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Scaling Foreign Exchange Volatility

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Abstract

When asset returns are normally distributed the risk of an asset over a long return interval may be estimated by scaling the risk from shorter return intervals. While it is well known that asset returns are not normally distributed a key empirical question concerns the effect that scaling the volatility of dependent processes will have on the pricing of related financial assets.

This study provides an insight into this issue by investigating the return properties of the most important currencies traded in spot markets against the U.S. dollar: the Deutsche mark, the Swiss franc, the Japanese yen, and the British pound, during the period from January 1985 to December 1998. The novelty of this paper is that the volatility properties of the series are tested utilizing statistical procedures developed from fractal geometry after adjusting for dependence in the series, with the economic impact determined within an option-pricing framework.

The results demonstrate that scaling risk underestimates the actual level of risk for all four series investigated. However while the call and put premiums on Deutsche mark, Swiss franc and yen options are too low, those on the English pound are not. In our discussion of the possible sources of option under-pricing both conditional heteroskedasticity and underpricing bias by the Black-Scholes model have been considered. However, these factors were unable to account for the very high levels of underpricing reported. In conclusion, these results highlight the complex behaviour of financial asset returns and the inability of general pricing models to completely and accurately describe this behaviour.

Keywords: Scaling volatility, long-term dependence, foreign exchange

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1. Introduction

When asset returns are normally distributed the risk of an asset over any return interval may be estimated by scaling the risk from other return intervals (Mandlebroot, 1963). However, while non-normality is a feature of many financial time-series (Alexander (1961); Fama, (1965); and more recently Pagan (1996)) many market valuation models require an annualized risk coefficient. The empirical question that must be addressed is whether the practice of annualizing risk is appropriate when the series is dependent.

This study provides an insight into this issue by investigating the return properties of currencies traded in spot markets against the U.S. dollar: the Deutsche mark (DMK/USD), the Swiss franc (SWF/USD), the Japanese yen (JPY/USD) and the British pound (GBP/USD) from January 1985 to December 1998. The novelty of this paper is that the volatility properties of the series are tested utilizing statistical procedures developed from fractal geometry (Mandelbroot, 1982; Batten and Ellis (forthcoming)) after adjusting for dependence in the series (Hurst, 1951; Lo, 1991), with the economic impact determined within an option-pricing framework.

The four currencies investigated constitute the largest foreign exchange markets in the world measured in terms of turnover, are highly liquid, and have low transaction costs. The BIS (1998) survey on foreign exchange market activity estimated that these currency pairs account for 69% of total daily turnover in the spot foreign exchange markets in 1998 of US$590 billion. ((DMK/USD (30%), JPY/USD (21%), GBP/USD (11%) and SWF/USD (7%)). Trading also occurs on a 24 hour basis, with almost instantaneous transmission of news items to market participants using computerised technology and on-line broking services. Consequently these markets are as close to the efficient market ideal as is currently possible. The timing of the sample period is significant only in that the end of the period corresponds with the introduction of the Euro as the official currency of the Federal Republic of Germany on 1 January 1999.

There is extensive evidence of non-normality in the distributions of daily and weekly returns for spot and forward currencies (Tucker and Pond, 1988; Kearns and Pagan, 1997). More recently the empirical literature has focused upon the presence of low order (short-term memory) and high order (long-term memory) correlation structures of time-series which if present would also be a breach of market efficiency. One approach is to determine if market-practitioners are able to exploit return anomalies using trend-following technical trading strategies. For example, research on stock market prices includes studies by Brock, Lakonishok and LeBaron (1992) and more recently Sullivan, Timmerman and White (1999). While these studies suggest abnormal trading returns are possible, Fama (1998) would argue that these results are statistical anomalies.

The significance of correlation structures in time-series may be determined, using an exponent named after the original developer Hurst (1951), for the presence of high-order or long-memory, or using the modification suggested by Lo (1991) to eliminate low order persistence. Findings have generally been in favour of the presence of non-linear dependence consistent with long-memory (Peters 1994; Van De Gucht, Dekimpe and Kwok 1996; Opong, Mulholland, Fox and Farahmand, 1999; Barkoulas, Labys and Onochie, 1999). Using the classical rescaled range method proposed by Hurst (1951), Peters finds evidence of significant positive dependence in USD/DMK, USD/JPY and USD/CHF daily exchange rates. Using a variety of univariate and multivariate persistence measures, Van De Gucht et al. similarly find that a number of exchange rate series exhibit long-run behaviour that deviate from a strict random walk. The results are consistent with those of Cheung (1993) who finds evidence of long-term memory in spot currency markets using spectral regression techniques. However the results of a recent study on the Japanese yen by Batten, Ellis and Fetherston (2000) casts doubt on the earlier Peter's result and supports Fama's (1998) conclusion: there was no evidence of long-term dependency with the results from earlier researchers being "methodological illusions".

Statistical dependence has particular implications for modelling the behaviour of financial market asset prices. Modelled as Brownian or fractional Brownian line-to-line functions[1], the principle of scale invariance suggests an observable relationship between asset returns across different time

[1] The definition of a line-to-line function is one where (as in the case of time-series observations) increments in the series are constrained not to move backward. A Brownian line-to-line function is

frequencies. Series exhibiting non-linear dependence (of the type associated with fractional Brownian motion) should scale by a factor equivalent to their Hurst exponent. By contrast, Gaussian series should scale by the factor H = 0.5. An example of a common application of scaling in financial time-series involves the annualising of risk in the Black-Scholes Option Pricing Model. Under the assumption of a normal Gaussian distribution the Hurst exponent H = 0.5 implies, for time-series data, that mean annual increments (or returns) should be 12 times the equivalent monthly mean and 52 times the equivalent weekly mean. Similarly, the standard deviation of annual increments should be $\sqrt{12}$ times that of the monthly increments. However, series generally exhibit higher peaks and fatter tails than are associated with the normal distribution and become more leptokurtic as the frequency at which the returns are measured increases.

The presence of long-term memory or non-linear dependence in currency time-series data also has implications for investors and risk managers. Analysing mean absolute logarithmic price changes for five foreign exchange rate pairs, Muller et al. (1990) find that intraday price changes scale at approximately H = 0.59. Measuring the standard deviation of interday returns as the proxy for risk, Holton (1992) demonstrates that when asset prices do not follow a random walk, annualising risk by the square root of time ($\sqrt{T}$) either overestimate or underestimate the true level of risk associated with an investment. Using Monte Carlo techniques, Diebold, Hickman, Inoue and Schuermann (1998) show that when asset returns follow a GARCH (1,1) process, scaling by $\sqrt{T}$ tends to overestimate fluctuations in conditional standard deviations, yet yields volatilities that on average are approximately correct. Applying a formula derived by Drost and Nijman (1993), Diebold et al. (1998) were able to correctly scale daily standard deviations to longer time horizons, but concede that the general result is conditional on first being able to correctly identify the conditional variance of the return's (GARCH) process. Developing a GARCH option pricing model, Duan (1995) finds that GARCH conditional variance processes provide one explanation for underpricing of out-of-the-money option contracts by the simple Black-Scholes model.

To reconcile these various empirical findings, initially we investigate the distributional qualities of the daily returns of long-term currency data and then determine the dependence relationships. Then we establish the relationship between the data's distributional qualities, and the appropriate scaling

therefore analogous to a Gaussian random walk through time.

relationships. The distributional qualities of the data confirm features of non-normality (leptokurtosis), though the lagged correlation structure does not suggest the presence of long-term memory. The series are then used to calculate the standard deviation of returns for return intervals from one day (t - (t-1)) to one year (t - (t-252)). Using Ordinary Least Squares (OLS) regression techniques, the rate of change in the standard deviations versus the returns interval is expected to be equal to 0.5 (the square root of time); the value of the scale exponent for a Gaussian time-series. The results show that all series except the GBP/USD, scale at a significantly faster rate than the square root of time.

The economic implications of these results are also significant. Employing a simple Black-Scholes model for pricing currency options, we find that the incorrect scaling of volatility leads to significant incorrect pricing decisions. Specifically, scaling risk by the square root of time underestimates the actual level of risk when the underlying time-series scaled at a faster rate than $\sqrt{T}$. Dependence in the conditional variance of the currency returns is suggested as one explanation for this result.

The paper is set out as follows: In the next section (2) the scaling and self-similarity of Brownian and fractional Brownian motion systems are discussed in more detail. Then Section 3 establishes the theoretical basis and research methodology for the empirical research. Section 4 describes the currency data and provides evidence on its distributional qualities. Section 5 presents our empirical results on the scaling relationships. The final section summarises the results and allows for some concluding remarks.

2. Scaling and Self-similarity in Brownian and Fractional Brownian Motion.

The concepts of standard Brownian motion (sBm) and fractional Brownian motion (fBm) may be defined in terms of the relative level of dependence between increments. One characteristic of these processes is their self-similar behaviour. Developing the principles of fractal geometry, Mandelbrot (1982) proposes the concept of fractional Brownian line-to-line functions to describe time-series that exhibit an underlying fractal distribution. Batten and Ellis (forthcoming) later combine the scaling properties of Gaussian series and the value of the Hurst exponent to develop a set of testable empirical conditions in time series data.

We begin by considering ordinary Brownian line-to-line motion B(t) in time t as characterised by small and mutually independent increments in B such that

$$F(x) = Pr\,(X < x) = Pr\left(\frac{B(t+\Delta t) - B(t)}{|\Delta t|^{H}} < x\right) \tag{1}$$

where F(x) is the probability distribution of x. For real values of x and t, and where the Hurst exponent H = 0.5, Equation (1) conforms to standard Brownian motion for all t and Δt. Distinguishing this from fractional Brownian line-to-line motion, $B_H(t)$, Mandelbrot relaxes the restriction of H = 0.5 (which is necessary for a Gaussian process) and allows any $0 \le H \le 1$. For $0 \le H < 0.5$ the series $B_H(t)$ is characterised as being anti-persistent. Anti-persistent series diffuse more slowly than ordinary Brownian (line-to-line) functions and will therefore appear "rougher" than a time series graph corresponding to a Gaussian process. Alternatively, persistent series with $0.5 < H \le 1$ will appear "smoother" than a time series graph of a Gaussian process of the same function. Thus time series with a H value close to one is said to have less fractal noise than a low H value series.

Defining the fractal dimension (D) of a line-to-line function as a measure of the degree of irregularity of the time-series graph Mandelbrot (1987) proposes an alternative interpretation of the value of the Hurst exponent as the amount of space a time series graph fills in two-dimensional space. Measured as

$$D = 2 - H \tag{2}$$

where the fractal dimension of an ordinary Brownian line-to-line function is D = 1.5. For the limiting values H = 0 and H = 1, the value of the fractal dimension is D = 2 and D = 1 respectively. The intuition of this result is that the graph corresponding to H = 0 will fill its space entirely. For the upper limit H = 1, the resulting graph will be linear, and hence only fill one dimension (length). In so far that Brownian motion, as originally defined by Bachelier (1900), implies an equal probability of incremental movement in *all* directions, the definition of the line-to-line function may be seen as a more precise description of time-series behaviour. These functions are explained in the following section, then evidence and implications of scaling relationships are provided.

The related concepts of self-similarity, self-affinity and scale invariance can be used to describe the relationship between the parts and the whole of any function. For standard Brownian and fractional Brownian functions, the relationship is expressed in terms of self-similarity. For line-to-line functions (time-series) the correct expression is self-affine.

Consider a function (S) made up of the points $X = [X_0, X_1,..., X_n]$, where the probability of incremental movement is unrestricted with respect to the direction of the movement. Changing the length of the function by a common factor $r<1$, such that $rX = [rX_0, rX_1,..., rX_n]$, will yield a new function rS, whose geometric length is less than that of the original function. For the appropriate value of r, self-similarity implies the original function S can be recovered by N times contiguous replications of the self-similar rescaled function rS. In other words, the function S is scale invariant, that is, it is invariant to the change in scale by the factor r.

For line-to-line functions measured with respect to time, Mandelbrot (1982) shows these functions will instead be self-affine. Consider the same function (S), measured now as a line-to-line function comprising the points $X(t) = [X(t_0), X(t_1),..., X(t_n)]$, in time t. Changing the time scale of the function by the ratio $r<1$, the required change in scale of the amplitude is shown to be r^H for a self-affine function.

Given the function S is a Brownian line-to-line function, the distance from $X(t_0)$ to a point $X(t_0 + t)$ is shown by Mandelbrot (1982) to be a random multiple of $\sqrt{t}$. Setting $t_0 = 0$ it follows for $t>t_0$ that

(3) $$X(t_0 + t) - X(t_0) \approx e\,|(t_0 + t) - t_0|^{0.5} \approx e\,t^{0.5}$$

where e is a random variable with zero mean and unit variance. Properly rescaled in time by r, and in amplitude by $\sqrt{r}$, the increments of the self-affine rescaled function $(rS)/\sqrt{r}$ will be

(4) $$\frac{X(t_0 + rt) - X(t_0)}{\sqrt{r}}$$

For the correct choice of scaling factor, the two functions S and (rS)/√r are statistically indistinguishable, such that they have the same finite dimensional distribution functions for all t_0 and all r>0. Consistent with the known value of the Hurst exponent for Gaussian series (H = 0.5), the scaling factor $\sqrt{r} = r^H$ is characteristic of all self-affine Brownian line-to-line functions. Allowing for $0 \leq H \leq 1$, $H \neq 0.5$ it follows for fractional Brownian line-to-line functions that Equation (3) can be generalised by

(5) $$X(t_0 + t) - X(t_0) \approx e\,|(t_0 + t) - t_0|^H \approx e\,t^H$$

Equation (4) can similarly be generalised by

(6) $$\frac{X(t_0 + rt) - X(t_0)}{r^H}$$

for a fractional line-to-line function. Self-affine line-to-line functions also appear the same graphically when properly rescaled with respect to H. Mandelbrot (1982) notes that fractional Brownian line-to-line functions should exhibit statistical self-affinity at all time scales. Independent of the incremental length (or frequency of observation) of $X(t_0 + t) - X(t_0)$, the relative level of persistence, or anti-persistence, should remain consistent. Peitgen, Jürgens and Saupe (1992), also reveal that the limit formula for the fractal dimension of Brownian and fractional Brownian line-to-line functions in Equation (2) can be proven to follow directly from the principles of scale invariance.

3. Research Method

Modeling spot currency time-series as a line-to-line function measured with respect to time, scaling invariance may describe the relationship between the moments of the distribution of the time-series at different time intervals. Derived from the premise that amplitude and time do not vary independently, but instead are dependent, the relationship can be described by the power law function

(7) $$y \propto x^H$$

Equation (7) demonstrates that the y-axis measure is proportional to time (x) raised to a scale exponent, H.

While the definition of self-affinity requires that *every* mathematical and statistical characteristic of the time-series under observation and its self-affine rescaled function be examined, in practice this proof can be "inferred from a single test that is only concerned with one facet of sameness" (Mandelbrot, 1982: 254). The first test conducted in this study is the relation described by Equation (7) for the standard deviation of returns for all time intervals k = 1, 2, …, 252 days. Using OLS regression techniques, the scale exponent will be estimated as the coefficient γ_1 in the regression

(8) $$\log(s_k) = g_0 + g_1 \log(k)$$

where k is the length of the return interval and σ_k is the standard deviation of k interval returns. This technique is similar to the one employed by Muller et al. (1990) for intraday foreign exchange rates. Under the null hypothesis that each currency series conforms to an independent Gaussian distribution, the expected value of the scale exponent is H = 0.5. Continuously compounded returns for four spot currency pairs, DMK/USD, SWF/USD, JPY/USD and GBP/USD, are measured using $X_k = \ln(P_t / P_{t-k})$ for each of the nominated values of k. The volatility of returns is estimated by the second moment of the distribution of each returns series.

The self-affine relation described by Equation (5) and Equation (6) is then tested. This second test describes scaling relations between the volatility of returns measured over time intervals of k = 1, 5, 22 and 252 days. The values of k correspond to daily (1), weekly (5), monthly (22) and annual (252) return intervals. Using the principles of scaling invariance, the volatility of returns at any time interval can be estimated from the volatility at any *other* interval such that for any combination of k and n, $\infty \geq k \geq n \geq 1$, where n is any interval length less than k

(9) $$[\, s^2(P_t - P_{t-k})\,]^{0.5} = (k/n)^H\,[\, s^2(P_t - P_{t-n})\,]^{0.5}$$

Where the time-series under observation conforms to a standard Brownian line-to-line function the value of the exponent H in Equation (9) is H = 0.5. For fractional Brownian line-to-line functions, the value of H should be $0 \leq H \leq 1$; $H \neq 0.5$. Based on the above principle, the Hypotheses tested in this study are:

H_0 : $\sigma(P_t\text{-}P_{t\text{-}k}) = (k/n)^{0.5}\,\sigma(P_t\text{-}P_{t\text{-}n})$

H_1 : $\sigma(P_t\text{-}P_{t\text{-}k}) = (k/n)^{H}\,\sigma(P_t\text{-}P_{t\text{-}n})$, where $1 \geq H \geq 0$, $H \neq 0.5$

Implied standard deviations for each interval (k = 5, 22 and 252) are estimated from the standard deviation of n interval returns (n = 1, 5 and 22) for all n<k, and the results compared to the observed standard deviations. The imputed value of the scale exponent, for which the standard deviation of k interval returns can be exactly estimated from the n interval standard deviations, is then calculated for each currency pair. The significance of the imputed scale exponent is that this number represents the value of H for which the implied k interval standard deviations equals exactly their observed values. The acceptance of the null hypothesis that the appropriate scale exponent (H) is H = 0.5 will imply that the series under observation conforms to a random Gaussian distribution. Observed values of H which are significantly different from H = 0.5, will imply the rejection of the null hypothesis. The existence of a stable scaling law for each currency series implies that the imputed value of the scale exponent in this second test will not be significantly different to the regression scale exponent γ_1.

With respect to the independence of consecutive returns $\ln(P_t / P_{t\text{-}k})$ for different values of the nominated lag length k, Muller et al (1990) show that the size of the overlap between successive return intervals (k = 1 to n) does not affect the value of the estimated scale exponent. However one distracter is that the R^2 of the regression model used to measure the scale exponent will be very close to one, due to the high correlation coefficient. Testing the robustness of our own results to changes in the length of the overlap between return intervals also shows no significant difference between either of the estimated scale exponent or the subsequent option premiums.[2]

In order to test the economic implications of our findings, implied annual standard deviations (k = 252) for each currency pair are estimated from the standard deviation of returns over daily (n = 1), weekly (n = 5) and monthly (n = 22) intervals. The results are then used to calculate the values of a series of in-the-money, at-the-money and out-of–the-money European call and put foreign currency options.

4. Data

The time-series properties of the natural logarithm (ln) of interday returns (P_t / P_{t-1}), for the four exchange rates (DMK, SWF, JPY and GBP against the US dollar) from 2 January 1985 to 31 December 1998 (n = 3516 observations) are presented in Tables 1 to 4. These are discussed in turn.

(insert Table 1 about here)

The first table (Table 1) provides information on the moments of the four time-series. For all series, the sample means are slightly negative but not significantly different to zero. However, the sum of the ln returns over the sample period (e.g -0.651 for the DMK/USD) shows the impact of the devaluation of the USD against all four currencies. For example, the DMK/USD ln return (-0.651) is equivalent to a nominal positive return of +42.9% from holding DMK instead of USD, excluding the effect of the interest rate differentials over the sample period. Allowing for the interest rate differentials produces less startling returns though the expected future spot prices suggest that the yen and the pound both appreciated more than expected based upon the interest rate differentials, while the Deutsche mark and the Swiss franc appreciated less. In all four cases technical trading strategies that exploited the long-term depreciation of the dollar (such as moving average system) would be expected to show positive returns during the sample period. However as Fama (1998) and Batten and Ellis (2000) observe, where one starts and ends the series will have an impact on these potential benefits. On the other hand which of the many technical trading strategies available to investors would provide the highest economic returns, given holding and transaction costs, provides scope for further analysis.

The four series are also non-Gaussian, due largely to their leptokurtic nature than due to skewness, which is close to zero. Three normality tests are reported: Anderson-Darling test (an empirical cumulative distribution test), Jacque-Bera (a chi-squared test) and Shapiro-Wilk (a correlation test) and each fails to accept normality in the four series. This result is not surprising since leptokurtic, non-normal distributions are common in financial time-series Pagan, 1996). Also, a note of caution should be exercised in interpreting these three test results since they are very sensitive to small deviations from normality when the sample size is large. However these small differences may impact

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[2] Results of estimated scale exponents using different overlaps are available from the authors by request.

on pricing and risk management practice due to the higher probability of observations being away from the mean when the series displays leptokurtosis. Overall the four series display different degrees of departure from normality, with the JPY/USD and the GBP/USD having higher chi-squared values (Jacque-Bera test scores of 2129.66 and 3597.05 respectively) than the DMK/USD and SWF/USD. However the Shapiro-Wilk correlations suggest the DMK-USD (correlation to normal distribution of 0.986) and the SWF-USD (correlation to normal distribution of 0.990) are closer to normal distributions. These phenomena may be partly explained by the correlation structure of the four currencies pairs presented in Table 2.

(insert Table 2 about here)

While the four currency pairs suggest significant positive correlation (consistent with the DMK, JPY, SWF and GBP all appreciating against the USD), the co-movement of returns differs between the various currency pairs. Not surprising given the currency interdependence of Europe, the highest positive correlations are between the DMK and SWF (0.916), DMK and GBP (0.731), and SWF and GBP (0.698), while the lowest are between the GBP and JPY (0.472). These results highlight the limited ability for portfolio managers to diversify currency portfolios during the sample period.

(insert Table 3 about here)

The four time-series are then tested for the presence of autocorrelation up to 252 lags, with the results for lags 1, 2, 5, 22 and 252 presented in Table 3. The results suggest slight positive short-term autocorrelation that dissipates after lag one. The significance of the autocorrelations is tested using t-statistics and Ljung-Box Q statistics (reported as LBQ in Table 3). The t-statistics are high (ie. $t > 1.25$) for all currency pairs at lag one, but not high thereafter. However the LBQ statistic is only significant for SWF/USD (p-value = 0.091) and GBP/USD (p-value = 0.003) at lag 1. Over the longer lag structure, three of the currency pairs (DMK/USD, JPY/USD and GBP/USD) have autocorrelations which can be judged to be significantly different to zero (at the 90% level of confidence) at lag 22 (LBQ p-values are 0.064, 0.068 and 0.052 respectively), while two of the currency pairs (JPY/USD and GBP/USD) have autocorrelations which can be judged as being significantly different to zero (at the 90% level of confidence) at lag 252 (LBQ p-values are 0.090 and 0.051 respectively).

The next table, Table 4, reports tests for mean stationarity in the four series. Values for the Augmented Dickey-Fuller (ADF) test at lags of 9 and 28, and for the Dickey-Fuller (DF) test (lag 0) are presented for each currency pair. Calculated using the natural logarithm (ln) of the returns series, the significantly high negative values for each currency indicate that all the returns series are stationary over the sample period. Critical values for the ADF and DF tests at the 1% and 5% levels are –3.96 and –3.41 respectively.

(insert Table 4 about here)

Though not reported in these tables, the four time-series are also tested for partial autocorrelations up to 252 lags. Only the t-statistics for the DMK/USD at lag 1 (correlation = 0.026, t-statistic = 1.471), and the GBP/USD at lag 1 (correlation = 0.052, t-statistic = 3.008) are significant. Given the series are unit-root stationary, and have significant partial and auto-correlations at lag 1, the four series are then tested as AR(1) and ARMA(1,1) processes. None of the four series have significant coefficients when tested as ARMA(1,1) processes, however the GBP/USD and SWF/USD both have significant coefficients when tested as AR(1) processes (i.e. GBP/USD coefficient is 0.026, t-statistic 3.010, p-value 0.003, and the SWF/USD coefficient is 0.029, t-statistic 1.690, p-value 0.091). This result is consistent with the significant autocorrelations at lag 1 for both series as provided by the LBQ statistic.

Overall, though the four series provide evidence of non-normality in the form of leptokurtic returns with a slight positive autocorrelation structure, there is no compelling evidence that the appropriate scale exponent should be other than $H = 0.5$ (ie. a Gaussian or random walk series). This result alone implies that the standard deviation of interday return's should scale at $\sqrt{T}$. The next section investigates this issue in more detail by determining the scaling properties of the four currency series. Then, the implications for the pricing of currency options when volatility is incorrectly scaled are determined using simple currency option pricing models.

5. Scaling Properties of Foreign Currency Returns

The scaling properties of daily foreign currency returns are studied using two tests. The first is an investigation of the existence of a power law for the standard deviation of returns over the return intervals k = 1, 2, 3,…, 252. The second is an examination of imputed scale exponents for standard deviations between the following discrete intervals: k = 1, 5, 22 and 252. The economic significance of the findings is then considered by estimating the Black-Scholes currency option pricing model using both observed and implied annual standard deviations. Each set of results is discussed in turn.

Standard deviations of k interval returns for each currency pair are depicted in Figure 1. The heavy line in the figure shows the observed k interval standard deviation, while the lighter line shows how these same standard deviations would appear under the assumption that risk scales according to the square root of time rule.

(insert Figure 1 about here)

Results in Figure 1 indicate that the standard deviation of returns for each currency pair, except the GBP/USD, scale faster than H = 0.5. Furthermore it may be seen that the divergence between the observed and implied k interval standard deviations increases as the return interval (k) increases. Regression scale exponents for each currency pair are presented in Table 5. Calculated using Equation (8), the regression scale exponent γ_1 is the rate of change in the observed k interval standard deviations for the intervals k = 1 to 252. The coefficient γ_1 is the equivalent of the slope of the heavy line in Figure 1 when plotted in log-log space. Values in Table 5 confirm that the standard deviations of all currency pairs except the GBP/USD, scale at greater than the square root of time. However all the values are substantially lower than shown by Peters (1994) for daily currency returns and by Muller et al. (1990) for absolute changes in intraday currency prices. One implication of this result is that short interval risk is a poor predictor of risk over long time intervals. Option prices based on implied annual standard deviations are therefore expected to be lower than those derived from the observed annual standard deviations.

(insert Table 5 about here)

The scaling properties between the standard deviation of k and n interval returns (k = 1, 5, 22, 252; $k \geq n \geq 1$), for each of the four foreign currency pairs are presented in Table 6, while results presented later in Table 8 demonstrate the economic significance of the empirical findings.

(insert Table 6 about here)

Results presented in Table 6 show the imputed values for the scale exponent H calculated from Equation (9). Using observed values of the standard deviation of k and n interval returns, the imputed scale exponent is the value of H where rescaled values, of the standard deviation of n interval returns, exactly equal the standard deviation of k interval returns. Under the null hypothesis H_0, the value of the scale exponent should be H = 0.5. Rejection of the null hypothesis implies that the currency series tested do not conform to a Gaussian random walk. Additionally, the value of the scale exponent should be expected to remain approximately constant for all return intervals if the distribution of returns is stable.

Imputed values of the scale exponent in Table 6 are typically closer to H = 0.5 when the difference between the return intervals k and n is small. That is, when the standard deviation of weekly returns (k = 5) are estimated by rescaling daily return standard deviations (n = 1), imputed H values are closer to H = 0.5, than when daily return standard deviations are rescaled to estimate the standard deviation of annual returns (k = 252). For example, the imputed value of H for estimating the annual return standard deviation from the standard deviation of daily returns for the JPY/USD is H = 0.546. Rescaling the daily return standard deviation to estimate the standard deviation of weekly JPY/USD returns yields H = 0.499. The significance of this finding is that it provides some support for the proposition that the distributions of currency returns are not stable for different return intervals. The result is consistent with Muller et al. (1990) who suggests that the distributions of intraday foreign exchange returns are also unstable.

The statistical significance of the imputed scale exponents is measured using the Significance statistic (S) where

$$S = \frac{H \text{ - } E(H)}{[\, s\,(E(H))\,]^{0.5}}$$

(10)

$$s\,(E(H)) = 1/N$$

The results from utilising Equation (10) are reported in Table 7.

The significance was first proposed by Peters (1994) an may be interpreted as the number of deviations of the imputed scale exponent (H) from its expected value. A generalisation of the square root of time rule, the relevance of the denominator in Equation (10) is that the variance of a Gaussian time-series should scale in direct proportion to increases in the value of the series length (N).

(insert Table 7 about here)

Test results in Table 7 are conducted using two alternate values for the expected value of the scale exponent E(H). Under the null hypothesis that the underlying returns series are independent, the expected value of the scale exponent is E(H) = 0.5. Significance values in panel (A) of the table are calculated according to the null hypothesis. Under the alternate hypothesis, the expected value of the scale exponent is E(H) = γ_1, where γ_1 is the slope coefficient in the regression $\log(\sigma_k) = \gamma_0 + \gamma_1 \log(k)$. Panel (B) of the table reports significance values using the alternate hypothesis. Values of the regression scale exponent γ_1 are provided in Table 5.

Panel (A) of Table 7 shows that the imputed scale exponents in Table 6 are significantly different from E(H) = 0.5 for all currency pairs except the GBP/USD, given values of n = 1, 5 and 22 and k = 252. As implied in Figure 1, the result implies that divergence from the square root of time rule are most significant over long time intervals. That is, while weekly (k = 5) and monthly (k = 22) standard deviations can be accurately estimated from the standard deviation of daily returns (n = 1) using H = 0.5, annual (k = 252) standard deviations are generally poorly forecast from short interval standard deviations (n = 1, 5, and 22) for H = 0.5. Results in panel (B) show that the imputed scale exponents are significantly different from the E(H) = γ_1 for n = 1 and k = 5. That is, divergence from E(H) = γ_1 is most significant over short time intervals. This result is again consistent with results depicted in Figure 1 for all currency pairs except the GBP/USD.

Tests of the significance of the imputed scale exponents for the DMK/USD, SWF/USD and JYP/USD currency pairs indicate that none of the series conform to a Gaussian random walk. However the rejection of the null is not sufficient to prove that the series exhibits long-memory. Acceptance of a long-memory alternate hypothesis must also adjust for possible (G)ARCH dependence in the conditional volatilities. Using the classical rescaled range technique, Peters (1994) finds that daily JPY/USD and DMK/USD returns scaled at H = 0.642 and H = 0.624 respectively. While these earlier findings are shown to be statistically significant, imputed values of the scales exponent in Table 6 are significantly lower than those reported by Peters (1994). Overall, lower values of the scale exponent for the GBP/USD at all interval lengths provide support for acceptance of the null hypothesis for the GBP/USD exchange rate.

(insert Table 8 about here)

Evidence of the economic significance of the scale exponent values is provided in Table 8. Using a variation of the Black-Scholes Option Pricing Model for foreign currency options[3], annualised standard deviation estimates for intervals of n = 1, 5, and 22 lags are used to price in-the-money, at-the-money and out-of-the-money call and put options for each of the four currency pairs. Annualised standard deviations are estimated using Equation (8). Under the assumption that each of the four currency pairs conforms to a Gaussian random walk, the value of the scale exponent in Equation (8) is H = 0.5. For the calculation of the Black-Scholes model, spot exchange rates for each currency are their actual value as at 31 December 1998. In-the-money and out-of-the-money exchange rates are set at -/+ 20.0% of the spot exchange rate. For this example the domestic (US) and foreign, risk-free interest rate is arbitrarily set at 8.0% and 10.0% each, and the time to maturity is one-half year (180 days based on a 360-day year). Prices in the table are expressed as 100 times the calculated price. Results for each currency pair using rescaled standard deviations for interval lengths n = 1, 5, and 22 are then reported in Table 8. Results using observed annual standard deviations (k

3 Option prices are for European calls and puts. The model employed is analogous to the Black-Scholes continuous dividend option pricing model where the dividend yield (q) is replaced by the foreign risk-free interest rate.

= 252) are also presented. Under the null hypothesis H_0, option values derived from the rescaled standard deviations should not be different from those using the observed annual standard deviations.

Except for options written on the GBP/USD, option values derived from the rescaled standard deviations consistently underestimated their real value, based on the observed annual standard deviation (reported as n = 252 last column in Table 8). Values for out-of-the-money call contracts are underestimated by a maximum of 84.0% (JPY/USD n = 5 call price of 0.128) to a minimum of 64.7% (SWF/USD n = 22 call price of 0.031). For out-of-the-money puts, it is a maximum of 88.1% (JPY/USD n = 5 put price of 0.044). At-the-money call contract values are underestimated by a maximum of 25.2% (JPY/USD n = 5 call price of 18.234) and a minimum of 16.7% (SWF/USD n = 22 call price of 1.961). Equivalent put option values are undervalued by a maximum of 19.7% (JPY/USD n = 5 put price of 25.181) and a minimum of 13.1% (SWF/USD n = 22 put price of 2.609). Finally, prices for in-the-money calls and puts derived from the rescaled standard deviations are undervalued by less than 1.0%. The result for in-the-money options is consistent with the option value being determined mostly by its intrinsic value rather than the level of volatility, with the reverse being the case for out-of-the money options. The results are also consistent with lower (higher) option vegas for in-the-money (out-of-the-money) positions. Across all contracts and for all currency pairs, option values derived from rescaled daily standard deviations are more highly undervalued than those based on rescaled monthly standard deviations. Considering the observed behaviour of k interval standard deviations in Figure 1, this result is not unexpected.

The Mean Percent Forecast Error (MPE) for the four currency pairs is given in Table 9. Calculated as the implied option value (based on linear rescaling) less the option value using observed annual standard deviations, the MPE represents the error due to linear rescaling in calculating option values. The statistical significance of the forecast error is also given in the table. The significance is estimated using the two-tailed t-test for differences between two means where, following the null hypothesis that H = 0.5 is the appropriate value of the scale exponent, the expected value of the MPE is zero. Negative values of the error term in the table show that scaling risk by $\sqrt{T}$ underestimated the value of the option. However despite the observance of significantly larger forecast errors for out-of-the-money put and call options, the t-statistics for the MPEs of these options are not significant. MPEs for in-the-money call options and at-the-money calls and puts are only significant at the 0.10 level.

The results may be largely explained by the large standard error of the MPE, that is due to the forecast errors for the GBP/USD being overall close to zero. Excluding the GBP/USD result from the analysis, MPEs for the three remaining currency pairs are all highly significant.

(insert Table 9 about here)

Given the well publicised tendency for the simple Black-Scholes model to underprice out-of-the-money options (Black, 1975; Gultekin, Rogalski and Tinic, 1982) some degree of underpricing should be expected, even for series which scale according to the Gaussian null, H = 0.5. However the high level of bias shown in the results described above is greater than reasonably expected, given the contract maturity of 180 days. For instance, using maturities of 1 to 9 weeks Gultekin, Rogalski and Tinic (1982) demonstrate that the Black-Scholes model underestimates the value of out-of-the-money call options by approximately 1.6% to 4.8% respectively. Based on these results the equivalent expected level of underpricing given a 26-week (180-day) maturity is be calculated to be less than 20%.

Dependence in the conditional variance of the distributions of the currency returns may provide an alternate hypothesis over dependence in the mean, in explaining rejection of the Gaussian null. Engle (1982) proposes that conditional variances using an ARCH specification can be modelled as a linear function of past squared errors. Extending Engle's original model, the GARCH specification of Bollerslev (1986) allows the conditional variance of past errors to follow an ARMA process. Time-series with conditionally heteroskedastic variances will not scale according to the linear rule when the series increments are measured over short time intervals, yet will revert to the linear rule as the returns interval lengthens. As such linear rescaling of variance may either underestimate, or overestimate, individual conditional volatilities, yet should be expected to yield correct estimates of the volatility of returns on average (see Drost and Nijman, 1993; Bollerslev, 1986). Bias in the Black-Scholes model due to changes in asset volatility over time were first suggested in preliminary test results by Gultekin et al. (1982).

Dependence in the conditional second moment is one characteristic of time-series with conditional heteroskedasticity. Test results for conditional heteroskedasticity in the four currency series are summarised in Table 10. Measured using the Lagrange Multiplier test of Engle (1982) and the asymptotic t-test for autocorrelation in large samples, the results show highly significant evidence of at least ARCH type disturbances in short interval returns, yet decreasing levels of significance at longer return intervals. The results in the table are consistent with the aforementioned tendency of simple models of conditional heteroskedasticity to revert to Gaussian models over longer return intervals.

(insert Table 10 about here)

Duan (1995) discusses some implications of conditional heteroskedasticity for option pricing, stating that employment of the Black-Scholes model when the underlying asset returns process is a GARCH process is equivalent to substituting the stationary variance of the GARCH model into the Black-Scholes formula. Developing a GARCH option pricing model, Duan shows that the Black-Scholes model will generally underprice both in- and out-of the money options. The level of underpricing is highest for options with a short time to maturity and high initial conditional volatility, yet declines to less than 0.5% for a maturity of 180 days.[4] Using figures provided by Duan, the levels of underpricing of out-of-the-money call options by the Black-Scholes model observed in this study would imply an initial conditional volatility approximately 340 to 380 percent above the stationary level for the currency returns series.[5] However empirical test results of the level of conditional volatility in the currency returns series easily fail to support this proposition.

6. Summary and Conclusions

The correct estimation of financial asset risk has important implications for investors using standard asset pricing models. Under the usual assumptions of independent and Gaussian distributed increments, traditional methods of estimating risk require an annualised risk coefficient which is

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4 Percentage bias in the Black-Scholes model is estimated by Duan for European call options with an exercise price of $1 and assets prices in the range (X-20%)<S<(X+20%). Maturities range from 30 to 180 days and the risk-free rate of interest is set at 0%. initial conditional volatilities are 20% above to 20% below the stationary level.

5 Following from Duan (1995) the exercise price is set to $1 and asset price $0.80. To adjust for biases due to differential interest rates, the foreign and domestic interest rates are set equal to one another. Results are based on a 180 day maturity and use implied versus observed annual volatilities as per Table 8.

calculated by linear rescaling of the variance from shorter time intervals. Only when the returns series under observation are independent will rescaling provide correct estimates of the underlying level of risk associated with an investment. Dependence between increments in the returns series will instead lead the investor to underestimate or overestimate their exposure to risk. The higher the underlying level of dependence, the greater the possibility of error in the estimation of risk.

This study examines long-term returns for four spot foreign currencies (DMK/USD, GBP/USD, SWF/USD and JPY/USD) from January 1985 to December 1998. The key objective of this study is to determine the statistical and economic implications for investors of rescaling financial asset risk. Using standard statistical tests, the distribution of returns for each of the four currency series is shown to be non-normal, though no significant evidence of dependence between series increments for the four currencies is found.

Estimating scaling relationships between the volatility of returns at different time intervals produces evidence of dependence not found using traditional statistical techniques, with three of the four series tested having scale exponents (H) greater than 0.5 for all return intervals (H = 0.5 being the expected exponent value for independent increments). Imputed scale exponents between the intervals k = 1, 5, 22 and 252 are not similar, implying that the distribution of currency returns is not stable across intervals of different lengths. While the exponent values of the currency return series are not significantly large enough to conclude in favour of statistical long-term dependence, the economic implication of the exponent values *are* significant. Using a simple Black-Scholes foreign currency option-pricing model, linearly rescaled volatility estimates are shown to misprice the option value by as much as 84.0% for out-of-the-money contracts. These results are significant since they demonstrate that small deviations from independence in asset returns can result in significant economic benefits or costs. Investors should therefore exercise caution when using short-term returns to estimate longer-term risk, so as to avoid underestimating their risk exposure.

In our discussion of the possible sources of option under-pricing both conditional heteroskedasticity and underpricing bias by the Black-Scholes model have been considered. While statistically significant evidence of dependence in the conditional variance of short interval returns was found in all the currency series, this alone was unable to account for the very high levels of underpricing reported. General underpricing bias in the Black-Scholes model similarly implied a significantly lower level of underpricing than was found in the currency option prices in this study. In conclusion, these results highlight the complex behaviour of financial asset returns and the inability of general pricing models to completely and accurately describe this behaviour.

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Table 1
Descriptive Statistics of the Spot Foreign Currency Interday Returns [a]

Currency N = 3516	DMK/USD	SWF/USD	JPY/USD	GBP/USD
Mean	-1.80E-04	-1.80E04	-2.10E-04	-1.22E-04
Standard deviation	7.08E-03	7.72E-03	6.86E-03	6.92E-03
Skewness	-9.60E-02	-2.10E-01	-4.10E-01	1.70E-03
Excess kurtosis	2.340	1.866	3.833	5.094
Sum of ln returns	-0.651	-0.653	-0.816	-0.378
Spot Price at start of series 1 USD =	2.3824 DMK	2.6840 SWF	255.60 JPY	0.8873 GBP
Spot Price at end of series 1 USD =	1.6670 DMK	1.3740 SWF	113.30 JPY	0.6045 GBP
Expected Future Spot Price based on interest rate parity	1.2068 DMK	1.1887 SWF	129.42 JPY	0.8085 GBP
Nominal return from short dollar strategy	+42.9%	+95.3%	+125.6%	+46.7%
Anderson-Darling	20.344 p = 0.000	14.623 p = 0.000	30.064 p = 0.000	35.661 p = 0.000
Jacque-Bera	763.84 p = 0.000	507.30 p = 0.000	2129.66 p = 0.000	3597.05 p = 0.000
Shapiro-Wilk	0.986 p<0.01	0.990 p<0.01	0.978 p<0.01	0.972 p<0.01

[a] The table presents the four moments of the natural logarithm of interday returns for the four currency pairs (DMK/USD, SWF/USD, JPY/USD and GBP/USD) from 2 January 1985 to 31 December 1998 (n = 3516). The sum of the logarithmic returns is also determined and the nominal return during the time period from holding the currency is then calculated. The non-Gaussian nature of the series is highlighted by the 3 normality tests: the Anderson-Darling test (an empirical cumulative distribution function); Jacque-Bera test (a chi-square) and Shapiro-Wilk test (a correlation). The various tests p-values indicate that, at α levels at least greater than 0.01, there is evidence that all the time-series do not follow a normal distribution; a consequence of the leptokurtic nature of the distributions.

Table 2
Correlations of the Spot Foreign Currency Interday Returns [a]

	DMK/USD	SWF/USD	JPY/USD
SWF/USD	0.916		
p-value	0.000		
JPY/USD	0.608	0.612	
p-value	0.000	0.000	
GBP/USD	0.731	0.698	0.472
p-value	0.000	0.000	0.000

[a] The table presents the Pearson product moment correlations of interday returns between each of the four currency pairs (DMK/USD, SWF/USD, JPY/USD and GBP/USD) 2 January 1985 to 31 December 1998 (n = 3516). The table also notes the p-values (two tailed test) for the individual hypothesis of the correlations being zero below each of the correlations. Since all the p-values are smaller than 0.001, there is sufficient evidence at $\alpha = 0.001$ that the correlations are not zero, in part reflecting the large sample size.

Table 3
Autocorrelations of the Spot Foreign Currency Interday Returns [a]

Lag k	DMK/USD	SWF/USD	JPY/USD	GBP/USD
Lag 1	0.03	0.03	0.02	0.05
t-statistic	1.47	1.69	1.21	3.01
LBQ	2.17	2.86	1.46	9.05
p-value	0.141	0.091	0.227	0.003
Lag 2	-0.02	0.00	0.02	0.01
t-statistic	-1.26	-0.26	1.23	0.82
LBQ	3.76	2.93	2.97	9.73
p-value	0.153	0.231	0.227	0.008
Lag 5	0.03	0.02	-0.01	0.02
t-statistic	1.61	1.17	-0.54	1.39
LBQ	7.87	4.75	5.14	19.60
p-value	0.164	0.447	0.399	0.002
Lag 22	0.00	-0.02	0.01	0.03
t-statistic	0.23	-0.87	0.71	1.78
LBQ	30.04	32.85	32.58	33.73
p-value	0.118	0.064	0.068	0.052
Lag 252	-0.016	-0.004	0.003	0.001
t-statistic	-0.90	-0.22	0.18	0.02
LBQ	273.71	258.22	282.61	289.71
p-value	0.166	0.380	0.090	0.051

[a] The table presents the autocorrelations of interday returns for each of the four currency pairs (DMK/USD, SWF/USD, JPY/USD and GBP/USD) from 2 January 1985 to 31 December 1998 (n = 3516) for various time lags. The various time series are tested for autocorrelation up to 252 lags with the lags k = 1,2,5,22, and 252 presented in the table. The table also notes the t-statistic associated with each autocorrelation and the Ljung-Box Q (LBQ) statistic with its corresponding p-value. The LBQ statistic may be used to test the null hypothesis that the autocorrelations for all lags up to lag k equal zero.

Table 4
Stationarity Tests of the Spot Foreign Currency Interday Returns [a]

	DMK/USD	SWF/USD	JPY/USD	GBP/USD
DF (0)	-56.592	-56.094	-56.906	-55.056
ADF(9)	-17.875	-17.837	-17.044	-17.942
ADF(28)	-10.305	-10.030	-10.077	-10.368

[a] The table presents the tests for mean stationarity of the interday returns for each of the four currency pairs (DMK/USD, SWF/USD, JPY/USD and GBP/USD) from 2 January 1985 to 31 December 1998 (n = 3516). The table shows the results from the Augmented Dickey-Fuller (ADF) test at lags of 9 and 28, and for the Dickey-Fuller (DF) test at lag = 0. Lag lengths are selected according to the Schwert (1989) model. The significantly high negative values for each currency indicated that all the returns series are stationary over the sample period. Critical values for the ADF and DF tests at the 1% and 5% levels are –3.96 and –3.41 respectively.

Table 5
Regression Scale Exponents for k Interval Standard Deviations, k = 1 to 252 [a]

	DMK/USD	SWF/USD	JPY/USD	GBP/USD
Regression scale exponent, γ_1	0.541	0.538	0.559	0.481
Standard error	(0.001)	(0.001)	(0.001)	(0.003)

[a] The table presents the tests of the regression scale exponent, γ_1 for the regression $\log(\sigma_k)=\gamma_0+\gamma_1\log(k)$. Standard deviations are measured for returns at intervals of k = 1 to 252 days. Under the null hypothesis H_0 the value of the regression scale exponent should be $\gamma_1 = 0.5$. Correlation between the dependent variables σ_k prevents testing the statistical significance of the γ_1 estimates using usual (parametric) means. The economic significance of the results in the table are presented using a variation of the Black-Scholes Option pricing Model for currency options.

Table 6
Imputed Scale Exponent (H) for Estimation of k Interval Standard Deviation from Standard Deviation of n Interval Returns [a]

Currency	Interval Length k =	Observed Standard Deviation	Imputed Scale Exponent (H)		
			n = 1	n = 5	n = 22
DMK/USD	1	0.007			
	5	0.015	0.495		
	22	0.032	0.507	0.521	
	252	0.133	0.534	0.550	0.567
SWF/USD	1	0.008			
	5	0.017	0.502		
	22	0.036	0.511	0.521	
	252	0.146	0.535	0.549	0.565
JPY/USD	1	0.007			
	5	0.015	0.499		
	22	0.034	0.521	0.546	
	252	0.142	0.546	0.565	0.576
GBP/USD	1	0.007			
	5	0.015	0.509		
	22	0.032	0.520	0.532	
	252	0.104	0.499	0.495	0.473

[a] This table shows imputed values for the scale exponent H. Using observed values of the standard deviation of k and n interval returns, the imputed scale exponent is the value of H for which rescaled values of the standard deviation of n interval returns exactly equal the standard deviation of k interval returns. Under the null hypothesis H_0, the value of the scale exponent should be H = 0.5. Rejection of the null hypothesis will imply that the currency series tested does not conform to a Gaussian random walk over the sample period. For example, the imputed value of H for estimating the annual return (k = 252) standard deviation from the standard deviation of daily returns (n = 1) for the JPY/USD is H = 0.546.

Table 7

Statistical Significance of Imputed Scale Exponent (H) [a]

(A) Significance versus E(H) = 0.5

Currency N = 3516	Interval Length k =	Significance of Imputed Scale Exponent (H) n = 1	n = 5	n = 22
DMK/USD	5	0.277		
	22	0.417	1.196	
	252	1.961††	2.880††	3.853††
SWF/USD	5	0.096		
	22	0.623	1.213	
	252	2.024††	2.816††	3.742††
JPY/USD	5	0.030		
	22	1.224	2.632††	
	252	2.638††	3.733††	4.370††
GBP/USD	5	0.512		
	22	1.133	1.830	
	252	0.065	0.302	1.534

(B) Significance versus E(H) = g_1

Currency	Interval Length k =	Significance of Imputed Scale Exponent (H) n = 1	n = 5	n = 22
DMK/USD	5	2.615††		
	22	1.920	1.142	
	252	0.376	0.543	1.516
SWF/USD	5	2.119††		
	22	1.593	1.002	
	252	0.191	0.601	1.526
JPY/USD	5	3.409††		
	22	2.154††	0.747	
	252	0.740	0.355	0.991
GBP/USD	5	1.626		
	22	2.247††	2.944††	
	252	1.048	0.812	0.420

†† indicates significant at the 0.05 level

[a] This table shows the statistical significance of the imputed values for the scale exponent H provided in Table 5. The significance value (S) is interpreted as the number of deviations that the imputed scale exponent is from its expected value. Under the null hypothesis H_0, the expected value of the scale exponent is E(H) = 0.5. Under the alternate hypothesis H_1, the value of the scale exponent should be E(H) = γ_1 in the regression $\log(\sigma_k)=\gamma_0+\gamma_1\log(k)$. Values of the significance estimate greater than S = 1.96 indicate that the imputed scale exponent is significantly different from its expected value (0.05

level). Significance values in panel (A) are estimated using the expected scale exponent value E(H) = 0.5. Those in panel (B) are estimated using the regression $\log(\sigma_k)=\gamma_0+\gamma_1\log(k)$.

Table 8
Foreign Currency Option Values Calculated Using Implied Annual Standard Deviations [a]

(A) In-the-money

Currency	Exchange Rate	Option Exercise	1[a]	5[b]	22[c]	252[d]
			Call			
DMK/USD	0.5997	0.4798	10.9541	10.9538	10.9550	10.9697
SWF/USD	0.7262	0.5809	13.2698	13.2701	13.2732	13.3048
JPY/USD	8.8652	7.0922	161.9322	161.9318	161.9796	162.3326
GBP/USD	1.6637	1.3310	30.3847	30.3857	30.3899	30.3844
			Put			
DMK/USD	0.5997	0.7196	12.1078	12.1070	12.1099	12.1412
SWF/USD	0.7262	0.8714	14.6749	14.6754	14.6821	14.7434
JPY/USD	8.8652	10.6382	178.9843	178.9831	179.0954	179.8036
GBP/USD	1.6637	1.9964	33.5783	33.5809	33.5915	33.5773

(B) At-the-money

Currency	Exchange Rate	Option Exercise	1[a]	5[b]	22[c]	252[d]
			Call			
DMK/USD	0.5997	0.5997	1.5082	1.4938	1.5467	1.8741
SWF/USD	0.7262	0.7262	2.0204	2.0263	2.0982	2.5203
JPY/USD	8.8652	8.8652	22.2716	22.2479	24.0257	29.7944
GBP/USD	1.6637	1.6637	3.9307	3.9974	4.2196	3.9041
			Put			
DMK/USD	0.5997	0.5997	2.0815	2.0671	2.1200	2.4474
SWF/USD	0.7262	0.7262	2.7147	2.7205	2.7924	3.2145
JPY/USD	8.8652	8.8652	30.7467	30.7230	32.5008	38.2696
GBP/USD	1.6637	1.6637	5.5212	5.5879	5.8101	5.4946

(C) Out-of-the-money

Currency	Exchange Rate	Option Exercise	1[a]	5[b]	22[c]	252[d]
			Call			
DMK/USD	0.5997	0.7196	0.0107	0.0100	0.0129	0.0442
SWF/USD	0.7262	0.8714	0.0262	0.0267	0.0334	0.0947
JPY/USD	8.8652	10.6382	0.1574	0.1562	0.2685	0.9766
GBP/USD	1.6637	1.9964	0.0185	0.0211	0.0317	0.0175
			Put			
DMK/USD	0.5997	0.4798	0.0037	0.0034	0.0046	0.0193
SWF/USD	0.7262	0.5809	0.0102	0.0105	0.0136	0.0452
JPY/USD	8.8652	7.0922	0.0547	0.0542	0.1020	0.4551
GBP/USD	1.6637	1.3310	0.0059	0.0069	0.0111	0.0056

[a] The economic significance of incorrect volatility scaling is demonstrated in this table using a variation of the Black-Scholes Option Pricing Model for foreign currency options. Here, annualised standard deviation estimates for intervals of n = 1, 5 and 22 and actual standard deviation for 252 lags are used to price in-the-money, at-the-money and out-of-the-money call and put options for each of the four currency pairs. Spot exchange rates for each currency are their actual value as at 31 December 1998. In-the-money and out-of-the-money exchange rates are set at -/+ 20.0% of the spot exchange rate. The domestic (US) and foreign risk-free interest rates are arbitrarily set at 8.0% and 10.0% each, and the time to maturity is one-half year (180 days based on a 360 day year). Prices are expressed as 100 times. The table provides four sets of call and put option prices: (a) option value using an implied annual standard deviation from the standard deviation of daily returns; (b) option value using an implied annual standard deviation from the standard deviation of weekly returns; (c) option value using an implied annual standard deviation from the standard deviation of monthly returns; (d) option value using observed standard deviation of annual returns.

Table 9
Mean Percent Forecast Error for Currency Option Prices using Implied versus Actual Annual Standard Deviations [a]

(A) In-the-money

	n = 1	n = 5	n = 22
		Call	
Mean Percent Forecast Error	-0.162%	-0.164	-0.142%
(standard error)	(0.001)	(0.001)	(0.001)
t-statistic	2.655†	2.661†	2.432†
		Put	
Mean Percent Forecast Error	-0.231%	-0.229%	-0.189%
(standard error)	(0.001)	(0.001)	(0.001)
t-statistic	2.417†	2.346	2.026

(B) At-the-money

	n = 1	n = 5	n = 22
		Call	
Mean Percent Forecast Error	-15.980%	-15.708%	-11.382%
(standard error)	(0.057)	(0.062)	(0.065)
t-statistic	2.8002†	2.5490†	1.7488
		Put	
Mean Percent Forecast Error	-12.425%	-12.231%	-8.965%
(standard error)	(0.044)	(0.048)	(0.049)
t-statistic	2.806†	2.576†	1.822

(C) Out -of-the-money

	n = 1	n = 5	n = 22
		Call	
Mean Percent Forecast Error	-51.403%	-47.132%	-24.722%
(standard error)	(0.203)	(0.247)	(0.366)
t-statistic	2.538†	1.909	0.676
		Put	
Mean Percent Forecast Error	-56.876%	-51.903%	-26.102%
(standard error)	(0.257)	(0.307)	(0.487)
t-statistic	2.210	1.689	0.536

† indicates significant at the 0.10 level

[a] The table presents the Mean percent Forecast Error (MPE), the standard error of the mean and its statistical significance. Panel (A) shows results for the implied in-the-money call and put values given in Table 8. The Percent Forecast Error is calculated as the implied option value using linear rescaling, less the value of the option as calculated using the observed annual standard deviation. The MPE is the mean error for all four currency series. Under the null hypothesis, the expected value of the MPE is zero. A negative value for the MPE indicates that the implied option values underestimated the real value of the option (using the observed annual standard deviation). The t-statistic shows the statistical significance of the MPE and is measured using the two-tailed t-test for the difference between two mean values. The

0.10 and 0.05 critical values for the t-statistic are 2.35 and 3.18 respectively. Values of the t-statistic in the table in excess of the critical values indicate that the MPE is significantly different from zero.

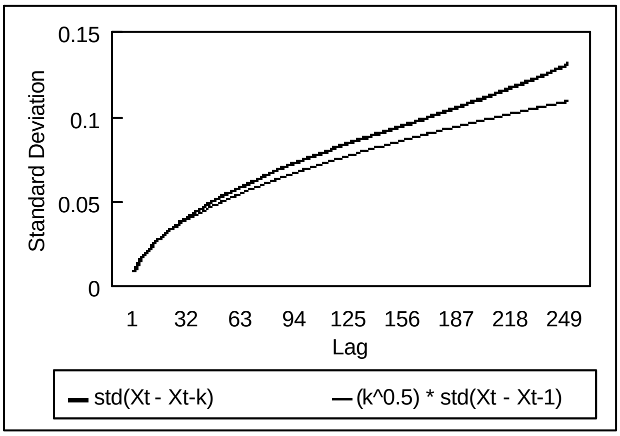
Table 10
Heteroskedasticity Tests of the Spot Foreign Currency Interday Returns [a]

Lag k	DMK/USD	SWF/USD	JPY/USD	GBP/USD
Lag 1				
LM	75.150††	54.713††	17.205††	31.956††
asymptotic t-statistic	8.669††	7.397††	4.148††	5.653††
Lag 2				
LM	156.388††	124.461††	109.920††	164.771††
asymptotic t-statistic	12.506††	11.156††	10.484††	12.836††
Lag 5				
LM	82.051††	87.615††	50.390††	71.573††
asymptotic t-statistic	9.058††	9.360††	7.099††	8.460††
Lag 22				
LM	31.729††	25.339††	7.961††	20.969††
asymptotic t-statistic	5.633††	5.034††	2.822††	4.579††
Lag 252				
LM	0.037	-	0.165	0.316
asymptotic t-statistic	0.192	-	0.406	0.562

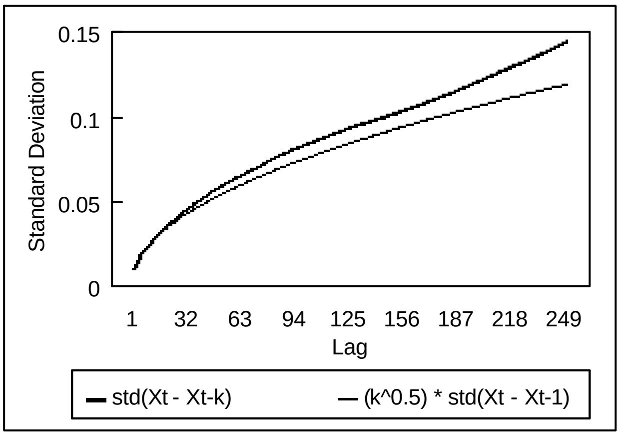
††indicates significant at the 0.05 level

[a] The table presents the tests for variance nonstationarity in interday returns for each of the four currency pairs (DMK/USD, SWF/USD, JPY/USD and GBP/USD) 2 January 1985 to 31 December 1998 for various time lags. The various time series are tested for ARCH(1) disturbances up to 252 lags with the lags k = 1,2,5,22, and 252 presented in the table. Two test statistics are employed; the Lagrange Multiplier (LM) test and the asymptotic t-statistic. The LM procedure tests for the explanatory power of the ARCH regression model and has an asymptotic χ^2 distribution. The asymptotic t-statistic is a large sample test for autocorrelation with an asymptotic standard normal distribution. Applied to the regressors in the ARCH model, a significant value for the asymptotic t-statistic implies that the ARCH regression model should have high explanatory power. Critical values for the LM test and asymptotic t-statistic at 5% level are 3.84 (upper tail) and 1.96 (two tail) respectively. Results in the table show significant evidence of ARCH disturbances in the short lag returns (k = 1,2,5,22) of all four currency pairs. The statistical significance of the both test statistics declines below the critical values at higher lag lengths.

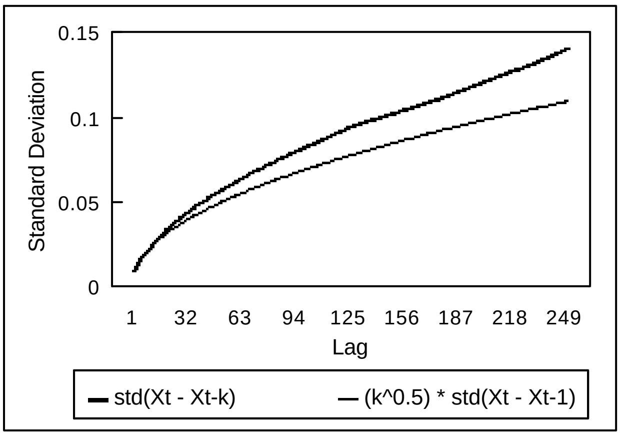
Figure 1
Observed Standard Deviation of lag k Returns versus Standard Deviation of 1-Day Returns Rescaled by $\sqrt{k}$ [a]



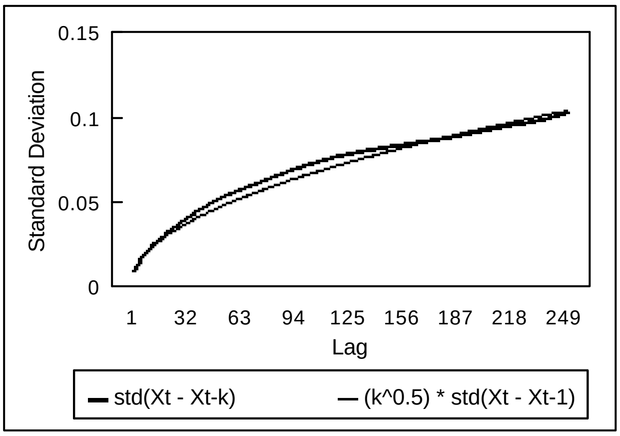
DMK/USD



SWF/USD



JPY/USD



GBP/USD

[a] The figure shows the relationship between the standard deviation of returns and the return interval (lag) k for the four currency series. The heavy line $(std(X_t - X_{t-k}))$ is the actual k interval standard deviation, the light line $((k^{0.5})*(std(X_t - X_{t-k}))$ shows the expected value of the k interval standard deviations, when risk scales according to the square root of time. The figure shows that all series except the GBP/USD scale at a faster rate than the square root of time. The estimated regression scale exponent (γ_1) is given in Table 5 for each series.