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School of Accounting & Finance
Deakin University
221 Burwood Highway
Victoria, Australia 3125

Are Long-Term Return Anomalies Illusions: Evidence From the Spot Yen

Jonathan Batten
Deakin University
School of Accounting and Finance
Burwood, Melbourne, 3125
Australia
Email: jabatten@deakin.edu.au
Fax: 61 3 9251 7243

Craig Ellis
University of Technology, Sydney
School of Finance and Economics
Haymarket, Sydney 2007
Australia

Thomas A. Fetherston*
University of Alabama at Birmingham
School of Business
Birmingham, Alabama 35294
USA
Fax: 1-205-975-4428

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* Corresponding Author

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Abstract

This study investigates the sensitivity of the long-term return anomaly observed in the USD/Yen currency market to sample and method bias during the period from 11 October 1983 to 21 July 1999. Initially the CUSUM statistic is employed to identify subperiods of sign shifts in the mean returns. Then the dependence in these subperiods is investigated using the Hurst (1951) and the Lo (1991) rescaled range statistic. The results suggest that rejection of the random null is conditional on both the procedure and the period being tested. We conclude that researchers may have inadvertently introduced sample and method bias into their studies of the time series properties of the spot USD/Yen.

Are Long-Term Return Anomalies Illusions:
Evidence From the Spot Yen

1. Introduction

A number of empirical studies (e.g. Booth, Kaen and Koveos, 1982; Cheung, 1993; Batten and Ellis 1996) employ the rescaled range statistical procedure, originally developed by Hurst (1951), to identify long-term return anomalies in currency markets. However, Fama (1998) argues that this type of anomaly may be sensitive to the method employed and will tend to disappear when alternative approaches are used. That is, they are "methodological illusions" (ibid. p.385). The objective of this paper is to provide an insight into this issue by further investigating the long-term return anomaly observed in the spot U.S. dollar to Japanese Yen (USD/Yen) currency market and to determine the extent to which the method of analysis may bias the final result.

For example, the study by Batten and Ellis (1996) observes that the USD/Yen logarithmic returns, over the period 3 March 1987 to 8 September 1993, displays long-term persistence when measured using the Hurst (1951) statistic. The presence of persistence favours a continued depreciation of the USD. That is, speculators holding a long (bought) Yen short (sold) USD position earn positive returns even after accounting for the interest rate differential between the lower yielding Yen investment and the higher yielding USD borrowing. This finding is contrary to theories of forward parity and weak-form market efficiency, but expected when a series displays long-term memory (Mandelbrot, 1971).

However, is the positive return on the USD/Yen a return anomaly or a statistical illusion? Given the scale of the spot USD/Yen trading[1] and the high level of information efficiency in foreign exchange markets one may be predisposed to favour the Fama (1998) view. One approach to the problem is to apply similar statistical methods but determine whether the return anomaly persists over different sample periods, or is specific to one or more subperiods. Our approach to this problem differs from other researchers in that we deduce from the data the subperiods for investigation.

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1 Of US$455.2 billion in daily global spot turnover against the USD, US$120.5 billion was against Yen. There was an additional US$27.7 billion in daily forward outrights The USD/Yen is the second major currency pair after the USD/Deutsche Mark which trades US$143.9 billion in daily spot markets and US$22.1 billion in daily forward markets (BIS Tiannual Survey, 1999).

Our method is threefold. First, we extend the sample period of the Batten and Ellis (1996) study to cover the period from 11 October 1983 to the 21 July 1999 (4,117 observations). The USD/Yen depreciated from 232.32 to 118.97 during this period. Second, we investigate and compare the time series properties of the overall sample period with the properties of specific subperiods selected on the basis of instability in the mean returns. To identify these subperiods we estimate the stream of CUSUM statistics for the return series. The CUSUM statistic, originally developed by Brown, Durbin and Evans (1975) to test for regression non-stationarity, is based on the recursive residuals (the one-step ahead forecast errors). However CUSUM may be applied to single series whose only basis for prediction is its mean, which in the case of a white-noise return series is zero. We utilise the Bos and Fetherston (1992) procedure that provides formulas for the critical values and also enables identification of the exact point in time a mean return sign-change occurs. This procedure enables the USD/Yen series to be decomposed into 6 consecutive (non-overlapping) subperiods with varying mean returns.

Preliminary analysis reveals that each sub-period has different moments and none has a mean of zero. Using the traditional unit root tests for stationarity (Dickey-Fuller and Augmented Dickey Fuller), the unit root null hypothesis is accepted for each subperiod individually, and for the whole sample period. The first subperiod (11 October 1983 to 21 June 1984), second subperiod (22 June 1984 to 1 July 1985) and sixth (last) subperiod (25 October 1995 to 21 July 1999) is characterised by USD appreciation against the Yen. However, the third subperiod (2 July 1985 to 15 March 1990), fourth subperiod (16 March 1990 to 14 August 1990) and fifth subperiod (15 August 1990 to 24 October 1995) are characterised by US depreciation against the Yen.

We also test the weak-form efficiency and normality of the daily return subperiods. Tests of weak-form efficiency may be categorised as either tests for patterns, or tests of trading strategies employing past data. Statistical tests for patterns incorporate the examination of correlation between successive price changes and runs testing. A non-parametric runs test on the entire series supports randomness of the USD/Yen return series. However when the runs test is conducted on the subperiods we accept randomness for all but the fifth time subperiod. We incidentally find all the series are statistically non-normal and leptokurtic which is consistent with empirical research in other currency time-series returns (Tucker and Pond, 1988; Kearns and Pagan, 1997). The presence of slight autocorrelation at lag 1 in four of the subperiods is also consistent with time varying short-term dependence in the series.

Next we investigate the presence of dependence in the various series using both the classical rescaled adjusted range statistic, developed by Hurst (1951) and the Lo (1991) modification to eliminate low-order or short-term dependence. The Hurst (1951) approach suggests the presence of time-varying long-term dependence, though this result was not supported when the Lo (1991) procedure was applied. This suggests that the significant Hurst (1951) result in some of the subperiods was due to low-order or short-term dependence in the series. Thus rejection of the random null is conditional on both the procedure used and the period being tested. We conclude that other researchers may have inadvertently introduced a sample or method bias in their studies of the time series properties of the spot USD/Yen.

The paper is structured as follows. Next in Section 2 the CUSUM and the rescaled range tests are explained in more detail. Then in Section 3 the results from these tests are presented. This Section also includes the descriptive statistics on the various subperiods. The final Section allows for some concluding remarks.

2. Methodology

(i) The CUSUM test for mean stability.

In this study we use CUSUM procedure introduced by Brown, Durbin, and Evans (1975) to test the mean stability of the USD/Yen returns. Brown et al. (1975) show that ordinary-least-squares recursive residuals (the recursively calculated and standardized one-step ahead prediction errors) are independent and standard normal. This property allows the creation of tests of the CUSUM of recursive residuals. In the case of misspecification of the regression equation this involves a test of whether the standardised prediction errors are white noise. Then if w_t are independently distributed standard normal deviates, the CUSUM test statistic is the sum of r independent standarised random normal deviates

$$W_r = \sum_{t=1}^{r} w_t \quad r = 1, 2, \ldots n \qquad (1)$$

Plots of the CUSUM of recursive residuals are useful in the determination of when and how instability occurs since it announces the incidence of instability when series cross critical boundaries. Bos and Fetherston (1992) show that these critical boundaries lead to a varying level of precision with respect to finding evidence of instability. To overcome this problem they estimate the 95th percentile of these crossings under the hypothesis of stability. Thus if the number of CUSUM crossings is greater than the corresponding 95th percentile of crossings estimated assuming stability, the hypothesis of stability is rejected. The estimate of the critical number of crossings for CUSUM is the critical number of crossings = 0.17772 + 0.31636 (total number of recursive residuals).

Nearly all the uses of the CUSUM procedure in the literature are in the area of regression stability. However since CUSUM statistics are based on the recursive residuals, the same can be done with a single series whose only basis for prediction is its average. Then for a white noise series about a fixed average

$$w_t = \frac{1/(t-1)\sum_{t=1}^{t-1} x_r}{\sqrt{(t/(t-1))}\sigma_x^2} \quad \text{for r =1, 2, . . . , n} \qquad (2)$$

where the numerator is the prediction error of the *t*th observation. This is the average of the observations up to but not including the *t*th observation (the prediction) minus the actual observed value in the period *t*. The denominator is the standard deviation of this type of prediction.

(ii) The classical and modified rescaled range statistics.

First proposed by Hurst (1951) the classical rescaled adjusted range $(R^*/\sigma)_n$ is calculated as

$$(R^* / \sigma_n) = (1/\sigma_n)\left[\underset{1 \le k \le n}{\text{Max}} \sum_{k}^{j=1} (X_j - M_n) - \underset{1 \le k \le n}{\text{Min}} \sum_{k}^{j=1} (X_j - M_n) \right] \qquad (3)$$

where M_n is the sample mean $(1/n)\sum_j X_j$ and σ_n is the series standard deviation

$$\sigma_n = \left[1/n \sum_{j}^{n} (X_j - M_n)^2 \right]^{0.5} \tag{4}$$

For a given series of length *N*, empirical estimates of the rescaled adjusted range are derived by first dividing the series length into subseries *n* of length $n \leq N$. Using Equation (3) and Equation (4) over each subseries length, the Hurst exponent can then be estimated by an OLS regression of the form

$$\log(R^* / \sigma)_n = \alpha + \beta \log(n) + \varepsilon \tag{5}$$

where the estimated β coefficient is the value of the Hurst exponent (H). Under the null hypothesis of no long-term dependence, the value of the classical rescaled adjusted range should be H = 0.5. Subseries lengths may be chosen to be either overlapping (G-Hurst) or contiguous (P-Hurst). Developed by Wallis and Matalas (1970), the G-Hurst technique generally requires that subseries lengths be evenly distributed in log space though contiguous subseries lengths are typically chosen from the set of factors of N.

Utilising the gamma distribution, the expected value of the classical rescaled adjusted range is modelled by Anis and Lloyd (1976) as

$$E(R^* / \sigma)_n = \frac{\Gamma(0.5(n-1))}{\pi^{0.5}\,\Gamma(0.5n)} \sum_{r=1}^{n-1} [(n-r)/r\ ]^{0.5} \tag{6}$$

where Γ (.) is the gamma variable. Estimated values of the classical rescaled range which exceed their expected value should be indicative of long-term dependence in the underlying series.

For a given series X_t across various lags (q) the modified rescaled range (Q_n) is defined by Lo (1991) as

$$\sigma^2_n(q) \equiv 1/n \sum_{j=1}^{n} (X_j - \overline{X}_n)^2 + 2/n \sum_{j=1}^{q} w_j(q) \{ \sum_{i=j+1}^{n} (X_j - \overline{X}_n)(X_{i-j} - \overline{X}_n) \} \tag{7}$$

$$= \sigma^2_x + 2 \sum_{j=1}^{q} w_j(q) \gamma$$

$$w_j(q) \equiv 1 - \frac{j}{(q+1)}$$

for $1 \leq k \leq n$ and $q<n$. This method replaces division by the observed sample standard deviation σ, with the denominator $\sigma_n(q)$. Critical values for the statistical significance of $V(q)_n$ are provided by Lo (1991).

By comparison with Hurst's original specification for the classical rescaled adjusted range, Lo's determination of the modified rescaled range in Equation (7) is equivalent to Equation (4) except for the addition of the denominator $\sigma_n(q)$. From Equation (7) it may be shown that $\sigma_n(q)$ is a determination of the series variance, σ^2_x plus the weighted autocovariance, $w_j(q)\gamma$, across various lags of q.

$$V_n(q) \equiv \frac{1}{\sqrt{n}} Q_n \qquad (8)$$

For short-term dependent series the denominator in the modified rescaled range statistic, $\sigma_n(q)$, will be non-zero. However, in support of Lo's (1991) assertion that the classical rescaled adjusted range is not well defined to accommodate general classes of short-term dependence, when $q = 0$, Equation (7) reduces to Equation (4) and Q_n equals R^*_N. Consequently, Lo (1991) proposed that the behaviour of Q_n in the limit corresponded to that of the classical rescaled adjusted range $(R^*/\sigma)_N$ only when $\sigma_n(q)$ and σ_N were asymptotically equivalent. However, given that estimates of the classical rescaled range are calculated using OLS techniques for several subseries $n<N$ (the modified statistic employs a single series length only), the asymptotic behaviour of $\sigma_n(q)$ and σ_N does not imply that $V(0)_n$ is equivalent to the classical exponent H.

Though Lo (1991: 1290) concedes that "little is known about how best to pick q" when the series X_t is finite, Andrews (1991) provides a data-dependent rule for choosing the lag length q. For example, when the series under observation was assumed to conform to an AR(1) process, the optimal lag (q^*) is given by Andrews as

$$[q^*] \equiv (3n/2)^{1/3} (2\hat{\rho}/(1-\hat{\rho}^2))^{2/3} \qquad (9)$$

where $\hat{\rho}$ was the estimated first-order autocorrelation coefficient and [.] denoted the largest integer less than or equal to q^*. Using Equation (9), the autocorrelation weights in Equation (7) was replaced by

$$w_j = 1 - \left| \frac{j}{k_n} \right| \tag{10}$$

For example given n = 1000 and $\hat{\rho}$ = 0.1, $[q^*]$ = 3 using the Andrews model (1991). For $\hat{\rho}$ = 0.9, $[q^*]$ = 51. It may be further observed that the optimal length of the lag in the Lo (1991) model increases directly with increasing values of the autoregressive coefficient.

3. Results

The natural log of daily spot USD/Yen return data was obtained from Datastream. The series selected was the midday New York price from the period 11 October 1983 to 21 July 1999 (4117 observations). Figure 1 provides a plot of the daily spot yen against its long-term trend using a quadratic trend model (y_t =248.856-0.09.962t + 0.00001755t^2). This model provides the lowest mean squared deviation of 292.438 from a number of alternatives including growth and Pearl-Reed logistic models. The difference between the actual spot rate and the trend line is drawn on the lower scale and may indicate periods of exchange rate overvaluation or undervaluation.

The sample period was also characterised by extensive Japanese and U.S. Government intervention in the exchange markets, initially to support the depreciating U.S. dollar in the period up to June 1995, and later to support the depreciating Yen in early 1998. Interest rate policy, specifically increasing the difference between U.S. and Japanese Government yields bonds, by lowering Japanese interest rates was also used to support the U.S. dollar at different times.

(i) Descriptive Statistics and Background

The Table reports the descriptive statistics and the CUSUM results for the different subperiods of USD/Yen spot daily log returns. The USD/Yen began the period (11-10-83) at 232.32 and ended the period (21-07-99) at 118.97. The USD therefore depreciated by 113.97 Yen in the period. The exchange rate gain or loss in value during each subperiod is reported with periods 3 (-96.10 Yen) and 5 (-47.05 Yen) having the largest values. There were three subperiods of gains and 3 periods of losses in value. The CUSUM value began the period at 0.00 and ended the period at 48.84. The shift in sign due to cumulative deviations from the mean marked the beginning of a new subperiod. The sum of the CUSUM values were highest in subperiods 3 and 6 and represented the periods of greatest departure from the zero mean. A white noise process should have a value of approximately zero since gains would tend to equal losses.

To establish the degree of randomness a non-parametric test a runs test was also conducted. The runs test p-value is recorded where for α levels > p-value the data is not random. This test accepts randomness for all subperiods except 5, and accepts randomness for the period as a whole. Subperiods 1, 2 and 5 display evidence of autocorrelation at lag 1. Subperiod 5 displayed the greatest level of negative autocorrelation. Overall the period displayed no autocorrelation. The standard deviation was lower in subperiods 1 and 2 and highest for 5 and 6 which also recorded the greatest range in values. All series were negatively skewed and leptokurtic. We conclude that while certain subperiods display evidence of non-normality and non-randomness these effects tend to offset over the entire sample period. In the case of the USD/Yen exchange rate the subperiod with the greatest departure from this white noise ideal appeared to be the fifth time period (15/08/90 to 24/10/95) which was also one of extensive central bank intervention.

DF and ADF test statistics are a t-test on γ from the regressions $\Delta Xt = a0 + \gamma Xt - 1 + a2t + \varepsilon t$ and $\Delta Xt = a0 + \gamma Xt - 1 + a2t + \sum \beta i \Delta Xt - i + \varepsilon t$ respectively, where X_t represents the continuously compounded single period yield. Values in parenthesis represent the number of lags used in the calculation of the DF and ADF statistics. Critical values for the ADF and DF tests at the 1% and 5% levels are –3.96 and –3.41 respectively. All of the estimated statistics in the Table are smaller than the critical values, indicating that each of the series under observation were stationary for each period tested.

(ii) Rescaled Range Statistics

Estimated values for the classical and modified rescaled range statistics for each subperiod, and their respective expected values are provided in Table 3. The values of both statistics for the whole sample period (11 October 1983 to the 21 July 1999) are also provided. Means and standard deviations of the classical and modified statistics for each subperiod are provided in Table 4. Estimated using bootstrap techniques, values in Table 4 provide an alternative test of the significance of the estimated values in Table 3, that is particularly effective for small sample sizes. Results for both the classical and modified rescaled range tests are discussed in turn.

Estimated values of the classical rescaled range statistic $(R/S)_n$ in Table 3 are not consistent across the six subperiods and the whole sample period. Relative to their respective expected values (shown in parenthesis) the results indicate significant positive long-term dependence during the second (22 June 1984 to 1 July 1985), third (2 July 1985 to 15 March 1990) and sixth (25 October 1995 to 21 July 1999) subperiods, as well as the whole sample period (11 October 1983 to 21 July 1999). Results for the first (11 October 1983 to 21 June 1984) and fifth (15 August 1990 to 24 October 1995) subperiods are not significant, and provide support for the null hypothesis of no long-term dependence during the subperiod periods. The low estimated $(R/S)_n$ value for the fourth subperiod (16 March 1990 to 14 August 1990) is indicative of significant negative long-term dependence in the series.

Batten and Ellis (1996) find that continued depreciation of the USD against the Yen is consistent with significant values of the classical rescaled range over the same sample period. Using the mean of the USD/Yen log first differences and the sum of log first differences as proxies for the level of appreciation and depreciation during each subperiod, there appears to be no relationship between the significance of estimated $(R/S)_n$ values in Table 3 and the overall direction of the USD/Yen during the period. For example the rank correlation coefficients for the mean of log first difference and the sum of log first differences versus $(R/S)_n$ (0.3214 and –0.2143 respectively) are not significant.

Values of the modified rescaled range $V(q)_n$ for all subperiod and the whole sample are calculated for lengths of the autocovariance lag q equal to 0, 3, 6, 9 and 12. Using the Andrews (1991) data dependent rule, estimates of $V(q^*)_n$ are separately calculated for the optimal lag length for each subperiod and the whole sample. Compared to $(R/S)_n$ values derived using the classical test, $V(q)_n$ values for all subperiods except the 2 July 1985 to 15 March 1990 subperiod are insignificant at the 0.05 level for all lengths of q. Only values of $V(0)_n$ and $V(3)_n$ are significant at the 0.05 level for this subperiod. Values for the optimal length of the lag q^* corresponding to the Andrews (1991) model were between $q^* = 0$ and $q^* = 3$ for all subperiods including the whole sample period. Consistent with the results just described, values of $V(q^*)n$ (not reported) are insignificant for subperiods except 2 July 1985 to 15 March 1990. Overall, the modified rescaled range statistic results support the null hypothesis of no long-term dependence.

One criticism of the use of the classical rescaled adjusted range methodology by Lo (1991) is the general failure by researchers to show empirical results derived from an equivalent random time-series. This failure is particularly relevant when the series under observation is small (ie. less than 1,000 observations). In this study, four of the six subperiods used are less than n = 1,000. Following Amrose Ancel and Griffiths (1992), each subperiod is scrambled 1,000 times. $(R/S)_n$ values for each iteration are then calculated and their mean and standard deviation used to estimate upper and lower confidence intervals. Means and standard deviations of $(R/S)_n$ are provided in Table 4.

In addition, given that critical values provided by Lo (1991) for the modified rescaled range are asymptotic, the above procedure is repeated for $V(q)_n$ estimates for each subperiod. Using this approach the null hypothesis of no long-term dependence is accepted at the 0.05 level if observed values of $(R/S)_n$ and $V(q)_n$ in Table 3 are within +/- 1.96 standard deviations of their respective mean values. Compared to their mean values in Table 4, the null hypothesis is accepted for every subperiod for both the classical and modified rescaled range tests. The result suggests that prior evidence of long-term dependence in some subperiods using the classical rescaled range in conjunction with the Anis and Lloyd (1976) significance test is spurious. Initial evidence of dependence using the modified rescaled range for the subperiod 2 July 1985 to 15 March 1990 is likewise rejected using the confidence interval approach. The result further highlights the asymptotic nature of Lo’s critical values.

4. Conclusion

This study investigates the sensitivity of the long-term return anomaly observed in USD/Yen currency market to method bias during the period 11 October 1983 to the 21 July 1999. This study differs from other time series studies in that initially the CUSUM statistic is employed to identify 6 subperiods characterised by a sign shift in the mean returns. Runs tests suggest all but the fifth subperiod (15 August 1990 to 24 October 1995) was random. We incidentally find all the periods are statistically non-normal and leptokurtic with the presence of slight autocorrelation being consistent with time varying short-term dependence in the series. The different statistical properties of the 6 subperiods highlight the significance of the sample period when determining the time series properties of exchange rates.

The presence of dependence in the various series was then established using both the classical rescaled adjusted range statistic, developed by Hurst (1951) and the Lo (1991) modification to eliminate low-order or short-term dependence. The Hurst (1951) approach suggests the presence of time-varying long-term dependence in some of the subperiods and for the whole sample period. However this result was not supported when the Lo (1991) procedure was applied to the various subperiods. This suggests that the significant Hurst (1951) result identified in some of the subperiods was due to low-order or short-term dependence in that particular series. Thus in line with Fama's (1998) suggestion that long-term anomalies may be due to "methodological illusions" we find that rejection of the random null in the spot USD-Yen market is conditional on both the procedure used and the period being tested. We conclude that other researchers may have inadvertently introduced a sample or method bias in their studies of the time series properties of the spot USD/Yen.

References

Ambrose, B.W., Ancel, E.W. and M.D. Griffiths. 1992. The Fractal Structure of Real Estate Investment Trust Returns: The Search for Evidence of Market Segmentation and Non-Linear Dependency. Journal of the American Real Estate and Urban Economics Association. 20 (1). 25-54.

Andrews, D.W.K. 1991. Hetroskedasticity and the Autocorrelation Consistent Covariance Matrix Estimation. Econometrica. 59 (3). 817-858.

Anis, A.A. and E. H. Lloyd. 1976. The Expected Value of the Adjusted Rescaled Hurst Range of Independent Normal Summands. Biometrika. 63 (1). 111-116.

Bank for International Settlements (BIS Triannual Survey). 1999. Central Bank Survey of Foreign Exchange and Derivatives Market Activity 1998. Basle March.

Batten, J. and C. Ellis. 1996. Fractal Structures and Naive Trading Systems: Evidence from the Spot US Dollar/Japanese Yen. Japan and the World Economy. 8. 411-421.

Booth, G.G., Kaen, F.R. and P.E. Koveos. 1982. R/S Analysis of Foreign Exchange Rates Under Two International Monetary Regimes. Journal of Monetary Economics. 10 (3). 407-415.

Bos, Theodore and Thomas A. Fetherston.. 1992. "Market Model Nonstationarity in the Korean Stock Market," Pacific-Basin Capital Markets Research, Volume III, ed. by G. Rhee and R. Chang. Amsterdam: North-Holland.

Brown, R. L., J. Durbin and J. M. Evans. 1975. “Techniques for Testing the Constancy of Regression Relationships over Time.” *JRSS*(B) 37:2, 149-163.

Cheung, Y. 1993. Long Memory in Foreign-Exchange Rates. Journal of Business & Economic Statistics. 11 (1). 93-101.

Fama, Eugene F. 1998. Market Efficiency, Long Term Returns, and Behavioural Finance, Journal of Financial Economics 49:283-306

Hurst, H.E. 1951. Long Term Storage Capacity of Reservoirs. Transactions of the American Society of Civil Engineers. 116. 770-799.

Kearns Phillip and Adrian Pagan 1997 Estimating the Density Tail Index for Financial Time Series, Review of Economics and Statistics, 79(2): 171-175

Lo, A.W. 1991. Long-Term Memory in Stock Market Prices. Econometrica. 59 (5). 1279-1313.

Mandelbrot, B.B. 1971. When Can Price Be Arbitraged Efficiently? A Limit to the Validity of the Random Walk and Martingale Models. Review of Economics and Statistics. 53 (3). 225-236.

Tucker, Alan and Lallon Pond. 1988. The Probability Distribution of Foreign Exchange Price Changes: Tests of Candidate Processes. Review of Economics and Statistics 70(4): 638-647.

Wallis, J.R. and Matalas, N.C. 1970. Small Sample Properties of H and K - Estimators of the Hurst Coefficient h. Water Resources Research. 6 (6). 1583-1594.

Figure 1
Time Series Plot of the Daily Spot USD/Yen

This figure plots the daily spot USD/Yen for the period from 11 October 1983 to 21 July 1999 against its long-term trend using a quadratic trend model (y_t =248.856-0.09.962t + 0.00001755t^2). The difference between the actual spot rate and the trend line is drawn on the lower scale and may indicate periods of exchange rate overvaluation or undervaluation. Generally when the USD/Yen was below the long-term trend line (yen "overshooting") central banks responded by buying US dollars against the yen.

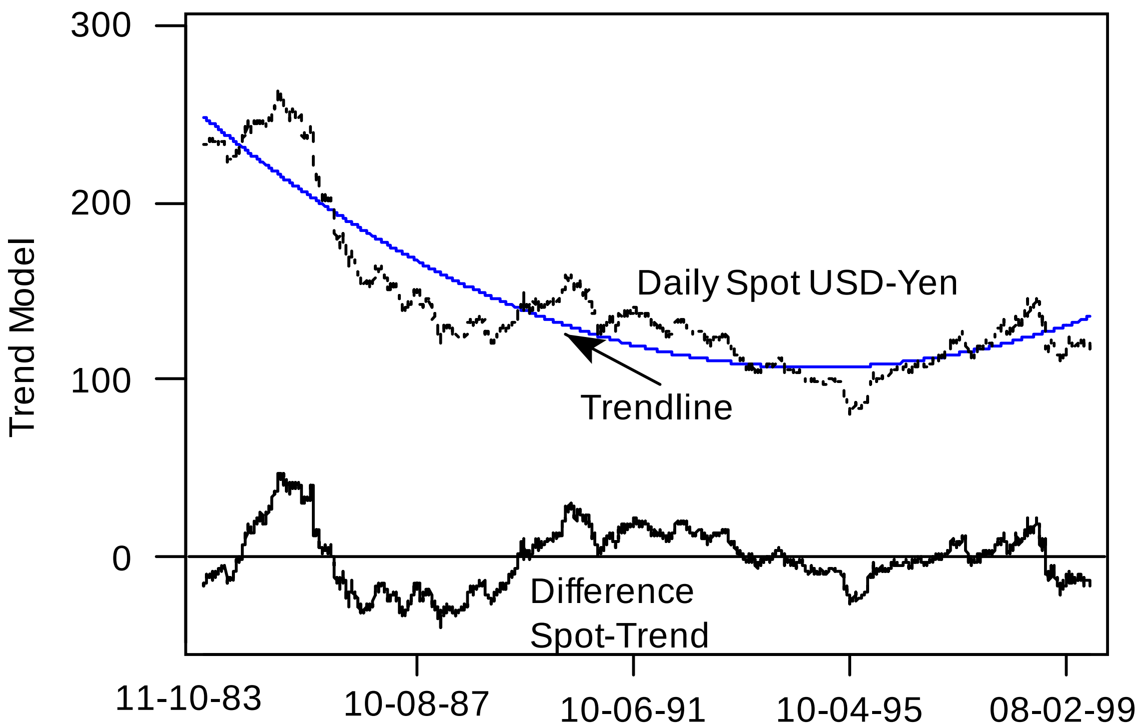

Figure 2
Time Series Plot of the CUSUM of Spot USD/Yen Daily Log Returns

This figure plots the stream of CUSUM statistics for the daily spot USD/Yen log returns for the period from 11 October 1983 to 21 July 1999. CUSUM is used to identify 6 periods of mean instability selected by a crossing or touching of the origin. Note that the statistic lies within the 95% confidence band calculated by Bos-Fetherston (1992). The test shows 47 crossings of the confidence band (at the start of the series) which is < the critical value of 1302.32. This result is consistent with mean stationarity over the sample period.

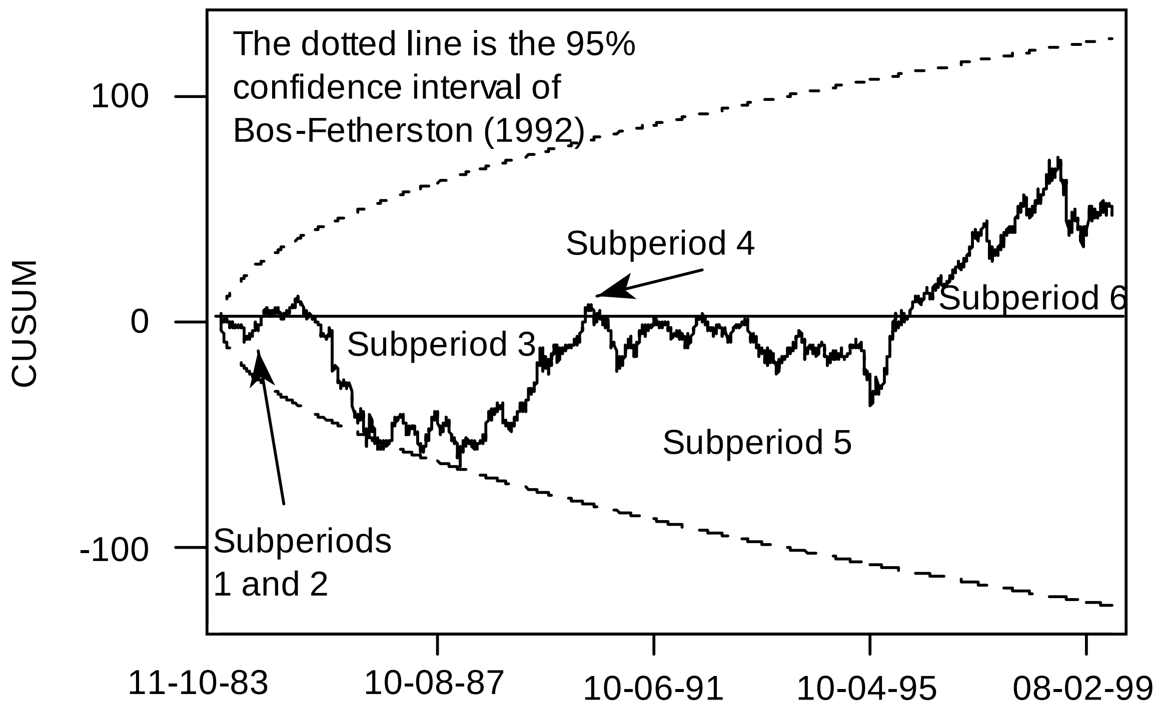

Table 1.
Summary Statistics of USD/Yen Log First Differences by Period

The Table reports the descriptive statistics and the CUSUM results for the different subperiods of USD/Yen spot daily log returns. The USD/Yen began the period (11-10-83) at 232.32 and ended the period (21-07-99) at 118.97. The gain or loss in value during each subperiod is reported. There were three periods of gains and 3 periods of losses in value. The CUSUM value began the period at 0.00 and ended at 48.84. The shift in sign marked the beginning of a new subperiod. The sum of the CUSUM values were highest in 3 and 6. A white noise process should have a value of approximately zero since gains would tend to equal losses. The runs test p-value is recorded where for α levels > p-value the data is not random. This test accepts randomness for all subperiods except 5. Subperiods 1, 2 and 5 display evidence of autocorrelation at lag 1. The standard deviation was lower in subperiods 1 and 2 and highest for 5 and 6 which also recorded the greatest range in values. All series were negatively skewed and leptokurtic.

Subperiod	**1** 11-10-83 to 21-06-84	**2** 22-06-84 to 1-07-85	**3** 2-07-85 to 15-03-90	**4** 16-03-90 to 14-08-90	**5** 15-08-90 to 24-10-95	**6** 25-10-95 to 21-07-99	**Total** 11-10-83 to 21-07-99
USD/Yen Start	232.32	235.95	248.30	152.70	147.50	101.50	232.32
USD/Yen End	234.65	247.85	152.20	148.70	100.45	116.15	118.97
Change	+2.33	+11.90	-96.10	-4.00	-47.05	+15.00	-113.35
CUSUM (value)	0.00(start)	0.9760	0.0008	0.4298	-0.7989	0.6107	48.84 (end)
CUSUM (sum)	-464	1195	-45,230	308	-13,185	36,027	-21,349
Runs Test (p-value)	0.8211	0.1212	0.9001	0.2739	0.0027	0.7690	0.1989
Autocorrelation at q =1, t-statistic	0.10 (1.40)	-0.07 (-1.21)	0.02 (0.59)	0.01 (0.07)	-0.04 (-1.61)	0.03 (0.98)	0.00 (0.08)
Mean	5.44E-05	2.11E-04	-3.99E-04	-2.15E-04	-2.94E-04	1.80E-04	-1.63E-04
Standard Error	3.07E-04	2.72E-04	2.01E-04	6.40E-04	1.97E-04	2.62E-04	1.12E-04
Standard Dev.	0.0041	0.0045	0.0070	0.0067	0.0072	0.0082	0.0071
Skewness	-1.7871	-0.5909	-0.3408	-0.8600	-0.6264	-1.2723	-0.8018
Kurtosis	10.1758	1.0227	4.3181	2.1017	6.8132	10.8649	8.1864
Max	0.0116	0.0109	0.0400	0.0148	0.0414	0.0395	0.0414
Min	-0.0260	-0.0151	-0.0491	-0.0262	-0.0542	-0.0769	-0.0769
Range	0.0375	0.0260	0.0890	0.0410	0.0956	0.1164	0.1182
Sum	0.0099	0.0565	-0.4894	-0.0233	-0.3985	0.1754	-0.6693
Count	183	268	1227	108	1354	977	4117

Table 2.
Stationarity Tests of USD/Yen Log First Differences by Period.

DF and ADF test statistics are a t-test on γ from the regressions $\Delta Xt = a0 + \gamma Xt - 1 + a2t + \varepsilon_t$ and $\Delta Xt = a0 + \gamma Xt - 1 + a2t + \sum \beta i \Delta Xt - i + \varepsilon_t$ respectively, where X_t represents the continuously compounded single period yield. Values in parenthesis represent the number of lags used in the calculation of the DF and ADF statistics. Critical values for the ADF and DF tests at the 1% and 5% levels are –3.96 and –3.41 respectively. All of the estimated statistics in the Table are smaller than the critical values, indicating that each of the series under observation were stationary for each period tested.

Period	11-10-83 to 21-06-84	22-06-84 to 1-07-85	2-07-85 to 15-03-90	16-03-90 to 14-08-90	15-08-90 to 24-10-95	25-10-95 to 21-07-99	11-10-83 to 21-07-99
DF (0)	-12.104	-17.686	-34.759	-10.125	-38.417	-30.283	-64.072
ADF (4)	-5.748	-8.048	-15.088	-4.8141	-16.267	-14.358	-28.300
ADF (22)	-	-	-7.7055	-	-6.510	-6.764	-12.569

Table 3.
USD/Yen Classical and Modified Rescaled Range Statistics by Period.

Results for the classical rescaled adjusted range are calculated using the OLS regression model are presented in the column headed $(R/S)_n$. Values in parenthesis correspond to the expected rescaled adjusted range $E(R/S)_n$ as calculated using the Anis and Lloyd (1976) model. $E(R/S)_n$ values are calculated separately for each subperiod. Values of $(R/S)_n$ larger then the corresponding $E(R/S)_n$ indicate that the series exhibited statistical long-term dependence during the subperiod. Values for modified rescaled range $V_n(q)$ using the Lo (1991) model are calculated for values of the autocovariance lag q = 0, 3, 6, 9 and 12. For all values of q, the 95% critical value of the modified rescaled range is 1.747. Observed values of $V_n(q)$ in excess of the critical value indicate that the series exhibited long-term dependence during the subperiod. Results attributable to the classical rescaled adjusted range indicate that four of the seven periods exhibited long-term dependence. Given values of the autocovariance lag q = 0 and q = 3, only one subperiod is significant at the 95% level when using the modified rescaled range statistic.

Period	Length	$(R/S)_n$	$V_n(0)$	$V_n(3)$	$V_n(6)$	$V_n(9)$	$V_n(12)$
11-10-83 to 21-06-84	183	0.5435 (0.5441)	1.2118	1.1926	1.2131	1.2138	1.2554
22-06-84 to 1-07-85	268	0.6049† (0.5410)	1.0961	1.1555	1.1757	1.1516	1.1550
2-07-85 to 15-03-90	1227	0.5706† (0.5298)	1.8625††	1.7807††	1.7252	1.7029	1.6693
16-03-90 to 14-08-90	108	0.1962 (0.5517)	1.0793	1.0786	1.1314	1.0977	1.0510
15-08-90 to 24-10-95	1354	0.5000 (0.5290)	1.0918	1.1444	1.6121	1.1732	1.1507
25-10-95 to 21-07-99	977	0.5499† (0.5312)	1.2615	1.2188	1.2314	1.2346	1.2348
11-10-83 to 21-07-99	4117	0.6257† (0.5238)	1.6378	1.6209	1.6167	1.6108	1.5874

† indicates observed values in excess of expected value.
†† indicates significant at the 95% level.

Table 4.
Mean and Standard Deviation of Classical and Modified Rescaled Range Statistics.

Mean values for the classical and modified rescaled range statistics are calculated using bootstrap techniques given 1000 simulations for each period. Using observed values for the classical rescaled range in Table 1, values of $(R/S)_n$ for all periods except the whole series are within +/- 1.96 standard deviations of the mean. Observed values for $V_n(q)$ in Table 1 are within 1.96 standard deviations of the mean for all periods, including the whole series. The results show that the null hypothesis of no long-term dependence is accepted at the 95% level for all the periods under observation.

Period	$(R/S)_n$	$V_n(0)$	$V_n(3)$	$V_n(6)$	$V_n(9)$	$V_n(12)$
11-10-83 to 21-06-84						
Mean	0.5260	1.1533	1.1604	1.1703	1.1822	1.1948
Std Dev	0.1794	0.2613	0.2403	0.2253	0.2127	0.1992
22-06-84 to 1-07-85						
Mean	0.5265	1.1630	1.1680	1.1767	1.1883	1.2010
Std Dev	0.1483	0.2591	0.2402	0.2285	0.2199	0.2123
2-07-85 to 15-03-90						
Mean	0.5254	1.2254	1.2278	1.2292	1.2306	1.2323
Std Dev	0.0728	0.2753	0.2719	0.2686	0.2644	0.2606
16-03-90 to 14-08-90						
Mean	0.5505	1.1318	1.1530	1.1734	1.1939	1.2168
Std Dev	0.3032	0.2469	0.2203	0.1994	0.1833	0.1703
15-08-90 to 24-10-95						
Mean	0.5269	1.1925	1.1936	1.1948	1.1965	1.1988
Std Dev	0.0695	0.2648	0.2606	0.2572	0.2544	0.2517
25-10-95 to 21-07-99						
Mean	0.5249	1.2094	1.2107	1.2128	1.2147	1.2168
Std Dev	0.0803	0.2639	0.2578	0.2527	0.2478	0.2438