If the sporting result was known with certainty the gambler should bet 100% of their available capital on the outcome of a sporting event. Therefore the value of a gambler's starting capital, $V_O$, would grow, $G$, over $n$ trials exponentially at the rate shown in Equation 1.

$$G = \operatorname*{Lim}_{n\to\infty} \frac{1}{n} \log \frac{V_n}{V_O} \tag{1}$$

If the probability, $p$, of an error is introduced, that is the information about the sporting result may or may not be accurate, the gambler then must adjust the betting strategy or else the probability of error would ultimately lead to a loss of all betting capital once the first loss was encountered. Consequently, for any non-zero value of $p$ the gambler will lose the entire stake with a probability of one. As the rational gambler is unlikely to intentionally lose the entire gambling stake, the gambler should then only invest a fraction, $l$, of the capital on any individual bet. Having introduced the probability of outcomes defined as either a Win, $W$, or a Loss, $L$, (where the probability if loss is defined as $q = 1 - p$) the value of the stake after $n$ trials is then:

$$V_n = (1 + l)^W (1 - l)^L V_O \tag{2}$$

[1] CBOT Margins, June 14, 2002, http://www.cbot.com/cbot/docs/26161.doc?report_id = 84

The growth, $G$, of the portfolio can be stated with a probability of one as:

$$G = \lim_{n\to\infty}\left[\frac{W}{n}\log(1+l) + \frac{L}{n}\log(1-l)\right]$$
$$G = q\log(1+l) + p\log(1-l) \tag{3}$$

While Kelly's (1956) paper produced a unique insight into the betting problem, the foregoing equations assume that wins and losses are of equal amounts and that the probabilities are known with certainty. Kelly's (1956) approach then needs to be modified for the futures trading context where probabilities of wins, losses and determining payoffs for a trading model must be estimated over some historical data set.

The application of Kelly's (1956) work on futures trading was primarily conducted by two authors, Gehm (1983) and Vince (1990). In Vince (1990), the technique described as 'Optimal $f$' was presented as a portfolio management tool for futures traders. It essentially aims to identify the optimum *fixed fraction* or the portfolio to bet on any individual outcome. Therefore, if $f = 0.2$ the trader would bet a fixed 20% of their portfolio allocated to futures trading on any individual trade – position sizes would increase as profits were accrued and decrease as losses were suffered.

At the *optimal f*, the rate of reinvestment is found that would maximize the geometric rate of return on the portfolio and so dominate all other betting strategies applied to trading activities. In one empirical study of the value of fixed fractional betting, Ziemba (1987) found that a fixed fraction approach dominated all other betting strategies when applied to a horse-racing data set. In the method proposed by Vince (1990) maximizing the geometric rate of return is achieved by modelling the largest observed loss and trading the portfolio reinvestment rate on this basis and determining which multiple of that largest loss would have produced the largest return on the funds invested.

To maximize the geometric growth one needs to identify the account capitalization required for each futures contract that produces the highest Terminal Wealth Relative (TWR) to the original investment per futures contract for a range of $f$ values. As the account capitalization per contract is a function of the $f$ value selected and the largest loss, it is defined as:

$$\text{Capitalization per contract} = \frac{\text{Largest Observed Loss}}{f} \quad (\text{for } 0 < f \leq 1) \tag{4}$$

From Equation 4, had the largest observed loss been \$1000 the optimal $f$ aims to determine which amount of capital should be applied per futures contract. At $f = 1.00$, the trader would allocate \$1000/1 = \$1000 per contract. As illustrated by Kelly (1956), if the bet is made equating to 100% of maximum loss, the probability of a loss of all trading capital is 1. Had the trader adopted a more conservative $f$ value, that is $f < 1$, then extra capital per contract would be allocated. For example, had a value of $f = 0.60$ been adopted, the capitalization per contract is \$1000/0.40 = \$2500 per futures contract traded.

The number of futures contracts traded on any given trade for a given level of portfolio capitalization is then a function of the optimum funding per contract divided by the account balance. Therefore, if the optimum funding per contract is \$10 000 and the trading account has \$100 000, the trader would then trade \$100 000/\$10 000 = 10 futures contracts on the next trade. The number of futures contracts to be traded is then defined in Equation 5 as:

$$\text{Number of Contracts} = \frac{\text{Account Balance}}{\text{Capitalization per Contract}} \tag{5}$$

The Optimal $f$ approach thus resolves two key issues for trading practitioners. First, how much money should be allocated for each futures contract traded and second, how many futures contracts should be traded at any one time with a given portfolio allocation to speculative futures trading activities.

The approach taken by Vince (1990) presented here is adapted into the following steps. First, select a trading rule and a market where the trading rule is to be tested. Second, run a simulation of the trading rule and report the results including a statement of profits and losses from each trade. Third, identify the largest loss that occurred during the test period. Fourth, calculate the profitability of trading from the trading rule where reinvestment occurs at the largest loss divided by an $f$ value ranging between zero and one. Finally, determine the optimal $f$ value – that is, the $f$ value which produces the highest TWR with profits reinvested according to the account capitalization per contract.

## III. APPLICATION AND RESULTS IN FUTURES MARKETS

To illustrate the performance of the Optimal $f$ technique, a publicly available trading model (the 'turtle model') from the website 'www.turtletrader.com' was applied into five different futures markets using a dataset covering the period 1990 and 1998. These were futures contracts over the S&P500, US T-Bonds, British Pound, COMEX Gold and Corn. The turtle model is essentially a basic 70-day moving average model using daily closing prices, $P$, with a $\pm 2$ standard deviation threshold. Specifically, the trading rule is specified as:

Buy when

$$P_t > \left[\frac{1}{70}\sum_{j=1}^{70} P_{t-j}\right] + 2\sigma \tag{6}$$

Sell when

$$P_t < \left[\frac{1}{70}\sum_{j=1}^{70} P_{t-j}\right] - 2\sigma \quad (7)$$

Trades are executed when the Closing Price at time $t$ crosses the 70-day moving average. The position sizes are then created by testing values of $f$ between 0.05 and 1.00 at increments of 0.05. Consistent with research in the futures trading rule area (for example, see Lukac *et al.*, 1988), transactions costs have been deducted at $100 per round-turn trade, that is $50 on the purchase and $50 on the sale for each futures contract. This is a conservative measure allowing for expenses associated with brokerage and poor execution or poor short-term liquidity.

Initially these models are applied into the five futures markets with no reinvestment and the basic reported dollar profits are displayed in Table 1. The table shows that positive Gross Profits were available in the US T-Bonds, Gold and Corn futures markets. Gross and Net Losses were reported for the S&P500 and British Pound futures markets. The determination of percentage returns could be achieved via some proxy, such as a riskless US Treasury security being used as the initial margin, but the problems with such an approach are discussed above. Accordingly, one applies the optimal $f$ technique to each contract in the context of the turtle model.

Table 1. *Basic no reinvestment profit results for the turtle system across five futures markets*

| | S&P 500 | US T-Bonds | British Pound | Gold | Corn |
|---|---|---|---|---|---|
| Number of trades | 62 | 43 | 52 | 40 | 35 |
| Per cent profitable | 24% | 40% | 21% | 40% | 51% |
| Maximum drawdown | 69 863 | 9438 | 42 550 | 5740 | 2563 |
| Gross profit | −7288 | 29 500 | −2088 | 5190 | 19 625 |
| Net profit | −13 488 | 25 200 | −7288 | 1190 | 16 125 |

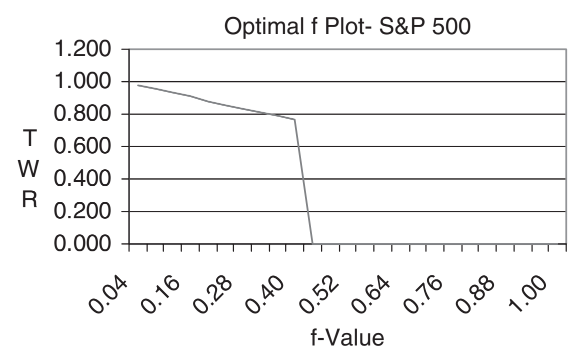


Fig. 1. *Optimal* f *plot for turtle trading of the S&P500 futures contract*

In the case of the S&P500 futures contract, Fig. 1 shows the Optimal $f$ plot where capitalization per contract has been determined across a range of $f$ values tested. Table 2 shows the account capitalization and TWR for each $f$ value tested. As Fig. 1 and Table 2 reveal, the model proved to be unprofitable at any $f$ value tested. It also very dramatically highlights that the trader relying on capitalization of less than $57 250 per contract would have experienced a loss of all trading capital. As the model produced losses in the S&P500 futures market during the test period, no benefits of money management via optimal $f$ were observed. While not definitive, it does suggest that inexperienced traders, with only limited knowledge of money management issues and relying on relatively small amounts of capital for speculative trading would have almost certainly faced considerable losses had they used this trading rule in S&P500 futures.

In the case of the US T-Bonds futures contract, Fig. 2 and Table 3 show the impact of optimal $f$ when trading according to the turtle model. These exhibits show that money management did produce an impact on the profitability and percentage returns produced during the test period. At an $f$ value of 1.00 the largest loss is shown to be $3475. As expected, betting an amount per contract to the maximum observed loss by definition produces a loss of all trading capital. But where different capitalization rates per contract are applied, the percentage returns vary considerably. For example, had the trader reinvested at an $f$ value of 0.56, representing a rate of one contract for every $6205 in the trading account, the trader's TWR would have been 1.141 or a return of 14.1% over the test period. Conversely, had the trader applied an $f$ value of 0.20, or capitalized the trading account to $17 375 per contract, the TWR would have been 2.344 or a return of 134.4% during the test period.

Figure 3 and Table 4 show the results for optimal $f$ when turtle trading the British Pound futures during the test period. The table shows that the trading rule was not profitable over the test period and that no matter what level of capitalization or reinvestment strategy was adopted, the unprofitable model could not be transformed into a successful trading model. It also shows that had the

Table 2. *Optimal* f/*TWR results for turtle trading the S&P 500 futures contract*

| *f*-value | 0.04 | 0.08 | 0.12 | 0.16 | 0.20 | 0.24 | 0.28 | 0.32 | 0.36 | 0.40 | 0.44 | 0.48 | 0.80 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Account size | $572 500 | $286 250 | $190 833 | $143 125 | $114 500 | $95 417 | $81 786 | $71 563 | $63 611 | $57 250 | $52 045 | $47 708 | $28 625 | $22 900 |
| Final balance | $559 013 | $272 763 | $177 346 | $129 638 | $101 013 | $81 929 | $68 298 | $58 075 | $50 124 | $43 763 | $- | $- | $- | $- |
| TWR | 0.976 | 0.953 | 0.929 | 0.906 | 0.882 | 0.859 | 0.835 | 0.812 | 0.788 | 0.764 | 0.000 | 0.000 | 0.000 | 0.000 |

trader capitalised the position at any less than $22 604 per contract it would have resulted in the total loss of all trading capital.

Figure 4 and Table 5 show the performance of the turtle trading rule at different capitalization levels in the COMEX Gold futures contract. Note that the percentage returns steadily increase for $f$ values between 0.04 and 0.44 but then face a precipitous decline. At $f=0.44$ the TWR of 1.240 shows that the trading rule was able to produce profits of 24% during the test period, but capitalization at levels less than $4955 resulted in the total loss of all trading capital.

Finally, Fig. 5 and Table 6 show the impact of money management in turtle trading Corn futures during the test period. At an $f$ value of 1.00 it is observed that the largest loss was $1325 per contract suggesting a relatively stable performance for the trading rule. This stability allowed profits to be reinvested with considerably beneficial results. Had the trader capitalized the trading positions at $2070 per contract the TWR was 20.991 or a closing account balance of $43 458. This represents a return on funds invested of an amazing 1999.1%.

## IV. SUMMARY AND CONCLUSIONS

This study identified the importance of money management via the Optimal $f$ technique for speculative traders in futures markets, when applying the turtle trading system. Some of the key findings highlighted the importance of capitalization and reinvestment for returns to futures traders. While in some markets, such as US T-Bonds, the results showed that returns of greater than 100% were achievable, the results were far more dramatic in Corn futures where returns of around 2000% were observed. Trade capitalization played a far less important role in those markets where losses were produced and no amount of money management was able to convert the unprofitable trading rules into a winning position.

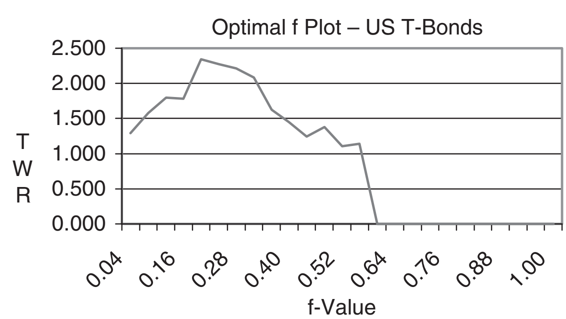


Fig. 2. *Optimal* f *plot for turtle trading of the US T-Bonds futures contract*

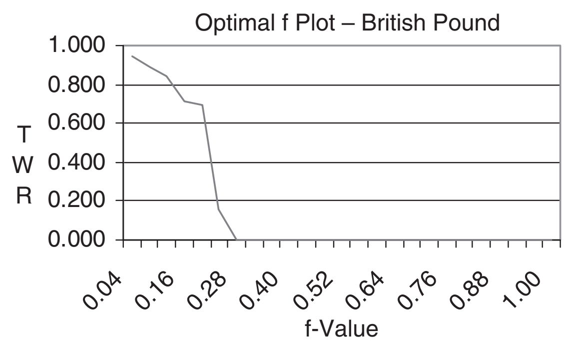


Fig. 3. *Optimal* f *plot for turtle trading of the British Pound futures contract*

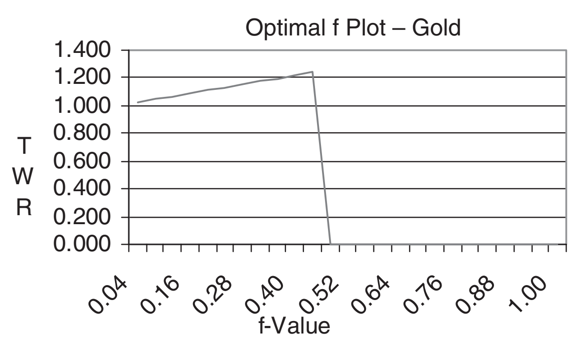


Fig. 4. *Optimal* f *plot for turtle trading of the COMEX gold futures contract*

Table 3. *Optimal* f/*TWR results for turtle trading the US T-Bonds futures contract*

| *f*-value | 0.04 | 0.08 | 0.12 | 0.16 | 0.20 | 0.24 | 0.28 | 0.32 | 0.40 | 0.52 | 0.56 | 0.60 | 0.80 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Account size | $86 875 | $43 438 | $28 958 | $21 719 | $17 375 | $14 479 | $12 411 | $10 859 | $8688 | $6683 | $6205 | $5792 | $4 344 | $3475 |
| Final balance | $112 075 | $68 638 | $51 996 | $38 719 | $40 731 | $32 910 | $27 467 | $22 622 | $12 575 | $7402 | $7080 | $- | $- | $- |
| TWR | 1.290 | 1.580 | 1.796 | 1.783 | 2.344 | 2.273 | 2.213 | 2.083 | 1.447 | 1.108 | 1.141 | 0.000 | 0.000 | 0.000 |

Table 4. *Optimal* f/*TWR results for turtle trading the British Pound futures contract*

| *f*-value | 0.04 | 0.08 | 0.12 | 0.16 | 0.20 | 0.24 | 0.28 | 0.32 | 0.36 | 0.40 | 0.44 | 0.48 | 0.80 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Account size | $135 625 | $67 813 | $45 208 | $33 906 | $27 125 | $22 604 | $19 375 | $16 953 | $15 069 | $13 563 | $12 330 | $11 302 | $6781 | $5425 |
| Final balance | $128 338 | $60 525 | $37 921 | $24 131 | $18 875 | $3517 | $- | $- | $- | $- | $- | $- | $- | $- |
| TWR | 0.946 | 0.893 | 0.839 | 0.712 | 0.696 | 0.156 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Table 5. *Optimal* f*/TWR results for turtle trading the COMEX gold futures contract*

| *f*-value | 0.04 | 0.08 | 0.12 | 0.16 | 0.20 | 0.24 | 0.28 | 0.32 | 0.36 | 0.40 | 0.44 | 0.48 | 0.80 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Account size | $54 500 | $27 250 | $18 167 | $13 625 | $10 900 | $9083 | $7786 | $6813 | $6056 | $5450 | $4955 | $4542 | $2725 | $2180 |
| Final balance | $55 690 | $28 440 | $19 357 | $14 815 | $12 090 | $10 273 | $8976 | $8003 | $7246 | $6640 | $6145 | $- | $- | $- |
| TWR | 1.022 | 1.044 | 1.066 | 1.087 | 1.109 | 1.131 | 1.153 | 1.175 | 1.197 | 1.218 | 1.240 | 0.000 | 0.000 | 0.000 |

Table 6. *Optimal f/TWR results for turtle trading the corn futures contract*

| *f*-value | 0.04 | 0.08 | 0.12 | 0.16 | 0.20 | 0.24 | 0.28 | 0.32 | 0.40 | 0.52 | 0.64 | 0.72 | 0.80 | 1.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Account size | $33 125 | $16 563 | $11 042 | $8281 | $6625 | $5521 | $4732 | $4141 | $3313 | $2548 | $2070 | $1840 | $1656 | $1325 |
| Final balance | $49 250 | $32 688 | $30 467 | $28 094 | $24 075 | $26 146 | $24 432 | $27 016 | $36 588 | $35 211 | $43 458 | $24 640 | $- | $- |
| TWR | 1.487 | 1.974 | 2.759 | 3.392 | 3.634 | 4.736 | 5.163 | 6.525 | 11.045 | 13.818 | 20.991 | 13.389 | 0.000 | 0.000 |

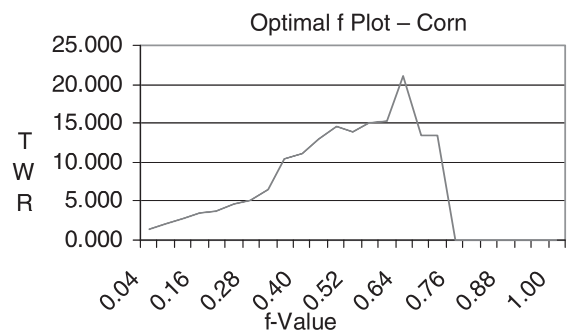


Fig. 5. *Optimal* f *plot for turtle trading of the corn futures contract*

One limitation in this work is that the results were all obtained *ex-post* and so our conclusions must contain an important caveat. Any profits generated in future tests may not reflect the past trading performance and so the capitalization issue may need to be treated more cautiously. Several alternatives are apparent including the use of very strict stop-losses on trading positions to ensure that the maximum observed loss is reflective of future trading activities. Similarly, the use of *f* values of less than one implies that some account drawdown is experienced and must be allowed for in the account capitalization issue and so perhaps more conservative *f* values should be applied.

For empirical researchers, the conclusion that capitalization and money management may play a role in determining the success or failure of speculative traders cannot be ignored. The results show that the propensity for account balances to be reduced to zero by undercapitalizing futures trading positions is quite high and may contribute to the failure of many speculators.

Finally, many works on the performance of technical trading models have reported results for a single futures contract for simplicity/comparability or for other reasons. The research presented here allows greater reconciliation between modelled results and the trading performance more likely to be encountered by traders with respect to risk attitudes observed in behavioural finance characteristics and the reinvestment of profits in speculative trading.

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