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Non-Monotonic Long Memory Dynamics in Black-Market Exchange Rates.

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List of Symbols

Φ : upper case Greek "phi"

μ : Greek "mu"

ϵ : Greek: "epsilon"

Δ : Greek: "delta"

σ : Greek: "epsilon"

ϕ : Greek "phi"

θ : Greek "theta"

Θ : upper case Greek "theta"

ϵ : Greek "epsilon"

∞ : infinity

$\prod$: product sign

Γ : upper case Greek "Gamma"

$\sim$: similar

λ : Greek "lambda"

π : Greek "pi"

β : Greek "beta"

ρ : Greek "rho"

$\sum$: summation sign

$\boldsymbol{y}$: bold y

$\boldsymbol{\sigma}$: bold Greek "sigma"

$\boldsymbol{\epsilon}$: bold Greek "epsilon"

$\boldsymbol{\theta}$: bold Greek "theta"

NON-MONOTONIC LONG MEMORY DYNAMICS IN BLACK-MARKET PREMIA

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ABSTRACT

The dynamic response of Black market premia to domestic shocks is an important issue in the design and implementation of stabilization and reform programs. We use a vector autoregressive fractionally integrated model to provide new evidence on the dynamics of the official and Black market exchange rates. We show that the official and Black market exchange rates in Hungary are cointegrated with a negative fractional order of integration in the cointegrating residuals. The new empirical finding means that the cointegrating residuals are positively autocorrelated in the short run due to autoregressive dynamics, but are negatively autocorrelated in the long run. The rich and complex dynamics of the premia suggests the existence of what we call long memory non-monotonicity.

KEYWORDS: Black Market Exchange Rate, Long-Memory Models, Overshooting

JEL CLASSIFICATION: F31, C22

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1 Introduction

The *sine qua non* of Black markets for foreign currency is restrictive foreign exchange and trade policies. Black markets for foreign currency are the rule rather than the exception in developing countries and the spread between the two rates—the Black market premium—is a useful indicator of real exchange rate misalignment.[1] A key policy objective in countries with over-valued exchange rates and large and well functioning Black markets is unification of the Black market and official rates. An important policy instrument in any attempt to unify the foreign exchange rate is nominal devaluations of the official exchange rate. However, despite the importance of devaluations as a means of reducing the Black market premium there is scant empirical evidence on the dynamics of the premium in response to a devaluation.

Investigating the dynamics of the Black market premium in response to a devaluation is important because movements in the premium can shed light on the risk and speculative behavior of agents participating in these markets. It is only by understanding how such agents respond to changes in their economic environment that we can design and implement effective policies.

This paper contributes to the empirical literature on Black markets for foreign exchange by investigating the response of the premium to devaluations in Hungary. We focus on Hungary for the following reasons. It maintained relatively close ties to the West despite the "iron curtain"; data on both the official and Black market exchange rates are readily available and convertibility and exchange rate policies are pressing issues for economies in transition to a market-based economic system.

An important novelty in our empirical analysis is the use of a multivariate maximum-likelihood estimator of ARFIMA processes developed by Sowell (1989a) and implemented by Dueker and Startz (1994) which estimates the fractional differencing parameter and the ARMA parameters simultaneously. This is important because by conducting joint estimation of the orders of integration of an input series and cointegrating residuals one is able to measure on a continuous scale the extent to which two series share a common stochastic trend. Failure to allow for a more general ARFIMA process and estimate all parameters jointly can bias the estimator of the fractional differencing parameter, as noted by Sowell (1992b).

We use a simple bivariate version of the more general model to test the hypothesis that the Black market and official exchange rate series have equal orders of integration. Our methodology differs from most studies of cointegration in which it is assumed, upon failure to reject the null hypothesis of a unit root, that both input series are integrated of the same order. The payoff to using the more general approach is that we are able to document more complex and richer dynamics in response to a devaluation.

To the best of our knowledge this is the first paper to examine long memory dynamics in Black market premia. There is however, a voluminous literature on Black markets for foreign currency to which the present analysis is broadly related.

The first strand of the literature has focused on investigating the determinants of the Black market premium, Dornbusch *et al.* (1983). The second strand of the literature is represented by Edwards (1989) and Kamin (1993) who investigate a large number of devaluation episodes in an attempt to determine whether there are any systematic patterns in the dynamics of Black market variables in response to a devaluation. The empirical evidence suggests a systematic increase in the Black market premium prior to a devaluation followed by an immediate decline. Agénor (1990, 1991) has provided evidence showing that the depreciation of the Black market rate is less than proportional immediately after a devaluation.

A third strand of the literature has investigated the extent to which the official and Black-market rates share a common stochastic trend. In this strand of the literature, Booth and Mustafa (1991) who find evidence of cointegration between the official and Black market rates in Turkey in the mid-1980s, using "integer" tests of the order of integration of the cointegrating residuals, comes closest in terms of methodology to the present analysis. There are however several significant differences between our work and Booth and Mustafa (1991).

First, we use monthly, rather than daily, data. When changes in the official exchange rate occur at approximately a monthly frequency, it is not clear how much advantage higher-frequency data offers. Moreover, Booth and Mustafa (1991) note that Turkey's official rate was often revised only on an annual basis. For our data, however, the official rate changes at least once a month for Hungary and on a monthly or bimonthly basis for Czechoslovakia. Second, our sample period spans a greater number of years, as Booth and Mustafa (1991) use daily data spanning a two-year period, whereas we use 80 monthly observations from May 1982 to December 1990. Third, we use a vector autoregressive fractionally-integrated moving-average (henceforth ARFIMA) model, rather than a vector autoregression. Hence, the model allows for fractionally-integrated moving-average processes, in addition to autoregressive processes.

The rest of the paper is organized as follows. Section 2 provides some background on the dynamics of the Black market for dollars in Hungary. Section 3 presents a brief exposition of fractionally integrated processes and fractional cointegration. In Section 4 we start by presenting the bivariate fractionally cointegrated empirical model that we use to explain the non-monotonicity phenomena and then proceed to discuss the empirical results and their implications. Concluding remarks are provided in Section 5. An Appendix lays out details on the econometric methodology.

2 The External Environment

In this section we provide a brief overview of the major features of the international trade and exchange rate regimes in Hungary that were relevant to the Black market for dollars.

For most of the post-war period restrictions on international trade transactions and currency convertibility in Hungary have lead to over-valuation of the official exchange rate and an active Black market for U.S. dollars. Figure 1 displays the time path of the Black-market exchange rates from 1955 to 1990. There appears to be relatively mild fluctuations the Black market for dollars until the late 1970s and 1980s, except for the discrete jump in the Black market premium associated with the Soviet invasion on Hungary in 1956.

[Place Figure 1 about here]

In Hungary important developments in the 1970s were steps taken to implement the New Economic Mechanism (NEM). Introduced in 1968 the NEM constituted the first comprehensive market oriented reforms by a socialist economy. However, the sharp increase in world prices for oil and other raw materials and expansive macroeconomic policies, led to a 20 percent appreciation in the exchange rate over 1974-1978, Balassa (1989).

Apart from Poland, Hungary's per capita debt surpassed that of any other socialist or developing country in 1978 rising to over 40 percent of GDP. The need to pay off the huge external debt built up over the 1974-1978 period resulted in several measures to promote exports to private market economies and regain access to international financial markets. These measures included the depreciation of the export exchange rate by 15 percent in real terms between 1983 and 1986, Balassa (1989).

The weakening of external markets, notably in the Middle East led to a worsening of the external environment between 1985 and 1987. This was manifested by an appreciation of the exchange rate. Structural adjustment and stabilization measures that were put in place in 1987 subsequently led to a steady depreciation of the exchange rate through 1990.

3 Long Memory and Fractional Cointegration

In this section we provide a brief introduction to long memory processes and fractional cointegration.

3.1 Long Memory Processes

Long-term memory or persistence is the term used to describe a time series whose autocorrelation structure decays slowly to zero, or equivalently whose spectral density is highly concentrated at frequencies close to zero. Such autocorrelation structure suggests that the process must depend strongly upon values of the time series far way in the past.

To fix ideas first consider the familiar ARIMA (p, d, q) model for a time series $\{y_t\}$ denoted

$$\Phi_p(L)\Delta^d y_t = \mu + \Theta_q(L)\epsilon_t \tag{1}$$

where p, d, q are non-negative integers, L is the backward shift operator (i.e. $Ly_t = y_{t-1}$) $\Delta = (1 - L)$ is the difference operator (i.e. $\Delta y_t = y_t - y_{t-1}$), Φ_p and Θ_q are polynomials of order p and q respectively and ϵ_t is a mean zero $i.i.d.$ sequence assumed to be Gaussian or at least to have a finite variance σ^2.

Assume that the polynomials $\Phi_p(z) = 1 - \phi_1 z - \ldots - \phi_p z^p$ and $\Theta_q(z) = 1 + \theta_1 z + \ldots + \theta_q z^q$ viewed as functions of a complex number z have no zero in the unit circle $|z| \leq 1$. This ensures that when $d = 0$ the time series $\{y_t\}$ is (i) *trend stationary*, (where μ is possibly replaced by some more general deterministic function of time), (ii) causal (depends only on past values of ϵ's and (iii) invertible (the ϵ's can be expressed in terms of the past y_t's.

When $d = 0$ the ARIMA (p, d, q) process is referred to as an ARMA (p, q) process. The covariance (or dependence) of the ARMA (p, q) $R_t = \mathbb{E}[y_t y_0]$ are mixtures of damped exponentials (i.e., they decrease relatively quickly). This behavior is often referred to as "weak dependence" or short memory. In general a process exhibits short memory if $\sum_{t=0}^{\infty} |R(t)| < \infty$.

The long memory or ARFIMA model generalizes the ARIMA (p, d, q) model by allowing d to take fractional values that may be either positive or negative.

Many features of ARFIMA models can be illustrated by studying the special case $p = q = 0$, ARFIMA $(0, d, 0)$. This time series model is called fractionally integrated noise and is denoted $\Delta^d y_t = \epsilon_t$ for d fractional as $y_t = \Delta^{-d}\epsilon_t$. Where $\Delta^{-d} = (1 - L)^{-d}$ by using the formal power series expansion $(1 - z)^{-d} = \sum_{j=0}^{\infty} b_j(d) z^j$ as follows

$$\Delta^{-d} = (1 - L)^{-d} = \sum_{j=0}^{\infty} b_j(d) L^j, \tag{2}$$

where L^j denotes the backward operator L iterated j times, $b_o(d) = 1$

$$b_j(d) = \prod_{k=1}^{j} \frac{k - 1 + d}{k} = \frac{\Gamma(j + d)}{\Gamma(j + 1)\Gamma(d)} \qquad j = 1, 2, \ldots. \tag{3}$$

and $\Gamma(\cdot)$ is the Gamma function.

The ARFIMA $(0, d, 0)$ model has an infinite moving average representation

$$y_t = \sum_{j=0}^{\infty} b_j(d) \epsilon_{t-j}, \tag{4}$$

where the b_j's are given as in (3).

For (4) to be well defined it is necessary that the square summability condition, $\sum_{j=0}^{\infty} b_j^2 < \infty$ be satisfied that is

$$-\infty < d < \frac{1}{2}, \tag{5}$$

The autocorrelation function is

$$\rho(k) = \frac{\Gamma(1-d)\Gamma(k+d)}{\Gamma(k-d+1)\Gamma(d)}, \qquad k = 1, 2 \ldots \tag{6}$$

When $0 < d < \frac{1}{2}$

$$\sum_{k=0}^{n} \rho(k) \to \infty \text{ as } n \to \infty. \tag{7}$$

When $-\frac{1}{2} < d < 0$

$$\sum_{k=0}^{n} \rho(k) < \infty \text{ as } n \to \infty. \tag{8}$$

When $d \in (0, \frac{1}{2})$ the autocorrelations are all positive, decrease hyperbolically and have an infinite sum . Such series are said to exhibit long memory because data in the distant past exerts small but non-negligible effects on the present.

When $d \in (-\frac{1}{2}, 0)$ the autocorrelations are all negative except at lag 0, converge hyperbolically to zero and have a convergent sum. Such series are said to be antipersistent.

In contrast to the ARMA (p,q) the covariance $R(k) = \mathbb{E}[y_o y_k]$ of an ARFIMA $(0,d,0)$ process satisfies

$$R(k) \sim C_d k^{2d-1} \qquad \text{as } k \to \infty \tag{9}$$

where $C_d = \frac{1}{\pi}\Gamma(1-2d)\sin \pi d$.

From (5) the spectral density equals

$$f(\lambda) = \frac{\sigma^2}{2\pi}|1 - e^{-i\lambda}|^{-2d} = \frac{\sigma^2}{2\pi}\left(2 \sin \frac{\lambda}{2}\right)^{-2d} \tag{10}$$

and satisfies

$$f(\lambda) = \frac{\sigma^2}{2\pi}|\lambda|^{-2d} \qquad \text{as } \lambda \to 0. \tag{11}$$

Since $\{y_t\}$ is well defined for $d < \frac{1}{2}$, we have $R(0) = \int_{-\pi}^{\pi} f(\lambda)d\lambda < \infty$ when $d < \frac{1}{2}$ which can be directly verified by using (11).

The autoregressive representation of the ARFIMA $(0,d,0)$ process is $\Delta^d X_t = \epsilon_t$, i.e.

$$\sum_{j=0}^{\infty} b_j(d) y_{t-j} = \epsilon_t \tag{12}$$

where the $b_j(d)$'s exactly compensate for the dependence structure of the time series $\{y_t\}$. It is easy to show that (11) is always defined for $-\frac{1}{2} < d < \frac{1}{2}$. The case of $-\frac{1}{2} < d < \frac{1}{2}$. is of particular interest. When $0 < d < \frac{1}{2}$ the process is covariance stationary and the covariances decrease so slowly that $f(0) = \sum_{k=-\infty}^{+\infty} R(k) = \infty$. In other words long memory behavior can be characterized by the unboundedness of the spectral density at frequency zero. When $d = 0$ the y_t's are uncorrelated. When $-\frac{1}{2} < d < 0$ they are weakly dependent because $\sum_{k=-\infty}^{+\infty} R(k) < \infty$. They are in fact "negatively" dependent because as (9) shows the covariances are negative for large k.

Relaxing the asssumption that $p = q = 0$ yields a general ARFIMA (p, d, q)

$$\Phi_p(L)y_t = \Theta_q(L)\Delta^{-d}\epsilon_t \tag{13}$$

where if $d < \frac{1}{2}$, then y_t is mean-reverting (or, more generally, reverts to its determinstic trend) and is covariance stationary. In practice, we difference y until the remaining differencing parameter is less than $\frac{1}{2}$ so the series we work with are covariance stationary. [2] The inclusion of AR and MA parameters in the model enables the ARFIMA (p, d, q) to better reflect both the short and long memory characteristics of the data. The ARFIMA representation is a parsimonious low-frequency generaliztion of the ARIMA class

3.2 Fractional Cointegration

Fractional cointegration (Granger 1986; Cheung and Lai 1993) enables us to measure the persistence of the stochastic trend common to two series on a continuous scale and thus provides more information as to whether two economic variables are related in ways suggested by theory.

If two series, x and y, are fractionally integrated with (continuous) orders of integration d_1 and d_2, then generally a linear combination $y + x\beta$ will be fractionally integrated of order $\max\{d_1, d_2\}$.[3]

If $d_1 = d_2$, a cointegrating vector $(1, \beta)$ might exist, however, such that the linear combination, $y + x\beta$, is fractionally integrated of order $b < d$.[4] In this case $d - b$ indicates the extent to which the series are "cointegrated". If $b = 0$, all long-memory components in the two series are common to both; if $b = d$, the series have distinct, unrelated stochastic trends. In the framework of fractional cointegration, integer cointegration tests become a joint test of $d = 1$ and $b = 0$. Note that the usual integer cointegration tests, which utilize unit root tests such as Dickey-Fuller [Dickey and Fuller (1981)], test whether $b = 1$ against the alternative that $b = 0$ and assume that the unit root in the input series is known with certainty ($d = 1$). To test cointegration hypotheses over the continuous metric $d - b$, it is necessary to have joint estimates of b and d.

It is possible for two series with fractional order of integration d to be cointegrated with residuals which are fractionally integrated of order $\frac{1}{2}d$, for example. With Dickey-Fuller tests on the residuals, inferences are limited to rejecting or not the hypothesis that the series share the same stochastic trend.

4 Empirical Analysis

In the empirical analysis we use monthly observations of the Hugarian florint for the period from May 1984 to December 1990. The Black market exchange rates are quoted as foreign currency units per U.S dollar and were obtained from various issues of the *World Currency Yearbook.* The Black market rates are end-of-month quotations. Official exchange rates for the Hungarian florint were obtained from the International Monetary Fund's *International Financial Statistics* data tapes.

Figure 2 displays time series plots of the respective series between 1982 and 1991

[Place Figure 2 about here]

4.1 Bivariate ARFIMA model with Fractional Cointegration

In this subsection we present the bivariate model with fractional cointegration that we use in the empirical investigation. Due to lengthy computation times, we did not pursue a formal model selection procedure, as Dueker and Startz (1994) found that models more heavily parameterized than the bivariate ARFIMA(1,d,1) model often proved to be overparameterized. For a discussion of model selection with ARFIMA models, see Sowell (1992b).

For a model of fractional cointegration, Granger (1986) generalized the usual bivariate error-correction mechanism (ECM) to include fractional integration and cointegration, shown here with the first-differences on the left-hand side:

$$\begin{aligned}\Phi_p(L)(\Delta x_t, \Delta y_t)' &= -(\tau_1, \tau_2)'[(1-L)^{b-d} - (1-L)](1, -\beta)(x_t, y_t)' \\ &\quad +\Theta_q(L)(1-L)^{-d}(\epsilon_{1t}, \epsilon_{2t})' \qquad (14)\end{aligned}$$

where x and y are $I(1+d)$, but have cointegrating residuals which are $I(b)$. One important feature of the ECM is its symmetry, which implies that changes in both variables are constrained by a long-memory process of past values of the cointegrating residuals if neither τ_1 nor τ_2 equals zero. This symmetry is present also in the usual $I(1)/I(0)$ cointegration, but when $d=0$ and $b=0$, the effects of lags of the cointegrating residuals on Δy and Δx decline geometrically with the terms in the inverted AR process, $\Phi_p(\boldsymbol{L})^{-1}$. When $b-d \neq 0$, that is, when there is fractional cointegration, both Δy and Δx respond with long memory to past deviations from the long-run relationship.

In some cases, it may not be desirable to assume that both variables respond symmetrically to distant past deviations from the long-run relationship. Moreover, Cheung and Lai (1993) note that estimation of the fractional ECM model of equation (14) is not straightforward. In our application, it does not seem appropriate to assume that the cointegration restriction constrains changes in the official rate, which is presumed to be at least partly a policy variable, the same way it constrains changes in the Black-market rate. To allow for fractional cointegration without forcing current changes in both variables to be long-memory processes of distant deviations from the long-run relationship, Sowell (1989a) proposed the following bivariate model of fractional cointegration:

$$\Phi_p(L)D(L)(\Delta x_t, y_t - \beta x_t)' \quad = \quad \Theta_q(L)(\epsilon_{1t}, \epsilon_{2t})' \tag{15}$$

where

$$D(L) = \begin{pmatrix} (1-L)^d & 0 \\ 0 & (1-L)^b \end{pmatrix}$$

By specifying the cointegrating residuals as a left-hand side variable in place of the variable that bears most of the long-run cointegration restriction, the changes in the other variable, Δx, become short-memory functions of deviations from the long-run relationship, according to $\Phi_p(\boldsymbol{L})^{-1}$. The changes in y, on the other hand, are implicitly a long-memory function of the deviations from the long-run relationship, because the cointegrating residuals themselves are a long-memory process. Note that there is no longer an error-correction term on the right-hand side, because the deviation from the long-run relationship is now one of the dependent variables in equation (15).

The bivariate ARFIMA model used is in (16) below. With all variables in logs our model of fractional cointegration between Black and official rates is

$$\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} L^0 - \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} L^1 \right] \begin{pmatrix} (1-L)^d & 0 \\ 0 & (1-L)^b \end{pmatrix} \times$$

$$\begin{pmatrix} \text{Official}_t - \delta_1 t \\ \text{Black Market}_t - \beta \text{Official}_t \end{pmatrix}$$

$$= \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} L^0 + \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix} L^1 \right] \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} \tag{16}$$

This framework enables estimation of the fractional differencing parameter b of the deviations from the long-run relationship jointly with the fractional differencing parameter of the input series. We can then formally test hypotheses regarding $d - b$, the extent to which the variables are cointegrated. The approach is general enough to allow for long-memory processes, unit roots and fractionally-integrated common stochastic trends of possibly lower order than the input series.

4.2 Estimation

Maximum-likelihood estimation of (16) for the Hungarian florint data was conducted using techniques described briefly in the appendix, in Sowell (1989a) and Dueker and Startz (1994).

With fractionally integrated time series no matter what the order of integration, it is always possible to difference a series enough times such that the order of integration lies in $(-\frac{1}{2}, \frac{1}{2})$. As we saw above, in this region, the series is covariance stationary with an invertible moving-average representation. For this reason, the official rate is differenced in the first row of (16). After differencing, the order of integration lies between $-\frac{1}{2}$ and $\frac{1}{2}$, whereas in levels the series is not covariance stationary ($d > \frac{1}{2}$).

For the official rate for the Hungarian florint, we find d to be close to $\frac{1}{2}$. When we estimated the model in first differences the estimate of $d - 1$ converged to the lower limit of $-\frac{1}{2}$. Consequently, we re-estimated the model in levels with a time trend as in (16). Thus, the data suggest that Hungary's official exchange rate has long-memory departures from a deterministic time trend. If in fact d were between $\frac{1}{2}$ and 1, then the series would still revert to the deterministic time trend, but its departures would be sufficiently persistent to make the series non-covariance stationary. Thus, the estimation issue is whether to estimate the model in levels or first-differences in order to have $d \in \{-\frac{1}{2}, \frac{1}{2}\}$. Interestingly, a Dickey-Fuller test does not reject a unit root, which illustrates the weak power of the test to fractional alternatives.

The apparent slow reversion to a linear trend in the series conforms with Cheung (1993) who finds that many nominal exchange rates have a fractional root less than one and with the finding of Booth and Mustafa (1991) that the interplay between official and Black market exchange rates includes a long-term overshooting of their long-run relationship.

Table 1 contains the parameter estimates for Hungary and shows that the official rate appears marginally covariance stationary ($d = .41$), although we cannot reject an order of integration above one-half. This implies that the departures from the linear trend are nearly persistent enough to cause the departures from the trend not to be covariance stationary.

A negative order of integration in the Black market premium ($b = -.465$) implies that low-order autocorrelations are persistently negative –a property called antipersistence. In the context of (16), a negative order of integration in the Black-market premium would simply imply that the long-run relationship will be restored following some overshooting in the premium: If a shock pushes the premium below average today, the long-run relationship will be restored via asymptotic decay of a *positive* deviation from the long-run relationship. This is what we call *long memory overshooting* because the effect of a shock declines at a slower rate than the usual exponehtial decay assocaited with the autocorrelation functions for the class of covariance stationary ARMA process.

Figure 3 illustrates the model-implied and sample autocovariances of Hungary's Black-market premium. Note that the model-implied autocovariances dip below zero at a lag of about five months and gradually return to zero.

The described long-memory overshooting has important implications for the dynamic relationship between the official rate and the Black-market premium. Our multivariate time series model also specifies expected covariances between lags of the official rate and the Black-market premium. These lags help trace the response of the Black-market premium to an increase in the official rate. Figure 4 shows that an increase in the official rate initially tends to be associated with smaller Black-market premia in the first five months, but the Black-market premium does return monotonically to its mean. Instead, the Black-market premium overshoots and goes persistently *above normal* before returning to its mean.

Table 1 contains the parameter estimates for Hungary and shows that the vector autoregressive process implied by Φ_p is relatively persistent with complex roots. The conjugate pair of roots is .785+/- .028. For this reason, it takes some time for the autocovariances of the Black-market premium to become negative in Figure 3.

[Place Table 1 about here].

5 Conclusions

We have examined the dynamics of the official exchange rate and Black market premium using a bivariate time series model that allows for fractional integration and cointegration. The results suggest that the official and Black-market rates in Hungary were cointegrated with a negative fractional order of integration in the cointegrating residuals.

The significance of negative orders of integration in Black-market exchange-rate premia is that the Black-market premium eventually overshoots in response to a devaluation of the official rate. The initial response to a devaluation is for the Black-market premium to fall below normal, as expected. The model and data suggest, however, that the long-run equilibrium is restored asymptotically following a devaluation by *reductions* in the Black-market premium from an above-normal level. Thus, along the way the Black-market premium overshoots its long-run average level due to long-memory, non-monotonic dynamics. The overshooting may result from market speculation against the official rate in anticipation of further rounds of devaluation. Such expectations were warranted in the 1980s in Hungary.

Appendix A

This appendix provides details on the estimation of the model.

Consider a vector of k variables, $(y_{1t}, ..., y_{kt})'$, denoted by $\boldsymbol{y}_t$ that is differenced enough times so that their orders of integration are all in $(-\frac{1}{2}, \frac{1}{2})$, then the covariance matrix, Σ, of the multivariate time series $\boldsymbol{y} = (\boldsymbol{y}'_1, ..., \boldsymbol{y}'_T)'$ is block Toeplitz where

$$\mathbb{E}[y_t y_{t+s}] = \Sigma(s) = E[y_t y_{t-s}]' = \Sigma(-s)'.$$

Assuming that $\boldsymbol{y}$ consists of a multivariate ARFIMA process with fractional differencing parameters $d_1, ..., d_k$,

$$\Phi_p(L)D(L)y \quad = \quad \Theta_q(L)\epsilon \tag{A. 1}$$

where $\boldsymbol{\epsilon}$ is assumed to be a vector of random *i.i.d.* disturbances, $\Phi_p(\boldsymbol{L})$ is an autoregressive polynomial of order p, $\Theta_q(\boldsymbol{L})$ is a moving-average polynominal of order q, and $\boldsymbol{D}(\boldsymbol{L})$ is a diagonal fractional differencing polynomial:

$$D(L) = \begin{pmatrix} (1-L)^{d_1} & 0 & \cdots \\ 0 & \ddots & \ddots \\ \vdots & \cdots & (1-L)^{d_k} \end{pmatrix}$$

Sowell (1989a) derives the autocovariances for the ARFIMA model, which do not have a closed form, as they are functions of hypergeometric functions. For this reason, we do not repeat the formulae here. Closed-forms exist for the ARFIMA(0,d,q) autocovariances, however, and we repeat those here:

$$\begin{aligned} \Sigma_{i,j}(s) \quad = \quad & \sum_{n=1}^{k}\sum_{r=1}^{k}\sigma_{nr}\sum_{m=0}^{q}\sum_{l=0}^{q}\theta_{i,n}(m)\theta_{j,r}(l) \\ & \frac{\Gamma(1-d_n-d_r)\Gamma(d_r+s+m-l)}{\Gamma(d_r)\Gamma(1-d_r)\Gamma(1-d_n+s+m-l)} \end{aligned} \tag{A. 2}$$

for element (i, j), where $\boldsymbol{\sigma}$ is the covariance matrix of $\boldsymbol{\epsilon}$, $\boldsymbol{\theta}(\boldsymbol{m})$ is the matrix in the moving-average polynomial corresponding with lag m, and Γ is the gamma function. When $p \neq 0$, in contrast, it is necessary to take a partial fraction decomposition with numerous evaluations of hypergeometric functions to obtain the model-implied autocovariances; see Sowell (1989a, 1992b).

Assuming that $\boldsymbol{\epsilon}$ is normally distributed, the log-likelihood function, given covariance matrix Σ, is

$$-\frac{1}{2}ln(2\pi) - \frac{1}{2}\ln \mid \Sigma \mid -\frac{1}{2}y'\Sigma^{-1}y, \tag{A.3}$$

The calculation of the inverse and determinant of the block toeplitx covariance matrix benefited from the algorithm discussed in Sowell (1989b). Sowell (1989b) suggests that the number of operations needed to invert a block toeplitz matrix using the algorithm he outlines is on the order of k^3T^2, where k is the number of variables and T is the number of observations. By comparison the number of operations needed to invert an arbitrary $(kT \times kT)$ matrix is k^3T^3.

Notes

[1]See Agénor for an excellent survey of the literature on parallel (Black) markets.

[2]See Hosking (1981) for a proof that one can always transform a fractionallyintegrated series of order higher $\frac{1}{2}$ into this range by taking a suitable number of integer deifferences.

[3]Because positive serial correlation is much more common than negative serial correlation in economic data, for convenience we assume that both d_1 and d_2 are positive. In estimation, however, it is sometimes necessary to overdifference the data to achieve stationarity, in which case one estimates negative fractional orders of integration.

[4]If the orders of integration, d_1, d_2, are not equal, a linear combination of the two series may still have less persistence than the two input series, however the cointegrating residuals cannot have the same fractional differencing parameterization that will be assumed in this article.

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Table 1: Bivariate ARFIMA(1,d,1) model of fractional cointegration between Hungary's official and Black-market foreign exchange rates (florint per dollar)

variable	*parameter*	*parameter value*	*stand. error*
Log-Likelihood		-282.87	
fractional root	d	.417	.270
fractional root	b	-.465	.160
coint. parameter	β	1.11	.102
	ρ_{11}	.812	.103
	ρ_{12}	.009	.014
	ρ_{21}	-.947	.504
	ρ_{22}	.758	.115
	θ_{11}	-.262	.138
	θ_{12}	-.134	.070
	θ_{21}	1.18	.491
	θ_{22}	.356	.162
time trend	δ_1	.233	.125
official rate	σ_1^2	6.88	1.18
coint. resids.	σ_2^2	22.3	3.77
	σ_{12}	-3.01	1.60

Figure 1

Hungary's Black-Market Exchange Rate With the U.S. Dollar

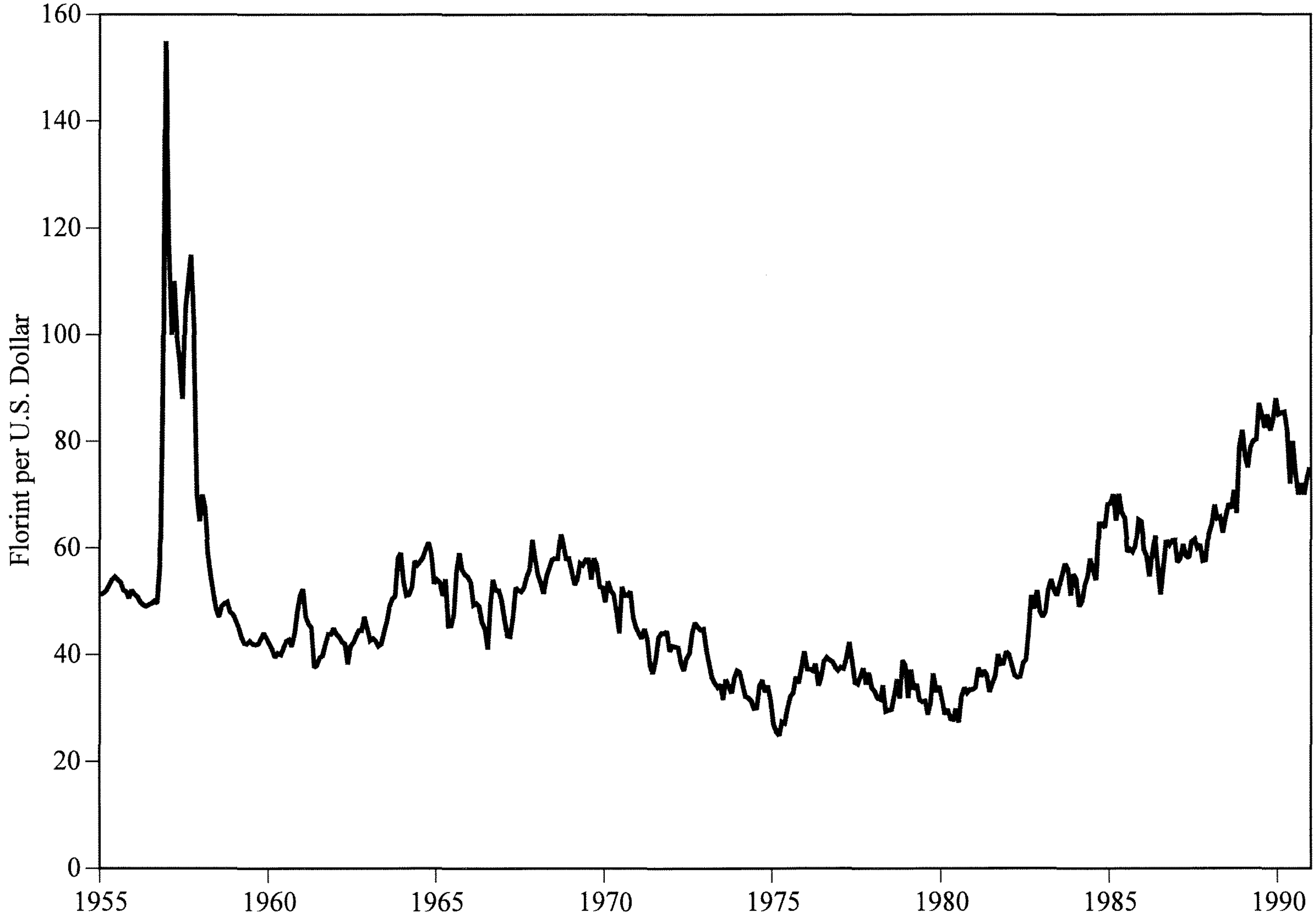


Figure 2

Official and Black Market Exchange Rates For Hungary

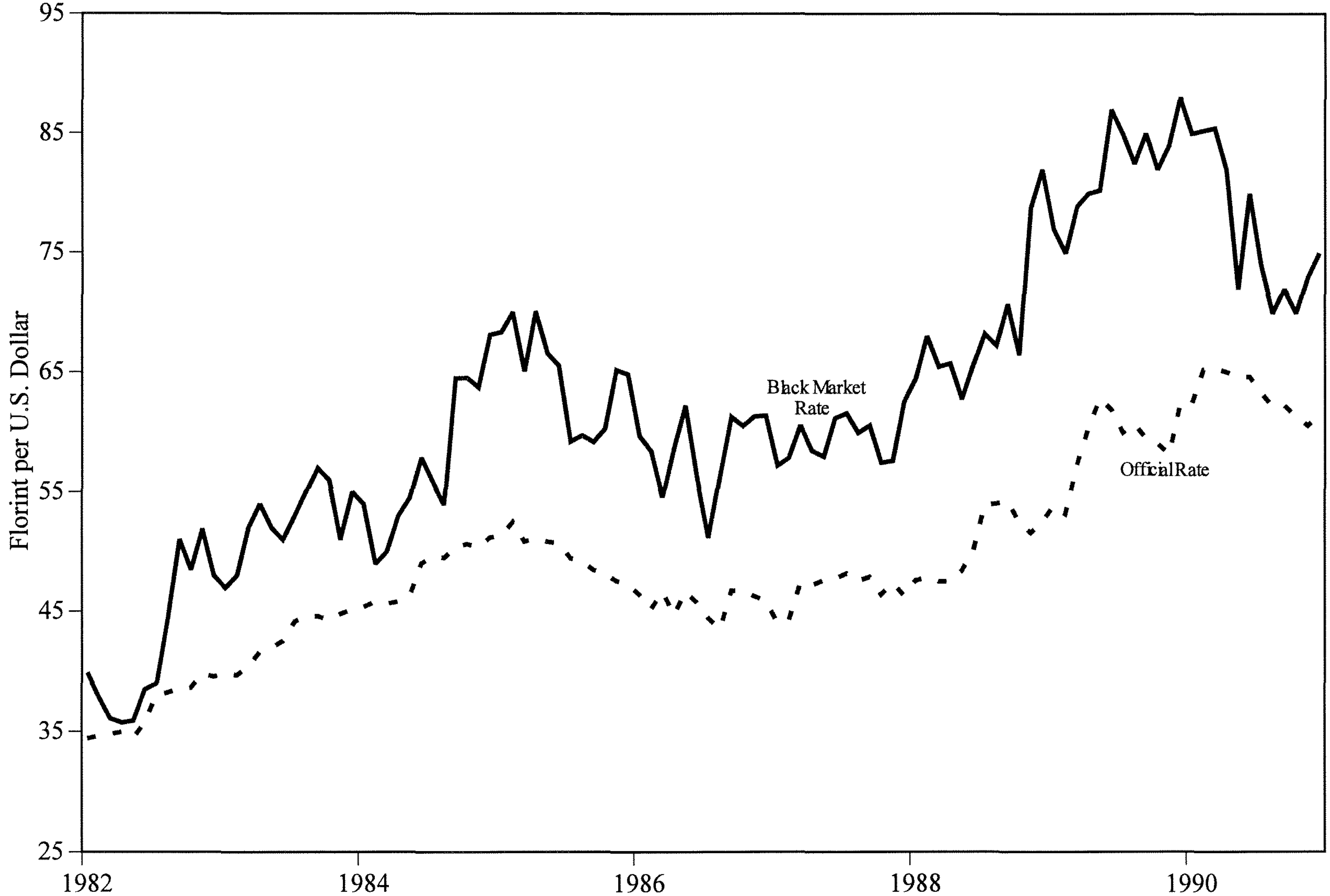


Figure 3

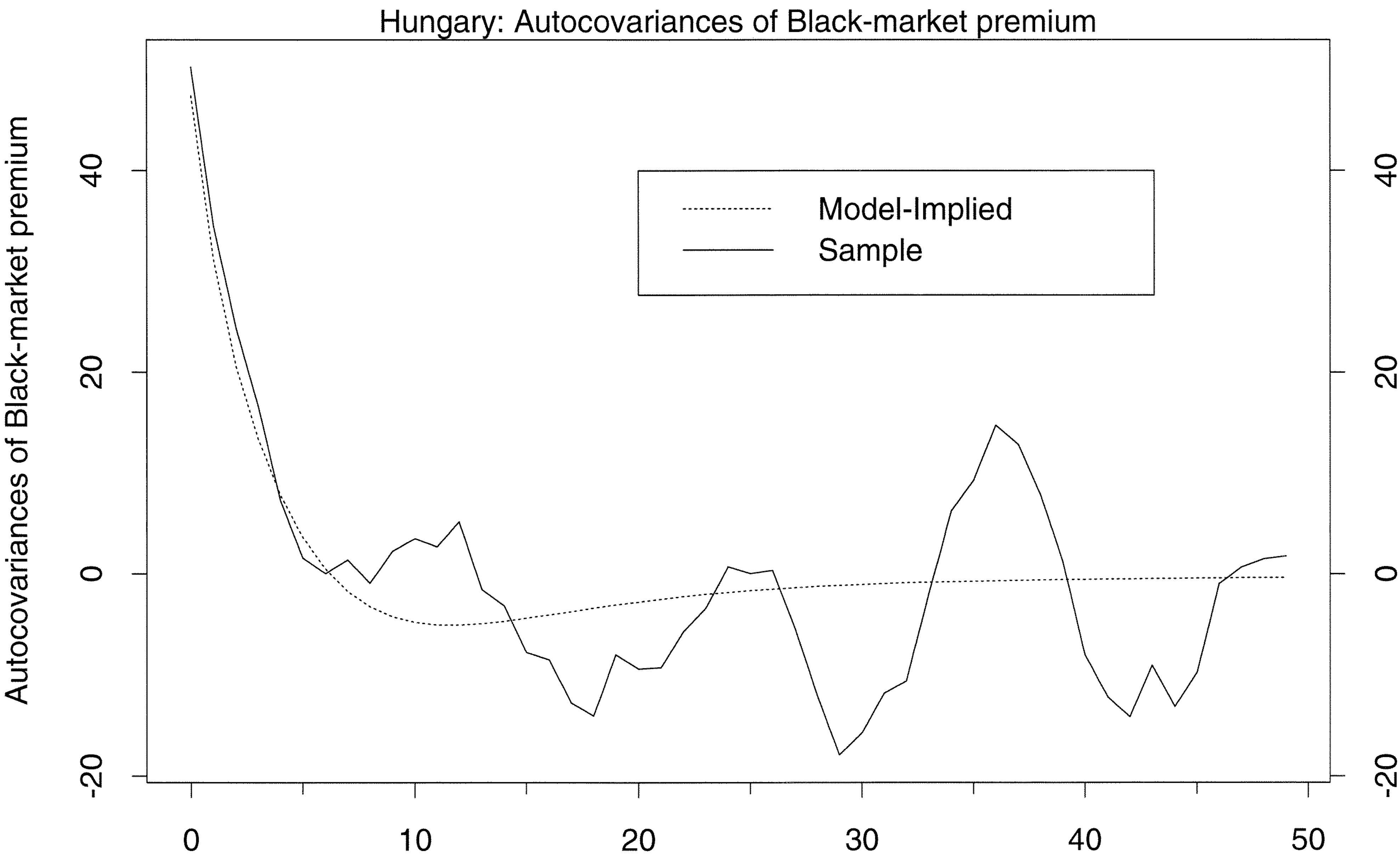
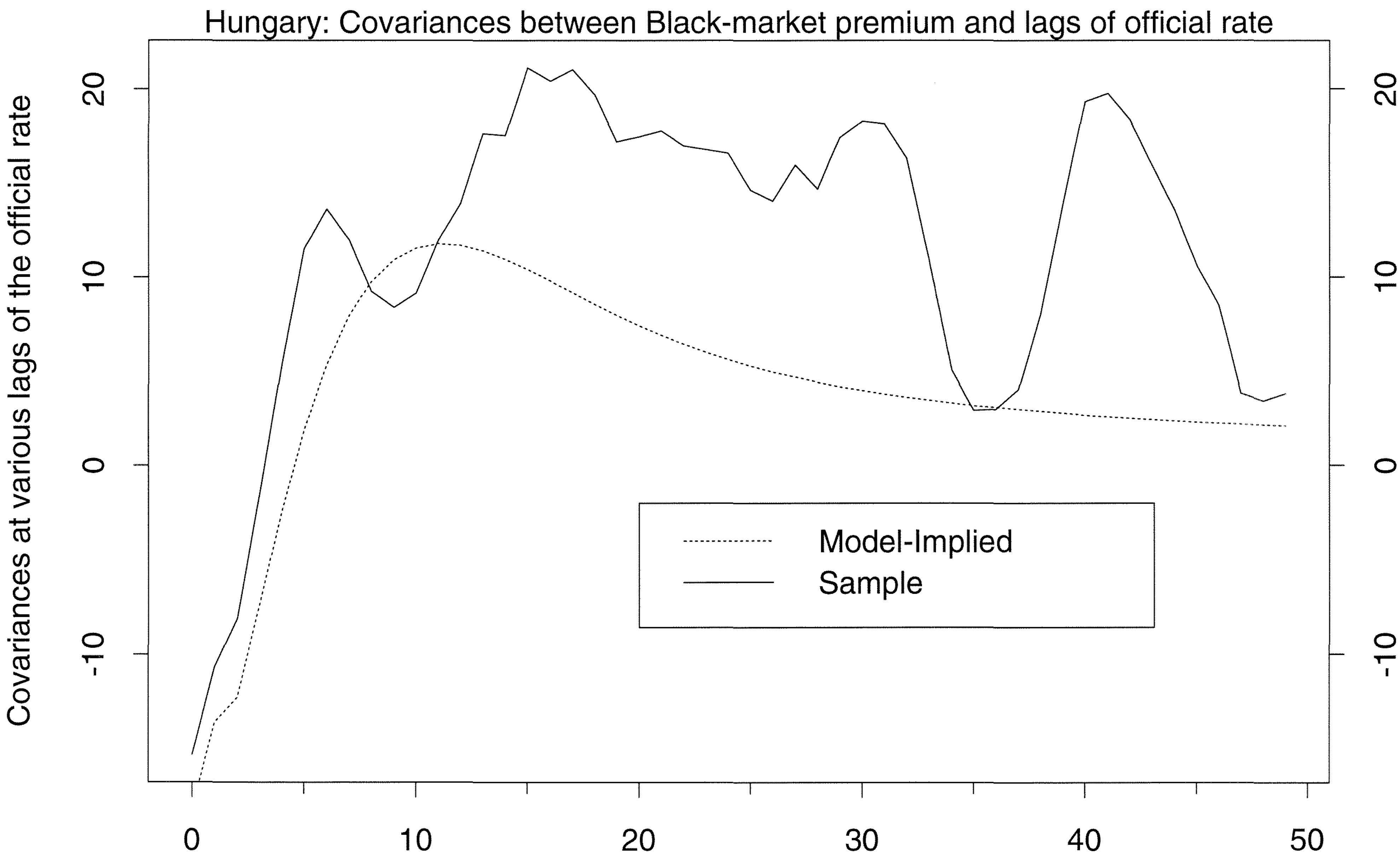

Hungary: Autocovariances of Black-market premium
Autocovariances of Black-market premium
Model-Implied
Sample
40
20
0
-20
40
20
0
-20
0
10
20
30
40
50

Figure 4



Hungary: Covariances between Black-market premium and lags of official rate
Covariances at various lags of the official rate
20
10
0
-10
0
10
20
30
40
50
Model-Implied
Sample