Long-Run Versus Short-Run Behaviour of the Real Exchange Rates*

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Abstract: This paper discusses the mean stationarity of real exchange rates by using new time series methods and new tests. The question whether the real exchange rates have a unit root or are level reverting is set in the general and flexible framework of fractionally integrated processes. The estimations and tests sustain the claim that real exchange rates may be non stationary and not revert to any *short-run* parity. However, estimations also suggest that real exchange rates behave differently on the short and on the long run and that they may revert to parity in a century-long period.

1 Introduction

The present paper discusses the stationarity and parity reversion behaviour of real exchange rates in light of new time series methods and new tests. The stationarity of exchange rates has been discussed extensively given its theoretical and practical implications, namely for the purchasing power parity hypothesis and for economic policy.

In fact, it is usually accepted that one major implication of the purchasing power parity is that real exchange rates are *mean stationary* (see, e.g., Baillie and McMahon 1989). In contrast, if they follow a random walk or, in general, if they have an autoregressive unit root, then there is no equilibrium value to which the real exchange rate returns and present shocks are expected to become permanent deviations.

Tests for the stationarity of real exchange rates have concentrated on the alternative between a unit root and a stationary autoregressive coefficient. Classic and Bayesian unit root tests have been applied by various authors with contradictory results. Frenkel (1981), Hakkio (1986), Baillie and Selover (1987), and others find support for the random walk hypothesis about real exchange rates. By contrast, Taylor and McMahon (1988), Abuaf and Jorion (1990), Witt (1992), and others cannot reject the purchasing power reversion. In this last work, arguing, with Sims (1988), that classical statistical tests for unit roots are fundamentally flawed, Witt applied the Bayesian Sims test to new long series he constructed and concluded for the existence of large autocorrelation coefficients, but not unit roots. Accordingly, he suggests that deviations from purchasing power parity “persist for a number of years, but they are not permanent.”

In a widely global market, among free market economies and on the long run, it is hard to believe that real exchange rates may follow a random-walk-type process. If this were true, in a finite future and with a positive probability it would be possible to buy an airplane in one country by exchanging the currency that buys a potato in another one. As Dornbusch and Krugman (1976) have said, “Under the skin of any international economist lies a deep-seated belief in some variant of the PPP theory of the exchange rate". The fact that unit root tests have often pointed to unit-root type models may well represent a shortcome in the economic and statistical methodology or the in the available data. We argue in this direction, as we apply a different time series approach and use much longer data sets than the ones usually studied. We claim that the alternative between a unit root and a stationary model is a limitative framework and we show that long data sets are necessary to test for a long-run concept of the PPP. Accordingly, we discuss the unit root issue in a more general and flexible setting by using fractionally integrated models. We develop an approach that builds upon previous work by Diebold, Husted, and Rush (1991), Baillie and Pecchenino (1991), Cheung and Lai (1993), and Wu and Crato (1995). These authors have argued that fractionally integrated models are better suited to analyze long-run parity reversion. Consequently, following Sowell (1992), they have discussed the parity reversion in terms of the degree of integration of the series and not in terms of the autoregressive unit roots.

We have studied the problem in the context U.S. and U.K. real exchange rates against the Portuguese currency. We have constructed two coherent sets of series. The first set consists of the monthly exchange rates for the last twenty years. The second set consists of the annual rates, with which we were able to construct with a century-long period worth of data.

For estimating the degree of integration of the series as well as other model parameters, we use the Whittle spectral likelihood method. This approximate maximum likelihood estimator has been receiving increasing attention in the literature given its computational simplicity and good practical performance—see, e.g., Crato and Ray (1996) and the references therein. As an output of the algorithm, we get estimates which allow a Wald-type test for stationarity or nonstationarity.

The results show a sharp contrast between the shorter series and the century-long series. While on the first series the estimated models are statistically undistinguishable from unit-root nonstationary and non-level-reverting models, on the second series the unit root hypothesis is rejected and the hypothesis of parity reversion holds.

For the recent period of flexible exchange rates, the estimates point to the nonstationarity of the series. However, estimates for the longer series suggest a degree of integration d greater than zero but smaller than the unity, while stationarity ($d < 1/2$) is not rejected. This is *compatible with the level reversion* implied by the purchasing power parity. Thus, in the framework of fractional models, the hypothesis of long-run parity reversion can be made compatible with the empirical evidence.

The plan of the paper is as follows. In Section 2, we discuss the impulse-response function of fractionally integrated models. In Section 3, we discuss the data and the transformations applied before testing. In Section 4, we describe the tests for fractional unit roots and apply them to Portuguese real exchange rates. The conclusions are summarized in Section 5.

2 Fractional Unit Roots

Let the time series under consideration be represented by (X_t). Traditional tests for the presence of a unit root essentially estimate an autoregressive model of real exchange rates. Following Witt (1992), the basic model can be written as follows

$$(X_t - \mu) = \phi(X_{t-1} - \mu) + u_t, \tag{1}$$

where μ is a constant and u_t is a zero-mean error term. In this model, the stationarity and parity-reversion are dependent on the value of the autoregressive parameter ϕ. If $\phi < 1$, then the present shocks u_t would be damped and, in absence of future shocks, future values of X_t would revert to μ, the unconditional mean of the process. But, if $\phi = 1$, then present shocks would persist forever and there is no tendency for (X_t) to converge to any constant.

If we allow (u_t) to be a moving average of a white noise, i.e., of a noncorrelated zero-mean process (ε_t), then model (1) can be seen as a special case of an autoregressive moving average model, an ARMA(p,q):

$$(X_t - \mu) - \phi_1(X_{t-1} - \mu) - \ldots - \phi_p(X_{t-p} - \mu) = \varepsilon_t + \theta_1\varepsilon_{t-1} + \ldots + \theta_q\varepsilon_{t-q} \quad (2)$$

Let B and ∇ represent the backwards and the differencing operators, respectively, i.e. $B^j X_t := X_{t-j}$ and $\nabla X_t := (1-B)X_t = X_t - X_{t-1}$. Let $\phi(B)$ and $\theta(B)$ represent the autoregressive and moving average lag polynomials in (2). The model (1) corresponds to an ARMA$(1,q)$. If $\phi = 1$ we have a *unit root* in the autoregressive polynomial $\phi(B)$. With this notation, the model (1), with a unit root, can be rewritten.

$$(1-B)(X_t - \mu) = \nabla(X_t - \mu) = \theta(B)\varepsilon_t$$

Classical Dickey-Fuller type tests or the Bayesian test of Sims, as in Witt (1992), consider the basic model (1). In contrast, our fractional unit root tests consider the general model

$$\phi(B)\nabla^d(X_t - \mu) = \theta(B)\varepsilon_t \quad (3)$$

where d is allowed to be any real value. If d is an integer then the differencing operator is defined in the usual way and (3) is the well-known ARIMA model. If d is any real number the *fractional difference operator* ∇^d is defined as

$$\nabla^d = (1-B)^d := \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k,$$

where $\binom{d}{k} = \frac{d}{k}\frac{d-1}{k-1} \cdots \frac{d-k+1}{1}$.

The model (3) is called an ARFIMA(p,d,q) process, i.e., an autoregressive fractionally integrated moving average process. It was introduced independently by Granger and Joyeux (1980) and by Hosking (1981), and have been proved as a valuable tool in various areas of econometric modeling (e.g., Diebold, Husted, and Rush 1991; Sowell 1992), forecasting (e.g., Geweke and Porter-Hudak 1983; Ray 1993), and financial time series analysis (e.g., Shea 1992; Cheung 1993).

An ARFIMA(p, d, q) model is nonstationary when $d \geq 1/2$. The existence of a simple (integer) unit root in the autoregressive polynomial corresponds to the particular case $d = 1$.

ARFIMA models have an interesting impulse-response function (see, e.g., Diebold and Nerlove 1990). In fact, for $d < 1.5$, the first differences of the ARFIMA process (X_t) have the following Wold representation

$$X_t - X_{t-1} = \sum_{k=0}^{\infty} \psi_k \varepsilon_{t-k},$$

where the sequence of moving average parameters (ψ_k) are the *impulse-responses*, parameters that track the response of future changes of X_t to an unitary shock. The *cumulative impulse responses*

$$S_j = \sum_{k=0}^{j} \psi_k \tag{4}$$

trace the response of future levels of X_t to the same unitary shock.

To analyze the long-run effects of shocks on the exchange rates, i.e., the limit value of S_j, it is convenient to use the spectrum of the differenced process, say $f(\omega)$. It is known that

$$f(\omega) = |1 - e^{-i\omega}|^{-2D} g(\omega), \tag{5}$$

where $g(\omega)$ is the spectral density of a causal and invertible ARMA process, hence finite and strictly positive, and $D = d - 1$. The values of $f(0)$ are determined by the difference power transfer function since $|1 - e^{-i\omega}| = 0$ for $\omega = 0$. There are three situations to consider. For $D = d - 1 < 0$, we have $\lim_{\omega \to 0} f(\omega) = 0$, which implies $f(0)2\pi/\sigma^2 = |\sum \psi_j|^2 = 0$ and $S_\infty = \sum \psi_j = 0$. For $D = d - 1 = 0$, we have $\lim_{\omega \to 0} f(\omega) = g(0)$, a positive finite number, which implies that S_∞ is a positive finite number. For $D = d - 1 > 0$, we have $\lim_{\omega \to 0} f(\omega) = \infty$, which implies that S_∞ diverges.

The long-term exchange rate dynamics are completely different according to each of these three situations. If $d = 1$, then S_j converges to a finite non-zero value. If $d \geq 1$, then S_j diverges. However, if $d < 1$, then $S_j \to 0$ as $j \to \infty$, i.e., level reversion exists in the long-run. The remarkable property of the long-run behavior of S_j in an ARFIMA model for the exchange rates, is that the nonstationary case ($d \geq 1/2$) is *compatible* with parity reversion ($d < 1$).

3 Portuguese Real Exchange Rates

The data we consider are the real exchange rates of the British pound and the U.S. dollar against the Portuguese escudo. Two sets of series were constructed: twenty years of monthly data and one hundred years of annual data. The monthly data span the period starting in January 1973 and ending in December 1994, which corresponds to the flexible exchange rates period and mostly corresponds to the recent political Portuguese history on the wake of the democratic revolution of April 1974. The annual data start in 1891 for the British pound and 1900 for the U.S. dollar; both series end in 1994. All exchange rates are *end-of-period*. Consumer price indexes (CPI) are used to compute the real rates.

In order to construct the series, raw data were obtained from the following sources. For the monthly series, the nominal exchange rates were obtained from *Banco de Portugal* and the CPI's from *International Financial Statistics*. For the annual series, nominal U.K. pound exchange rates were obtained from Mata and Valério (1994)—period 1891–1950—, from Neves (1994)—period 1951–1992—, and from *Banco de Portugal*—period 1993–1994. Annual U.S. dollar exchange rates were obtained from Neves (1994)—period 1900–1930—, from Mata and Valério (1994)—period 1931–1990—, and from *Banco de Portugal*—period 1991–1994. The annual CPI's were obtained from Siegel (1992)—period 1891–1990—and were extended up to 1994 by using the data in *International Financial Statistics*. In all cases, the data were made compatible but using the overlapping periods.

In order to compute the logarithmic *real* rates, X_t, the following formula has been used

$$X_t := \ln\left(\frac{E_t}{I_{P,t}/I_{F,t}}\right), \tag{6}$$

where E_t is the nominal exchange rate, and $I_{P,t}$ and $I_{F,t}$ represent the Portuguese and the foreign price indexes at period t.

The logarithmic transformations are standard. They stationarize the variance of the process and rescale the observations in order to deal with rates of change.

The real rates are depicted in the Figures. By visually inspecting the movement of the exchange rates, we do not see striking changes in their behaviour. Nevertheless, a question may arise regarding their structural stability, namely for the long data sets, which span a pre- and post-flexible regime. After fitting the models, it could be worthwhile to test for structural breaks in the data. However, the post-flexible regime includes only 20 observations. Having decided to adopt a fractionally integrated methodology, this short number of data points precludes any reliable estimation and testing for this later period.

— — *INSERT FIGURES ABOUT HERE* — —

Formula (6) can be rewritten as stating a relation between the nominal logarithmic value of the foreign currency in terms of the national currency, i.e., the nominal logarithmic exchange rate e_t, and the logarithmic ratio of national to foreign price indexes, p_t:

$$e_t = p_t + X_t.$$

The most common version of the purchasing power parity hypothesis corresponds to the stationarity in mean of the residual process X_t. There is, however, a less restrictive version which corresponds to the stationarity in mean of the residuals given by the regression equation

$$e_t = \alpha p_t + X_t, \tag{7}$$

where the constant α is not bound to be one.

Simple tests indicate that all log raw series under consideration are not stationary. On Table 1 we show the results of the Geweke and Porter-Hudak spectral estimator (GPH) for $\hat{d}$. The estimates were computed on the differenced series obtaining $\hat{D}$. Then, we computed $\hat{d} = \hat{D} + 1$ for the original series. The GPH estimator is based on the form of the spectrum of a process with integration parameter d, as exhibited in (5). After taking logarithms on both sides, we obtain an equation, which is linear in d. The regression is then performed by using the finite-sample counterpart of the spectrum, the periodogram. For robustness of the results, we have used various truncations for choosing the number m of lower periodogram ordinates to use. The standard deviations were computed from the regression results. For details, see Geweke and Porter-Hudak (1983) or Brockwell and Davis (1991).

All the estimates shown on Table 1 are compatible with processes with a unit root and strongly reject the stationarity hypothesis at all conventional levels of significance.

— — *INSERT TABLE 1 ABOUT HERE* — —

Next, we estimated the parameter α in equation (7). The results are presented on Table 2. Using conventional levels of significance, in three cases the estimates were significantly different from the unity, although very close to it. In one case, the monthly US series, the hypothesis $\alpha = 1$ was not rejected. These results showed that the observed indexes could be taken as relatively good proxies for the theoretical price variables, but imperfect ones, at least in three series. Consequently, in the three cases in which the homogeneity condition $\alpha = 1$ could not be sustained, we computed the real exchange rate series

X_t as the residuals from the regression on equation (7). For the US monthly series, we simply used formula (6).

— — *INSERT TABLE 2 ABOUT HERE* — —

The next steps were performed with the resulting series.

4 Tests and Results

Our approach is based on the Fox and Taqqu (1986) frequency-domain approximate maximum likelihood. Using a Whittle approximation to the log-likelihood function (Brockwell and Davis 1991, p. 529, equation (13.2.26)), the function

$$\mathcal{L}(\mathbf{x}|\beta) = \frac{2}{n}\sum_{j=1}^{[n/2]} \frac{I_n(\omega_j)}{f_\beta^*(\omega_j)} + \frac{2}{n}\sum_{j=1}^{[n/2]} \log f_\beta^*(\omega_j), \tag{8}$$

is minimized, where $\mathbf{x}$ represents the data and β the parameters of the ARFIMA model being estimated. The function $(2\pi/\sigma_\varepsilon^2)f_\beta^*(\omega_j) = f_\beta(\omega_j)$ is the corresponding spectrum and $I_n(\omega_j)$ is the periodogram of the series. The parameters in the vector β are the $p+q+1$ coefficients ϕ_j, θ_j, and d. The estimated vector of parameters $\hat{\beta}$ is the one which minimizes $\mathcal{L}(\mathbf{x}|\beta)$. The mean of the process is estimated as the sample average of the series. The variance-covariance matrix of $\hat{\beta}$ is estimated by using the Hessian matrix of $f(\omega_j)$ at the optimum.

For each series, we estimated a set of ARFIMA(p,d,q) models with p and q orders in the range $p,q = 0,1,2,3$, and selected the estimated model by using the AICc criterion (Brockwell and Davis 1991, p. 304). Simulation results in Crato and Ray (1996) show that this criterion is particularly appropriate in presence of a mixed model ($p, q \neq 0$).

The results of the estimations are presented in Table 3. The test on the parameter d is performed as a usual Wald test, using the estimated standard deviation.

— — *INSERT TABLE3 ABOUT HERE* — —

In all cases, we have obtained nonstationary models ($d > 1/2$). However, there are significant differences between the monthly and the annual series.

The monthly U.K. and U.S. real exchange rates strongly point into the direction of a unit root process, i.e., an ARIMA($p, 1, q$), thus a nonstationary and a non-parity-reverting process. A simple graphical analysis of the monthly series shows an extreme randomness, various level shifts, and apparent short-period trends. The existing reversions are not enough to suggest any mean level. This could indicate that the PPP hypothesis cannot be empirically validated with these series, similarly to what a number of authors have found with other data.

The annual data presents a completely different picture. For both the U.K. and the U.S. annual series, the unit-root hypothesis ($d = 1$) is rejected, the hypothesis of parity reversion ($d < 1$) is reinforced, and the hypothesis of stationarity ($d < 1/2$) is not rejected. From the graphical analysis of these longer annual series, we note that reversions are more noticeable and we may suspect that a long-term tendency to parity can exist.

This contrast between short- and long-term series strengthens a time-range argument previously made by several authors. As Diebold, Husted and Rush

(1991), among others, have argued, the purchasing power parity is a long-term concept and short-term series are not large enough to empirically test its validity. What we believe it is clear from our results is that, in this context, long-term does not mean a couple of decades but at least a century-long series.

5 Conclusions

We have constructed two CPI based real exchange rate series corresponding to two most important currencies: the British pound and the U.S. dollar. In each case, twenty-year-long monthly series and century-long annual series were analyzed. By using novel time series tools derived from the analysis of fractionally integrated models, we have tested the stationarity and parity reversion of these series.

The fractionally differencing parameter d of the series was estimated by maximizing the spectral likelihood. In all cases, we found d estimates outside of the stationary range. In the long-run series, however, the estimates did not reject parity reversion neither the stationarity, since the values of $\hat{d}$ pointed to level-reverting models and are compatible with stationary fractional processes.

Our results reinforce the claim that short-term series are not appropriate for analyzing the long-run behavior of real exchange rates. Consequently, the empirical discussion of the purchasing power parity hypothesis should only be based on the long-term analysis of exchange rate behavior. Long-memory models such as the fractionally integrated ARFIMA provide both a flexible framework and an appropriate setting for analyzing this long-term economic behavior.

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Table 1: Estimated degree of integration d for the log raw series

	Monthly Series				Annual Series			
	UK£	UK CPI	US$	US CPI	UK£	UK CPI	US$	US CPI
$m = [n^{0.5}]$	1.147	1.041	1.090	1.029	1.150	1.152	1.124	1.023
	(0.075)	(0.046)	(0.063)	(0.028)	(0.215)	(0.223)	(0.187)	(0.189)
$m = [n^{0.6}]$	1.153	1.038	1.124	1.037	1.076	1.033	1.077	0.980
	(0.058)	(0.030)	(0.045)	(0.024)	(0.135)	(0.130)	(0.112)	(0.099)
$m = [n/2]$	1.026	1.007	1.040	1.009	1.027	1.019	1.038	1.017
	(0.020)	(0.012)	(0.018)	(0.009)	(0.050)	(0.045)	(0.043)	(0.036)

The estimates were obtained by the GPH method. The number m of different periodogram ordinates considered were two truncations and $[n/2]$, which corresponds to the whole periodogram. The symbol $[\cdot]$ represents the greatest integer function.

Table 2: Regression estimated parameter α for the equation (7), relating exchange rates and price indices

	Monthly Series		Annual Series	
	UK Pound	US Dollar	UK Pound	US Dollar
$\hat{\alpha}$	1.059	1.000	1.065	1.059
$\hat{\sigma}_{\hat{\alpha}}$	(0.018)	(0.017)	(0.014)	(0.017)

Table 3: Estimated degree of integration d for the AICc selected models

	Monthly Series		Annual Series	
	UK Pound	US Dollar	UK Pound	US Dollar
ARFIMA Model	(1,d,0)	(1,d,1)	(0,d,1)	(1,d,0)
$\hat{d}$	0.885	1.009	0.578	0.575
$\hat{\sigma}_{\hat{d}}$	0.111	0.109	0.127	0.245